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# **Size effect of concrete in compression**

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**Size effect of concrete in compression**

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## 1 Introduction

Size effect is known as a relative change (decrease) of the structural properties (peak resistance, ductility, etc.) when the structure size increases. In quasi brittle materials such as concrete, this is a recognized phenomenon.

For the concrete material most focus on size effect has been connected to the tensile states of stress. After the development of the Fictitious Crack Model (Hilleborg et al 1976), there have been large research activities within the field of fracture mechanics applied for concrete, particular for tensile states of stress. Based on this work it is now generally accepted that tensile failure is localized to a limited zone and that this failure localization is the source of the so-called size effect. In the new Eurocode 2 this size effect is implemented for punching and shear using a size factor  $k = 1 + \sqrt{200/d}$ , where  $d$  is the member depth in mm.

Compressive failure is, as in tensile failure, found to be localized to certain zones and gives rise to a size effect (Hillerborg (1988), Bažant (1989), Markeset (1993, 1994), Markeset and Hillerborg (1995), Bigaj and Walraven (1993), Janson and Shah (1997), Walraven (2007), van Mier (2007)).

The compressive behaviour of concrete is one of the fundamental parameters of structural design as most load-bearing concrete elements, such as beams, columns and slabs, experience compressive strain gradients where the compressive strain at the critical section is in the post-peak (softening regime) of the stress-strain curve at failure. The presently used codes of practice do not limit their application field to some selected range of member dimensions although experimental studies have shown that size effect of concrete loaded in compression exists.

In this report the size effect of compressive failure of concrete members exposed to uniaxial compression as well as compressive strain gradients is discussed.

## 2 Stress-strain curve in compression and damage localization

From the work of RILEM TC 148SSC (van Mier et al (1997)), it was found that the specimen strength increased with decreasing slenderness when using rigid steel loading platens. In contrast, when friction-reducing measures are taken, the specimen strength becomes independent of the slenderness ratio. Moreover, the pre-peak stress strain behaviour was found to be independent on the specimen slenderness when low-friction loading platens were used. This specimen-machine interaction is illustrated in figure 2.1.

In the softening regime (post-peak), however, an increase of ductility (in terms of stress and strain) with increasing specimen slenderness is observed (van Mier (1984), Rokugo and Koyanagi (1992), Vonk (1993), Markeset (1994), van Vliet and van Mier (1996), Janson and Shah (1997)) and the deformations after peak load is found to be localized to certain zones. The length of the damage zone has been estimated to be of the order 2- 3 times the width of the specimen at the end of the test.



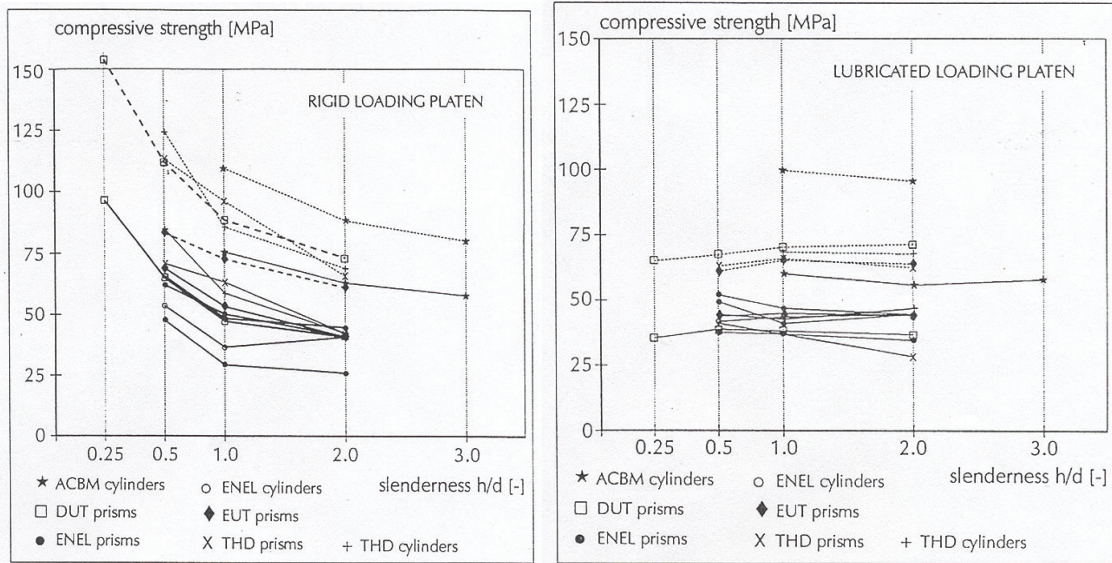


Fig. 2.1: Compressive strength using rigid and lubricated loading platen, respectively (van Mier et al (1997))

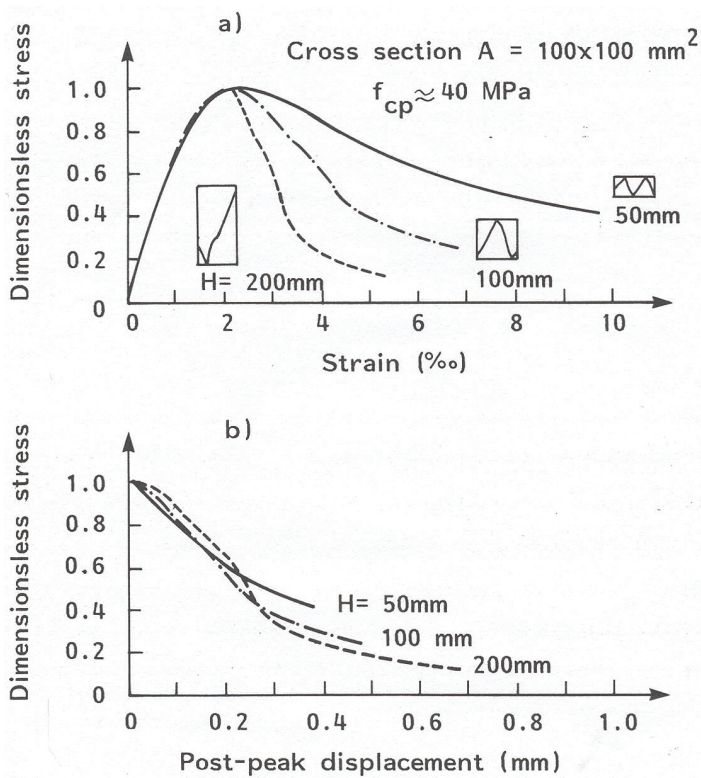


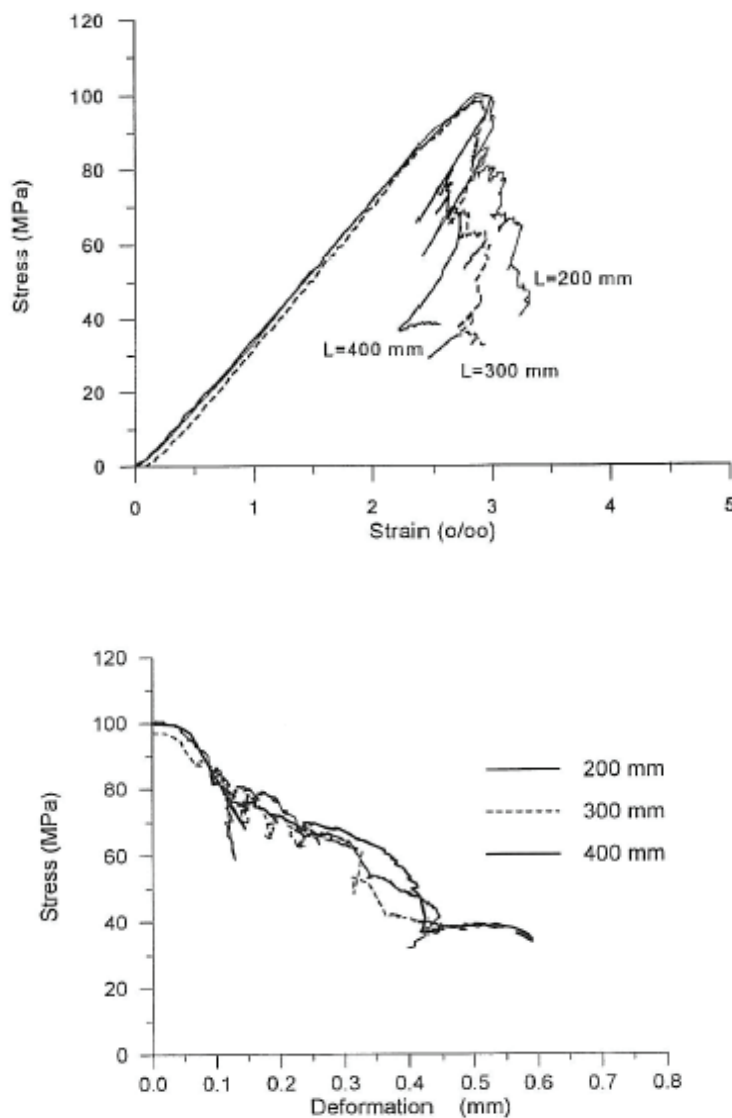
Fig. 2.2: Influence of specimen length on the stress-strain curve obtained on normal strength concrete after van Mier (1984). a) Stress-strain relationship, b) Post-peak stress-deformation relationship





The stress-strain response after peak load becomes thus dependent on specimen size and slenderness and the complete stress-strain curve obtained on centrally loaded specimen is thus not a pure material property.

The influence of the specimen length on the stress-strain curve is illustrated in figures 2.2 and 2.3. As seen, the steepness of the descending branch increases with increasing specimen length for both normal strength and high strength concrete. Due to the damage localization the post-peak behaviour should be described by a stress-deformation relationship.



*Fig. 2.3: Influence of specimen length on the stress-strain curve obtained on high strength concrete after Markeset (1994). Stress-strain relationship and post-peak stress-deformation relationship*



### 3 Compressive failure of concrete members and size effect

#### 3.1 Centric loading

Several experimental studies of axially loaded reinforced concrete columns, e.g. Bažant and Kwon (1994), Şener et al (1996), Şener et al (2004) have confirmed that the failure loads exhibit a size effect, i.e. the nominal stress at maximum load (failure load divided by cross-section area) decreases as the specimen size increases. The observed size effect may be illustrated by means of the size-effect law by Bažant and Planas (1988), see figure 3.1.

As is well known, the compression failure of concrete columns without strong lateral confining reinforcement is very brittle. In Caner and Bažant (2002) it was found that softening in the load-deflection diagram of spiral columns can be fully suppressed only if the reinforcement ratio exceeded 14 %. If mild softening is allowed, the reinforcement ratio must still exceed 8 %. They concluded that: *“If the steel ratios used in designing spiral columns are not increased, one needs to pay attention to the localization of softening damage, accept the size effect engendered by it, and ensure safety margins appropriate for protection against sudden explosive brittle collapse. This is of particular concern for the design of very large columns”*.

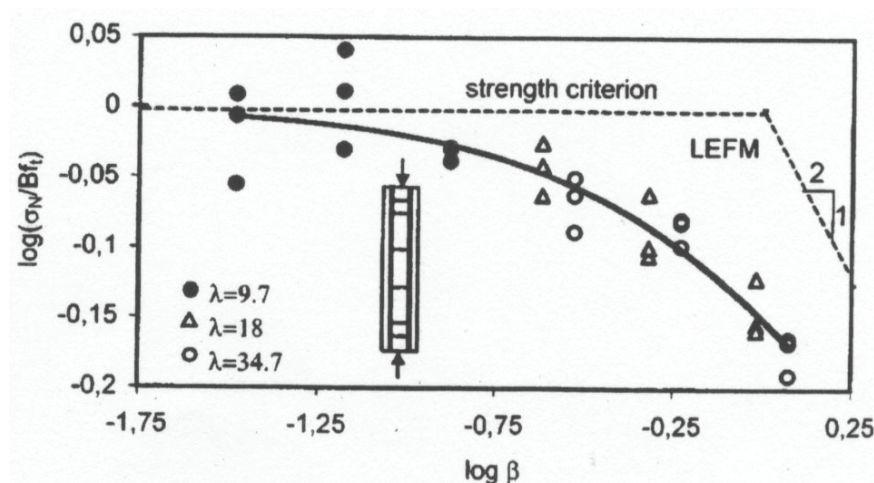


Fig. 3.1: Size effect in reinforced columns with different slenderness illustrated by means of the so-called “Size-effect law” (Şener et al (2004))

#### 3.2 Eccentric loading

Behaviour of eccentrically loaded plain and reinforced columns of different sizes has been experimentally and numerically investigated by Meyer (1997), Ožbolt and Li (1998). The experimental study was carried out for different concrete qualities and for different reinforcement ratios. For the constant columns thickness of  $b = 160$  mm and slenderness  $h/d = 2.5$  the width of the columns was varied from  $d = 160$  to  $480$  mm. Columns were loaded by deformation control of the end cross section. For the RC columns the concrete cover was kept constant. The numerical study of the same columns was performed by the use of the three-dimensional finite element code (MASA). Figure 3.2 shows the nominal strength as a function of the columns size obtained experimentally and numerically.



The reason for the relatively weak size effect observed in figure 3.2 was explained by unstable crack growth after reaching the nominal/ultimate strength. In the experimental investigation it was also found a moderate decrease in ductility with increasing member size as well as a decrease of ultimate strain (i.e. the strain in the most compressed fibre at maximum load) with increasing depth of the compressive zone.

The eccentricity of the load gives rise to a deformation gradient where the depth of the compressed area decreases with increasing eccentricity. Experimental and theoretical investigations have shown, e.g. Markeset (1996), Debernardi and Taliano (2001), that the compressive strain in the extreme fibre increases with increasing degree of eccentricity of the loading, i.e. decreasing depth of the compression zone. They both concluded that due to the strain gradient effect no unique stress strain relationship exists.

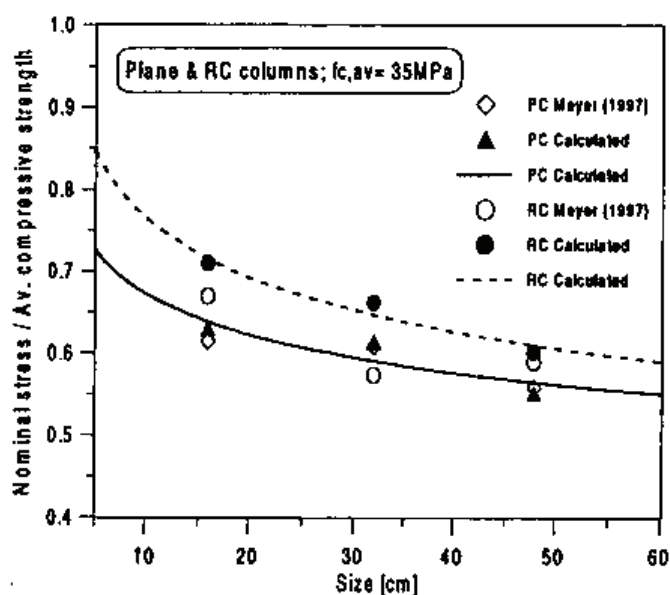


Fig. 3.2: Relative nominal strength as a function of size for eccentric loaded columns (Ožbolt and Elgehausen (1998))

### 3.3 Flexural loading

#### 3.3.1 Reinforced beams

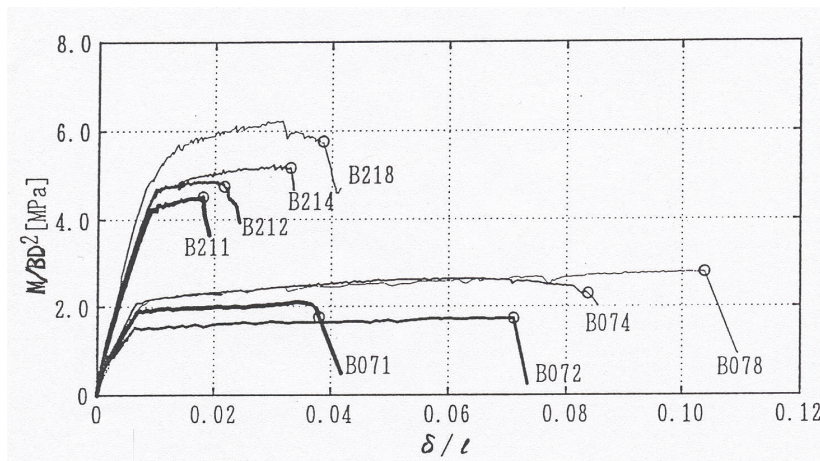
The size effect on strength and ductility of reinforced concrete beams failing in flexure was studied by Adachi et al (1995). Normalized moment displacement curves for the different beams tested are given in figure 3.3. From the figure it can be seen that there is a distinct size effect on the ultimate strength for the beams with the highest reinforcement ratio. For the specimen with the lowest reinforcement ratio a pronounced influence of size on the ductility is observed. Most likely, the size effect observed in ultimate strength is caused by a combination of strain localization in the concrete compression zone and the strain hardening in the tensile reinforcing steel.

The size effect on the rotational capacity was studied by Bigaj (1999). Figure 3.4 shows the relation between member size and its rotational capacity. The experimental curves for normal



strength concrete showed for a reinforcement of 0.28% a twofold increase of the rotational capacity as the effective height of the beam decreases from 450 to 180 mm, while for the reinforcement ratio of 1.12% a fourfold increase of the rotational capacity was observed with the effective height of the beam decreasing from 450 to 90 mm. The general trend of increasing value of rotational capacity with decreasing beam height is thus verified both for members which fail due to exceeding deformation capacity of the steel bars and the members in which crushing of concrete prevails after yielding of steel. The explanation of this phenomenon is likely to be found in the strain localization in the hinge region.

Name of specimen	Size of specimen BxDxL (mm)	Reinforcement ratio (o/o)	Yield strength/tensile strength
B07 1	300x600x5400	0.62	385/566
B21 1		1.64	361/570
B07 2	150x300x2700	0.52	364/503
B21 2		1.77	378/551
B07 4	75x150x1350	0.50	439/561
B21 4		1.51	439/561
B07 8	37.5x75x675	0.75	368/497
B21 8		2.23	348/485



*Fig. 3.3: Normalized bending moment–displacement curves (Adachi et al (1995)). The beam notations and dimensions, steel reinforcement ratio and yield strength/tensile strength of the tensile reinforcement are also given.*





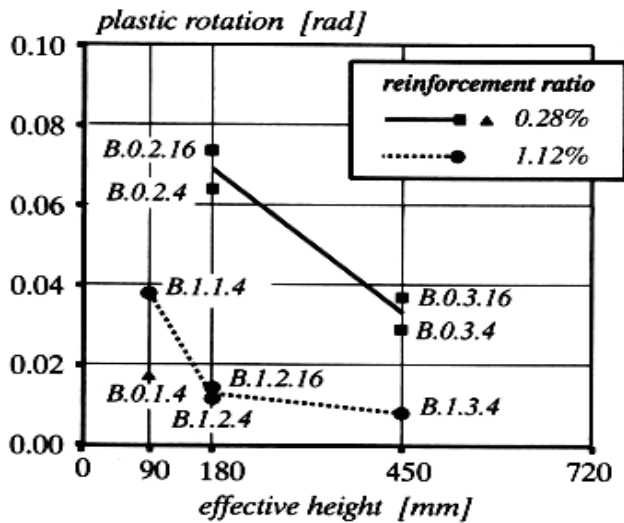


Fig 3.4: Plastic rotational at peak load versus effective beam height after Bigaj (1999)

Borges et al (2004) studied the flexural response of reinforced concrete beams under four-point loading and observed that after peak load was reached, damage began to localize in the uniform/constant moment region. The deformation (or strain) that was measured was strongly dependent on specimen length (in the constant moment area) as well as the length of the gage section used for measuring the response. The strain profile in the top fibre along the uniform moment zone length measure by nine 20 mm electrical resistance strain gauges glued to the beam surface is illustrated in figure 3.5. Figure 3.6 shows typical experimental moment-strain response for the beams with different lengths of uniform moment region. The moment is normalized with respect to the maximum moment and the strain is normalized with respect to the strain at maximum moment. The length effect on the post-peak ductility is obvious. While these test results are consistent with the expected length effect, it is to be recognized that only relatively small specimens have been tested (cross section 100 mm by 200 mm).

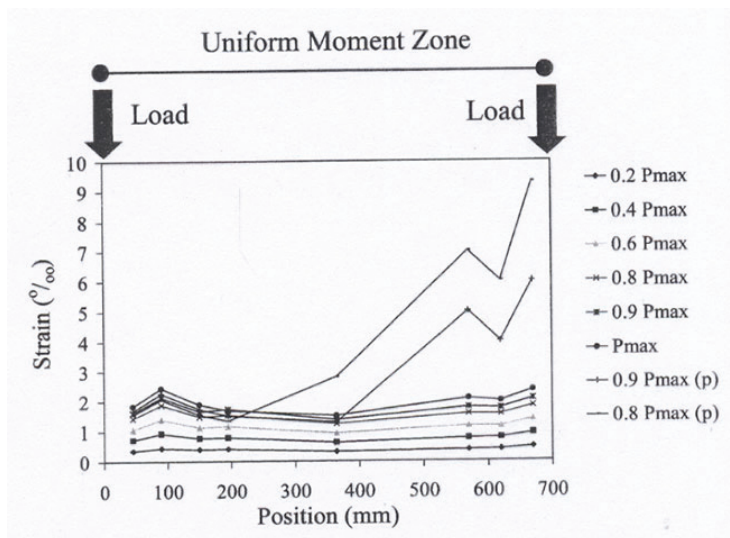


Fig. 3.5: Strain profile along top surface of longer beam (Borges et al (2004))



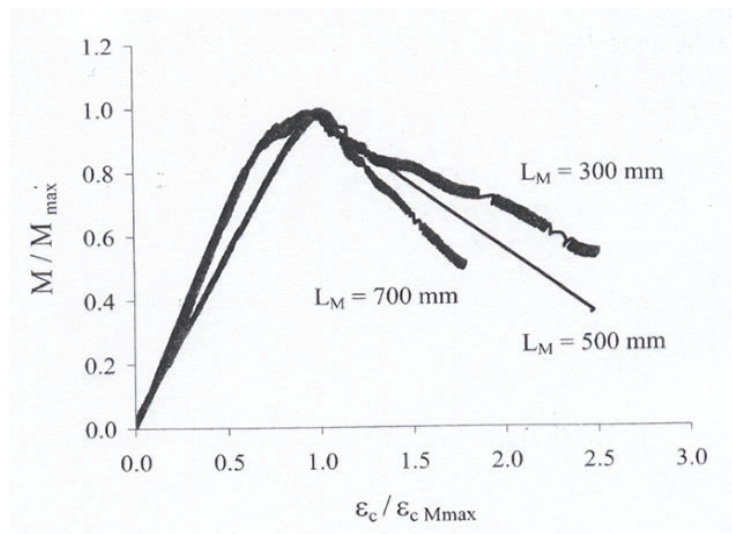


Fig. 3.6: Experimental moment-strain curves for beams with constant moment zone lengths of 300, 500 and 700 mm, respectively (Borges et al (2004))

### 3.3.2 C-shaped concrete specimens

Reinforced concrete beams subjected to flexural loads, the size, length and depth effect cannot be evaluated systematically due to change in the location of the neutral axis of the cross section as member sizes, reinforcement ratios, applied loading increments, loading point locations etc. vary. To solve this problem a series of experiments for C-shaped concrete specimens subjected to axial load and bending moment has been performed by Kim and Yi (2002). The shape of the specimen and test procedure is very similar to those by Hognestad et al (1955). In these tests the position (depth) of the neutral axis was kept fixed by continuously monitoring strains on the one surface of the C-shaped specimen and adjusting the eccentricity of the applied force so that the strain at the neutral surface remains zero. In figure 3.7 the relationship between  $\epsilon_{cu}/\epsilon_{co}$  (ultimate strain versus ultimate strain for specimen depth  $c = 20$  cm) is plotted versus the beam depth  $c/20$  cm. As seen in the figure, the ultimate compressive strain decreases as the specimen size increases.



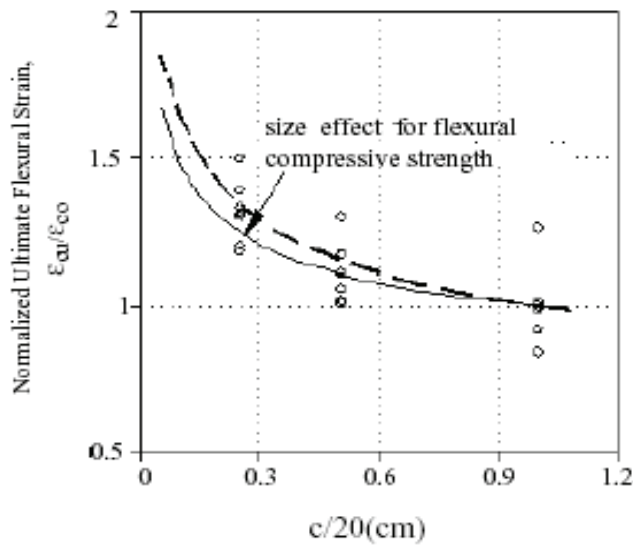


Fig. 3.7: Comparison of ultimate compressive strain with beam depth by Kim and Yi (2002 )

## 4 Fracture mechanical modelling

In chapter 2 and 3, size effect is described from an experimental point of view. In order to take the issue further and estimate effects on general structures a numerical model describing compressive behaviour which includes “size effect” is needed. For practical use such a model should be on a macro level description of the compressive behaviour.

It is increasingly being accepted that the mathematical modelling of such behaviour should be based on principals of fracture mechanics. Such material models involve energy quantity (called fracture energy) or an equivalent length quantity (characteristic lengths) and may predict the size dependent behaviour of geometrical similar specimens of different sizes.

The Compressive Damage Zone model (CDZ-model) by Markeset (1993) takes the localization of damage into account and describes the failure as a combination of axial splitting and shear sliding. The model is applicable for centric compression as well as when a strain gradient prevails (eccentric loading and bending). This CDZ-model has been further developed to include effects from stirrups and steel fibres as well as strain rates effects, (Han (1996), Meyer (1997), Bigaj (1999) and Schumacher (2006)). The CDZ-model is illustrated in figure 4.1.



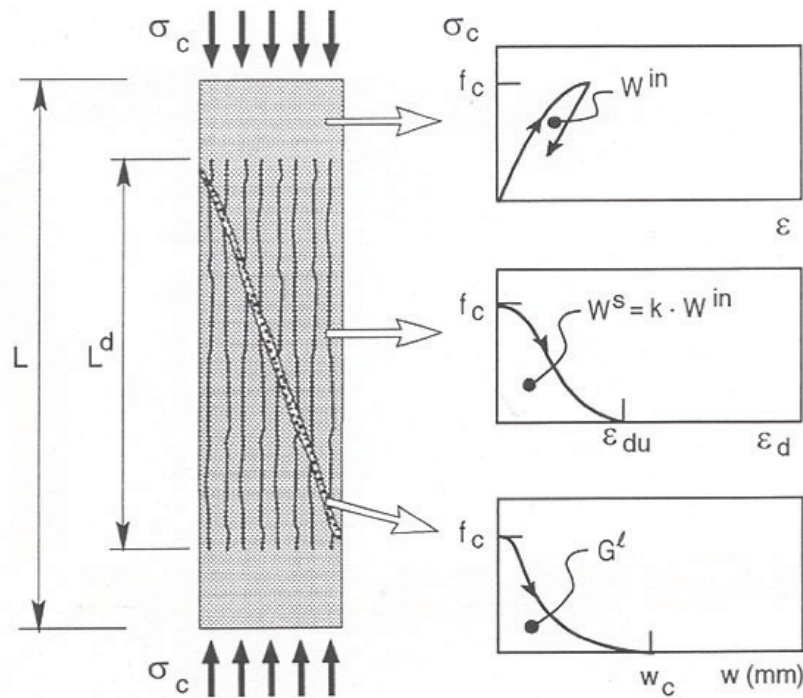


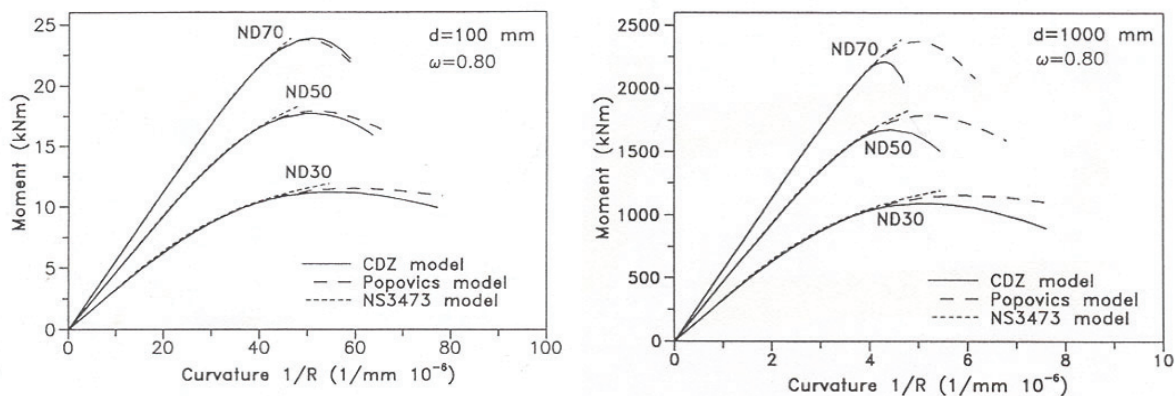
Fig. 4.1: Illustration of the CDZ model on a specimen loaded in uniaxial compression (Markeset (1993))

The CDZ model has been used to calculate the moment-curvature relationship of over-reinforced beams with effective beam depth 100 mm and 1000 mm, respectively. The results from the CDZ model is compared to that obtained using the stress-strain curve according to NS 3473 (2003) and Popovics (1973), see figure 4.2. As seen, for beam depth 100 mm there is no practical difference between the three models. However, for beam depth 1000 mm the CDZ-model predicts both lower capacity and ductility than the two other models. The stress-strain relationship given in NS 3473 is independent of the member size, and can therefore not predict the observed size effect.

The CDZ model in Figure 4.2 predicts a higher size effect for high strength concrete than for normal strength concrete due to the fact that the fracture energy of the concrete does not show the same rate of increase as the compressive strength (Markeset, 1993). The same conclusion has been drawn by Ožbolt et al (2000). They found that the ultimate compressive strength of reinforced-concrete beams of different concrete types that fail in compression was not proportional to the uniaxial concrete strength and that normal strength concrete was more efficient than high strength concrete.







*Fig. 4.2: Calculated moment-curvature relationships for effective beam depths 100 mm and 1000 mm*

## 5 Summary and conclusions

Size effect is known as a relative change (decrease) of the structural properties (peak resistance, ductility, etc.) when the structure size increases. By today, it is recognized that size effects will be encountered in all failures of concrete structures in which the failure is initiated within the concrete rather than in the reinforcement steel. This is not only valid for concrete failing in tension (direct tension, shear, pull out, bond, torsion) but also for concrete failing under compressive loading.

As discussed in chapter 2 the localization of damage (or strain) causes a size effect in the softening regime of the stress-strain curve in compression. The stress-strain response after peak load becomes dependent on specimen size and slenderness and the complete stress-strain curve obtained on centrally loaded specimen is thus not a pure material property.

The compressive behaviour of concrete is one of the fundamental parameters of structural design as most load-bearing concrete elements, such as beams, columns and slabs, experience compressive strain gradients where the compressive strain at the critical section is in the post-peak (softening regime) of the stress-strain curve at failure. The damage localization phenomenon gives rise to the so-called size effect in the structural response both with respect to strength and ductility, as described in chapters 3 and 4.

So far, design and numerical material models for compressive failure of concrete are based on laboratory tests of small/moderate size specimens, where uniform deformation over the concrete specimen is assumed. For instance, in NS 3473 the ultimate compressive strain (in the horizontal branch) of the stress-strain relationship is given as a function of the inelastic behaviour of the ascending branch. Accordingly, the ultimate compressive strain is assumed to be a material property and independent of member size.

In general, the presently used codes of practice do not limit their application field to some selected range of member dimensions. Properly accounting for the observed reduction in ultimate strength and ductility of large specimen is therefore crucial for ensuring safe design of large and slender concrete structures.



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