

Wall and Roof Design for Hot Climates

By Dr. techn. ROLF SCHJØDT
Norwegian Building Research Institute
and KJELL BLEGEN B.Sc.
Norwegian Computing Center

NORWEGIAN BUILDING RESEARCH INSTITUTE



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ROLF SCHJØDT and KJELL BLEGEN

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by

Rolf Schjødt ¹⁾

and

Kjell Blegen ²⁾

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When a wall or roof design shall be judged as to appropriateness in a hot climate, there are many questions that should be answered.

It is evident that strength, durability and protection against rain and wind must be sufficient, here as elsewhere, and that the aspect should be pleasing. But there are other important questions also, such as:

1. Given an outside daily variation of temperature and solar radiation, what will be the inside temperature for the different designs?
2. What is the effect of the colour of the surface, for instance, how much hotter will the house be inside with a black asphalt-covered terrace, compared to a grey or white covering?
3. How much heat capacity is needed to insure an even inside temperature?
4. When light insulating material is used to improve conditions, should it be placed on the inside of the wall, on the outside, or perhaps in the middle?
5. The always increasing use of coolers and air-conditioners raises one more question, namely, what amount of heat insulation and heat capacity is economically justified?

Some information on these questions will be found in the following.

1) Dr.techn., Norwegian Building Research Institute, Oslo.

2) Norwegian Computing Center, Oslo.

The method of calculation of optimum heat insulation for stationary conditions is well known. However, to find answers to the questions above for transient conditions has been difficult, since the combinations and variations of meteorological conditions and of designs made a mathematical solution complicated. Now, however, solutions can be found with the help of computers, as shall be shown in the present paper. A preliminary study on the subject has already been published by the author, see [8].

Some aspects of the problem have been examined. The solar heating of various surfaces has been measured, see [1] and [6], also the temperature inside buildings under variable external conditions, [2], [3]. Model studies and calculations have been made, see [2], [3], [4] and [5]. Analogue computers have mostly been used in the calculations, taking advantage of the fact that the equations for heat- and electricity- transmission are mathematically identical. A useful discussion of the various methods of calculation and analogue methods is given in [6].

Replies to the important questions raised above are not easy to find, however, in spite of the basic work mentioned. Further, the possibilities of variations in outside temperature, radiation, air-exchange, dimensions, materials and colours are so great that a general method is needed, that would make it possible to evaluate the thermal qualities of any design under any condition.

The basis for the calculations is the Fourier equation for heat transmission (for one-dimensional flow):

$$\lambda \frac{\delta^2 g}{\delta x^2} + W = c\rho \frac{\delta g}{\delta t} \quad (1)$$

g is the temperature, in centigrades

t time, in hours

x position coordinate, in metres ($\delta x \rightarrow \delta_n$)

λ thermal conductivity, kcal/mh⁰C

c specific heat, kcal/t⁰C

ρ density of material, t/m³

W heat source, kcal/m³h (in (2) and following, kcal/m²h)

Equation (1) states that the difference between the amounts of heat entering and leaving a section, plus the heat coming from some exterior source (here, radiative heating or cooling of the wall, or heating or cooling of the interior air-space), equals the heat stored in the section.

For numerical calculation, by hand or computer, equation (1) must be written as a difference equation. With constant values through the wall for λ , c and ρ , it is usually given as:

$$\frac{\lambda}{\delta_n} (\vartheta_{n-1,m} + \vartheta_{n+1,m} - 2\vartheta_{n,m}) + W_{n,m}\delta_n = \delta_n \rho_n c_n (\vartheta_{n,m+1} - \vartheta_{n,m}) \frac{1}{\Delta t} \quad (2)$$

See Fig. 1 for the numeration of the sections, the second indices numerate the time intervals. The first term in equation (2), however, is an expression for steady state heat conduction. With a finite length of the time interval $m \rightarrow m+1$, the results will be misleading for transient conditions. In order to take into account the time variation of the temperature in the term for conduction also, and not only in the term for capacity, equation (2) should be written:

$$\begin{aligned} & \left[\left(\frac{\vartheta_{n-1,m} + \vartheta_{n-1,m+1}}{2} - \frac{\vartheta_{n,m} + \vartheta_{n,m+1}}{2} \right) \frac{\lambda_n}{\delta_n} - \right. \\ & \left. \left(\frac{\vartheta_{n,m} + \vartheta_{n,m+1}}{2} - \frac{\vartheta_{n+1,m} + \vartheta_{n+1,m+1}}{2} \right) \frac{\lambda_{n+1}}{\delta_{n+1}} \right] + \\ & + W_{n,m} \frac{\delta_n + \delta_{n+1}}{2} = \frac{1}{2} (\delta_n c_n \rho_n + \delta_{n+1} c_{n+1} \rho_{n+1}) (\vartheta_{n,m+1} - \vartheta_{n,m}) \frac{1}{\Delta t} \end{aligned} \quad (5)$$

Equation (3) is written for variable sections through the wall, the form of the simplified equation for a homogeneous wall, as in (2), is easily seen.

When calculating "by hand", it is advantageous to write (3) as:

$$\vartheta_{n,m} = \vartheta_{n-1,m} \frac{1}{A_n} + \vartheta_{n+1,m} \frac{\beta_n}{\alpha_n} \cdot \frac{1}{A_n} + \frac{C_{n,m-1}}{A_n} + \frac{W_{n,m}(\delta_n + \delta_{n+1})}{2} \frac{\delta_n}{\lambda_n A_n} \quad (4)$$

$$\begin{aligned}
 A_n &= 1 + \frac{\beta_n}{\alpha_n} + 2x_n \\
 B_n &= 1 + \frac{\beta_n}{\alpha_n} - 2x_n \\
 C_{n,m} &= \vartheta_{n-1,m} - B_n \vartheta_{n,m} + \frac{\beta_n}{\alpha_n} \vartheta_{n+1,m} \\
 \alpha_n &= \frac{\delta_{n+1}}{\delta_n} \\
 \beta_n &= \frac{\lambda_{n+1}}{\lambda_n} \\
 x_n &= \frac{\delta_n(\delta_n \rho_n c_n + \delta_{n+1} \rho_{n+1} c_{n+1})}{2\lambda_n \Delta t}
 \end{aligned}$$

When using a computer, the calculation will start from (3).

When the temperatures in a building are to be calculated, the outside temperatures $\vartheta_{o,m} = \vartheta_{b,m}$ are usually given. The span from maximum to minimum temperature of the daily cycle is important for the amount of heat capacity needed in a house. The difference between day and night temperature may be as much as 25°C, and perhaps more. In the calculations following, a temperature curve from Baghdad is used, with a variation of 20°C.

Apart from conduction, roof and walls are exposed to solar heating and radiative cooling. The amount of this appears as "W" in the formulas, numerical values can be found in [1] and [7]. "W" is given as kcal/m²h (or Btu/ft²h), or in [1] as temperature rise of test specimens. This heat quantity is here assumed to be absorbed by a thin surface layer,

$$\frac{\delta_1 + \delta_2}{2} \quad \text{or} \quad \frac{\delta_{b-1} + \delta_b}{2} \quad \text{in Fig. 1.}$$

The coefficient of absorption of the surface, mainly dependent on the colour, must of course be taken into account when W is calculated. Values for this coefficient, also, are found in [1].

Equations (1) to (4) are valid for one unit of wall or roof area. For the heat balance of the whole house, the relation between wall and roof area must be taken into account. This can be done by multiplying, for instance, the thermal conductivities of the wall

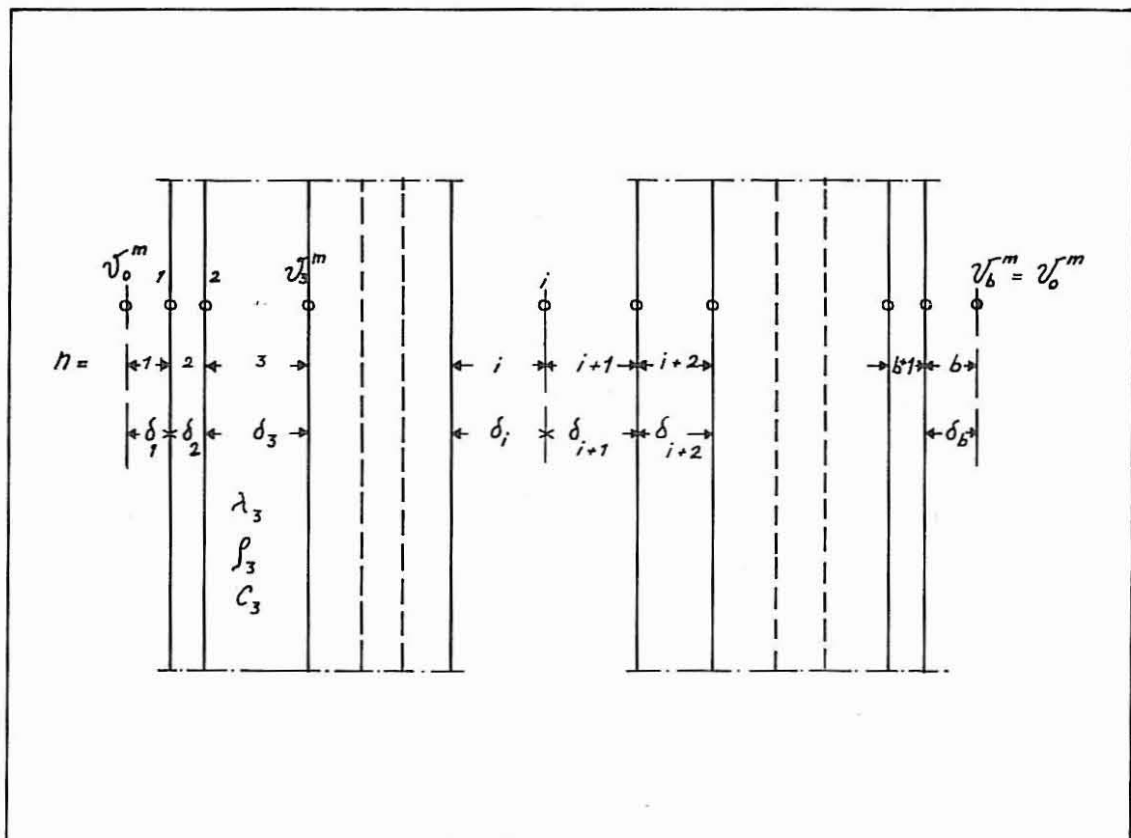


Fig. 1.

Numeration and nomenclature for temperature calculation.

with the relation between wall and roof area, and using the new values thus obtained in the calculations.

In the building's interior, section i , there will occur a number r of air changes per hour. The quantity of heat introduced (or taken away if $\vartheta_i > \vartheta_o$) will be:

$$W = (\vartheta_o - \vartheta_i) \rho r V c_i , \quad (5)$$

where V is the volume of the building. This heat quantity will be assumed to be distributed equally over the whole interior surface.

The heat capacity of the floor, and of inside walls, can have some influence on the resultant inside temperature. This may easily be taken into account by increasing the value of c_i correspondingly.

The value of r may vary over a wide interval. 1 - 2 is usual in residences in temperate climates, 10 may be used in assembly rooms, and up to 30 in special rooms. The value depends on the temperature differences indoor - outdoor, and is therefore small in hot climates. Without cooling or air-conditioning, and with doors and windows shut in order to keep the heat out, r may be only 0,2 in a well-built house (even 0,1 has been measured). In this paper, $r = 0,5$ will be used, and 5,0 for comparison.

δ_1 and δ_b (Fig.1) are equivalent lengths calculated from the coefficient of heat transfer outer air to structure. The wall material is assumed to continue for a length δ , to compensate for the surface air-layer.. This length depends on the wind velocity, usually 2m/sec is used, which gives 20 kcal/m²h^oC for the coefficient. δ_i and δ_{i+1} are calculated in the same way from the coefficient of heat transfer for indoor air to structure, usually 5 - 6,5 kcal/m²h^oC. One finds, therefore:

$$\delta_1 = \frac{\lambda_2}{20} , \quad \delta_b = \frac{\lambda_{b-1}}{20} , \quad \delta_i = \frac{\lambda_{i-1}}{6} , \quad \delta_{i+1} = \frac{\lambda_{i+2}}{6}$$

With a cooling system (not air-conditioning), the amount of heat removed must be introduced in (5). This may be done by introducing a correction in the value of ϑ_o in (5). This correction will depend on air humidity and temperature and may correspond to a decrease of ϑ_o of 10 - 18^oC.

Where air-conditioning is used, the indoor temperature will usually be given (by a thermostat), and an equivalent for the amount of electricity used may be introduced instead of g_i . This will not be considered here.

A solution for equation (5) is found with the help of a computer, as said before. Mr. Blegen of the Norwegian Computer Center will explain the programming.

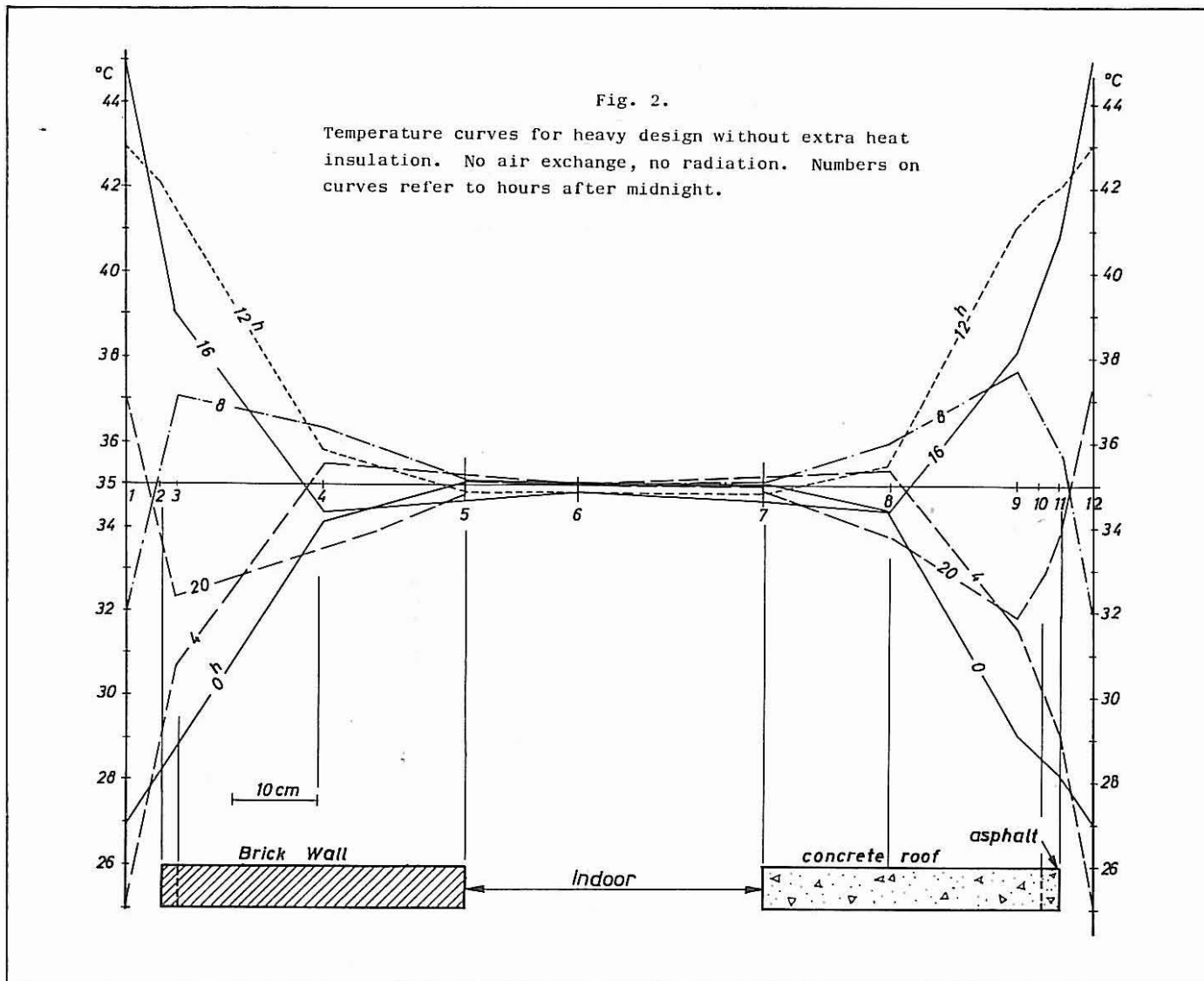
Calculations have been carried out for the roof and wall designs shown in Fig. 2 - 6, without air exchange and radiation, with 0,5 and 5,0 air exchanges per hour, and with radiation on roof and wall.

The values for solar heating have been taken from [7], and correspond to incident solar energy at latitude 42° . The maximum heating on a horizontal surface is taken as $850 \text{ kcal/m}^2\text{h}$. (310 Btu/hft^2). The corresponding radiative cooling has been found in [1], the maximum is taken as $160 \text{ kcal/m}^2\text{h}$.

The absorption coefficient for the roof has been taken as 0,9, black asphalt, and for the wall 0,7, corresponding to a grey surface as for instance crushed stone. The roof in Fig. 5 was also calculated with an absorption coefficient of 0,3, for a white or near white surface.

The results can be summarized as:

1. The needs for heat capacity are lower, and for heat insulation higher, than provided for in most designs. This is especially true for the roof. 15 cm (6") of heavy material will give sufficient heat capacity to give a reasonably constant indoor temperature under ordinary conditions.
2. When the rate of air exchange is low, a wall with light insulating material needs no heavy material to give heat capacity, see Fig. 5. With a higher rate of air exchange, some heat capacity is desirable.
3. In the last case, the insulating material should preferably be placed on the inside on the roof when the covering is black, and outside on the wall. (See below)



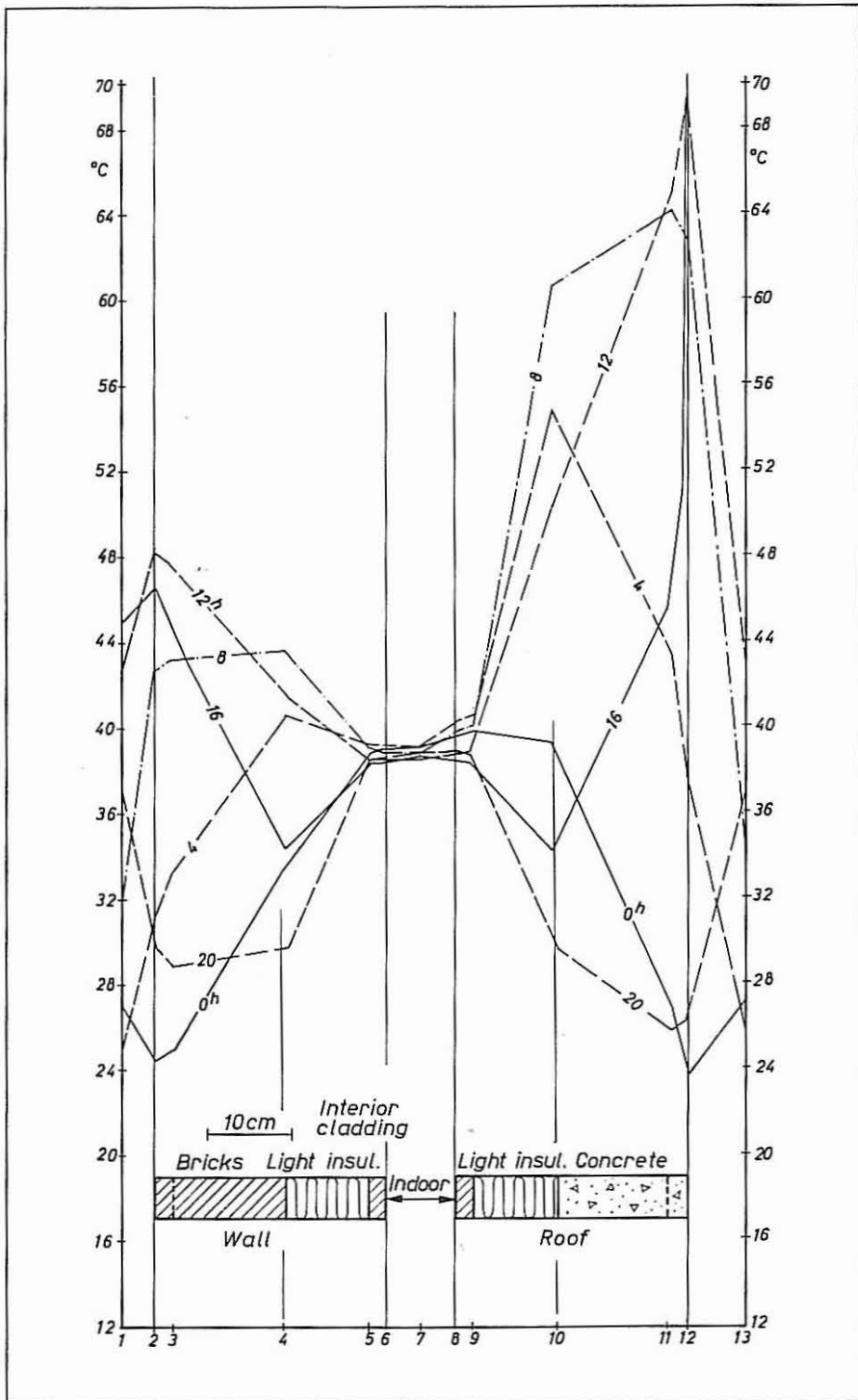


Fig. 3.

Brick wall and concrete roof with inside light (f.i. rock-wool) insulation. 0,5 air exchanges per hour, radiation on roof and wall.

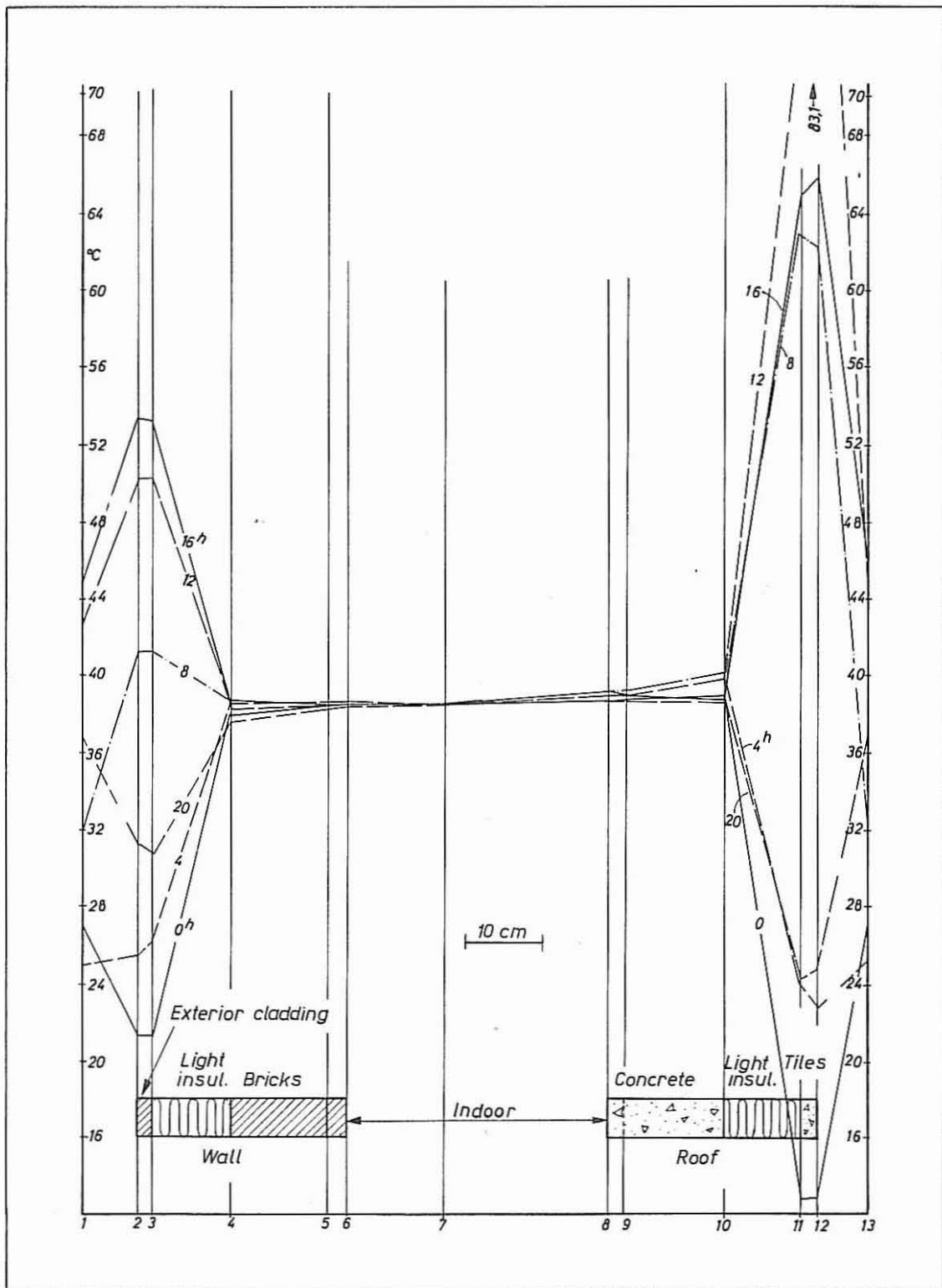


Fig. 4.

Brick wall and concrete roof with exterior light insulation. Air exchange and radiations as in Fig. 3.

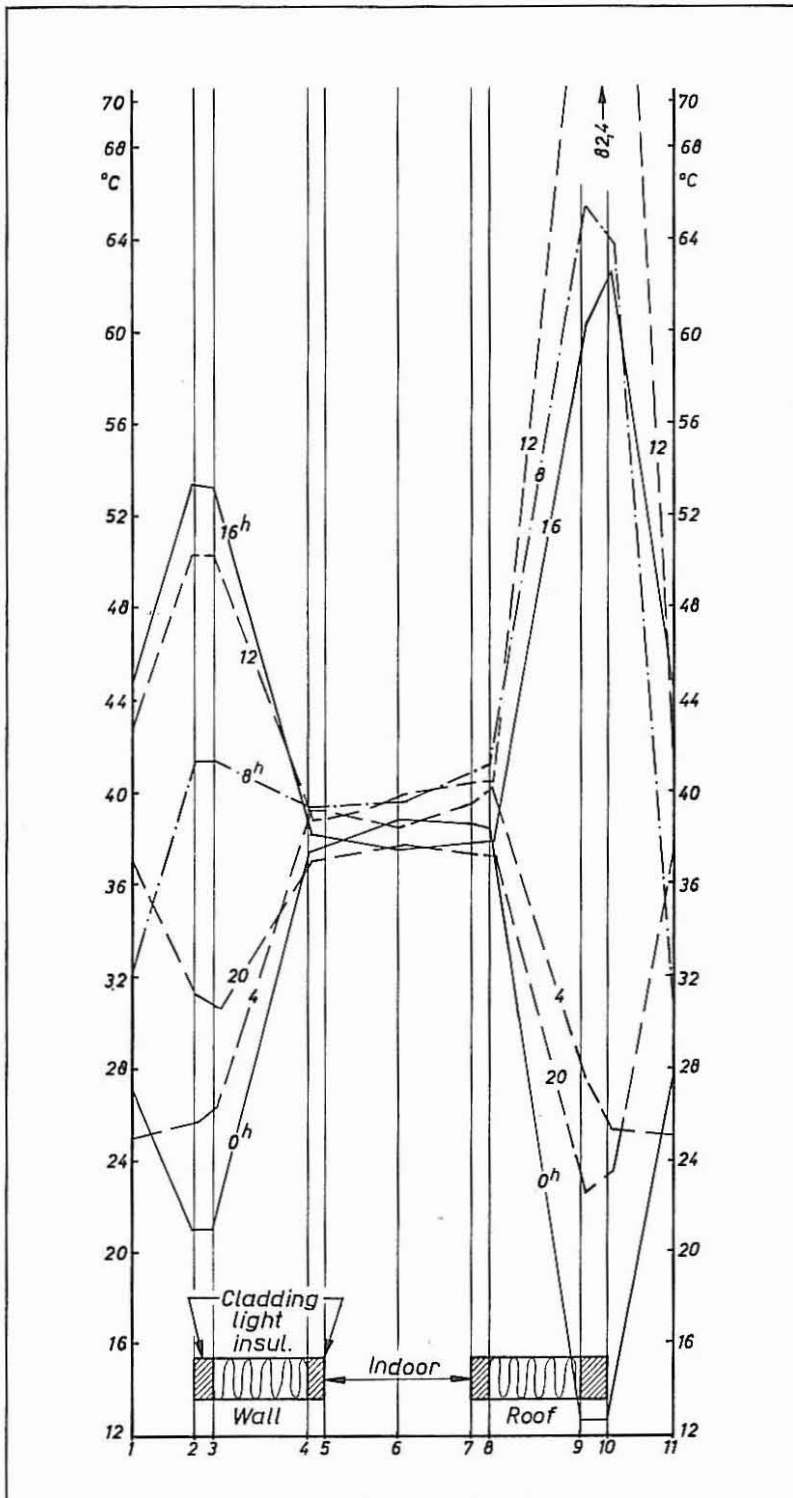


Fig. 5.

Wall and roof with only light insulation. Air exchange and radiation as in Fig. 3.

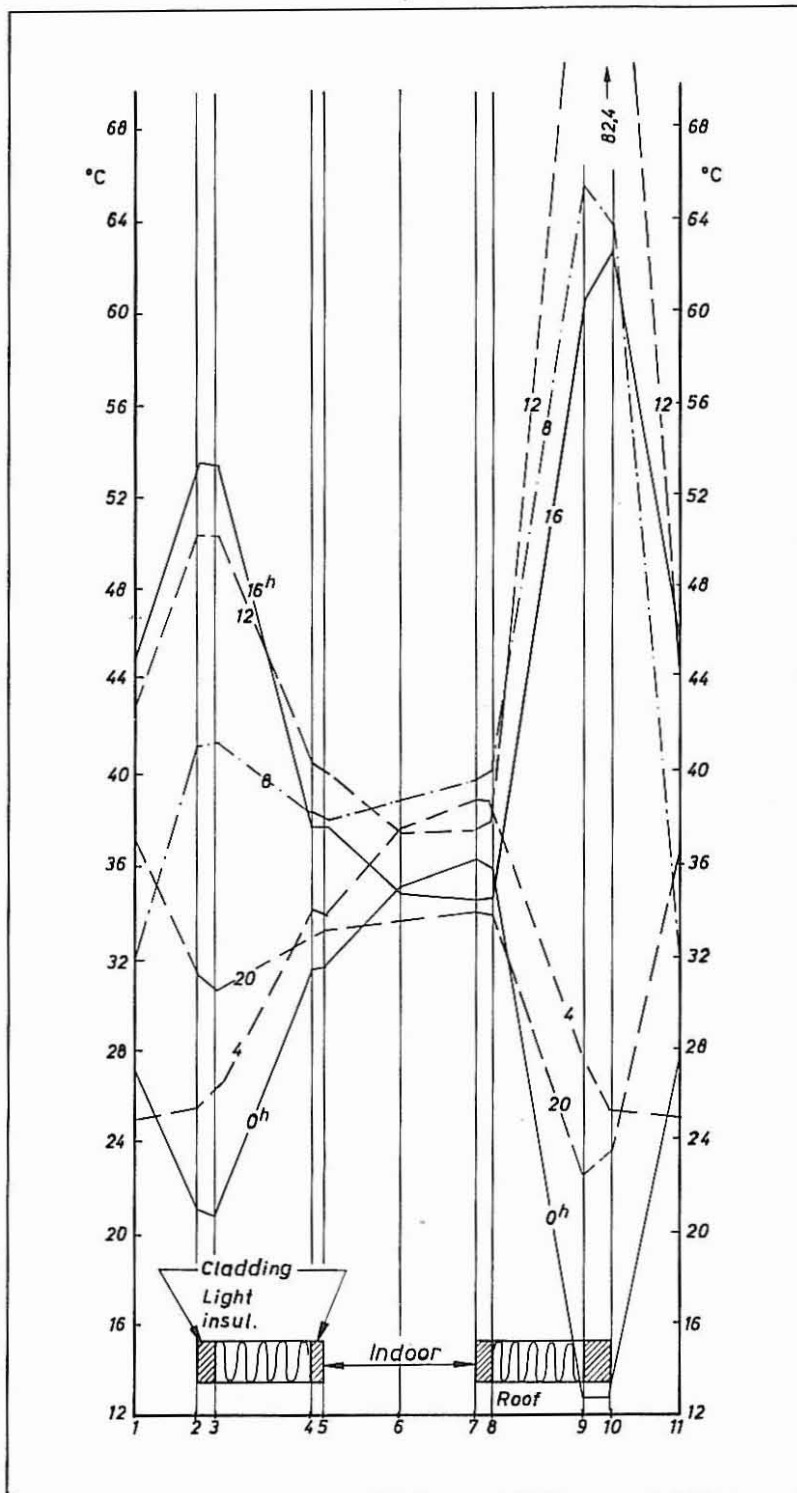


Fig. 6.

As Fig. 5, but with 5 air exchanges per hour, radiation on roof and wall.

With a white or near white colour on the terrace, the light material may be placed on the outside here also.

4. Exchanging white asphalt for black lowered the average inside temperature by 3.1°C ($6,2^{\circ}\text{F}$) in the design shown in Fig. 2, with the radiation as given above.

When the insulating material is placed outside on the roof, the variation of the roof temperature will be very great with a black surface, see Fig. 4, 5 and 6. Here is shown a range from $12,5$ to 83°C or 70°C variation. The temperature is found to be 38°C (76°F) hotter and 12°C (21°F) colder than the ambient temperature. This agrees with the results reported in [1]. With only heavy materials, as in Fig. 2, the temperature range was found to be 50°C with a black surface and 26°C with a grey surface, also in agreement with [1].

The very great variation found with the insulating material outside will of course cause considerable thermal movement, the results of which are evident in many houses in a hot climate. A correct design of all details can diminish the damage, but deterioration of the materials is inevitable. The radiation falling on a wall is much less, and the colour is usually brighter, so that here it seems justified to take advantage of the better indoor temperature conditions obtained with this solution.

A design with relatively heavy outside and inside layers, and the insulating material in the middle has also been calculated. As might be expected, the results lie between the other two. Such a solution can be recommended, for instance, two half-brick walls with insulating material between, and a concrete roof slab carrying light insulating material, covered with two inches of concrete and two inch tiles.

Cooling and air-conditioning introduce the question of the economically justified insulation and heat capacity. The reply to this will depend on the local climate, and on the local cost of fuel. Muncey [9] has examined Australian conditions, a discussion of the problem for Baghdad conditions was made in [8]. In these climates a k-value (heat flow through the design, $\text{kcal}/\text{h}^2\text{m}^{\circ}\text{C}$) of $0,5 - 0,6$ was

found to be desirable. This means a wall, and more particularly a roof, having much better insulating qualities than those generally used. Similar conclusions will be reached in most hot climates.

It has always been supposed that a certain amount of weight, that is, of heat capacity, is necessary in a wall or a roof. The need for insulation, which to some degree is also given by the heavy materials, caused this need to be exaggerated. On the other hand, with the modern light, highly insulating materials, there has been a tendency to ignore the need for heat capacity. It is hoped that the method of calculation given here will be of assistance in finding a well balanced design.

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APPENDIX

by

Kjell Blegen,
Norwegian Computing Center

This appendix is written to explain a computerprogram written at Norwegian Computing Center on request of Dr. Rolf Schjødt for use in his paper "Wall and roof design for hot Climates". The program is written in FORTRAN for the UNIVAC 1107.

In the referenced paper the physical problem in question is reduced to the mathematical model of heat flow through a composite wall along an axis normal to the face of the wall, ruled by boundary conditions which vary periodically with respect to time.

The wall is composed of a series of plates whose properties and system of identification are given in Fig. 7, where:

The index N , running from $N=1$ to $N=NB$ identifies the plate surfaces.

Any plate is identified by the index of the surface to its right.

1	2	3			NI			N-1	N	N+1			NB	
λ_2	λ_3							λ_N	λ_{N+1}				λ_{NB}	Thermal conductivity.
σ_2	σ_3							σ_N	σ_{N+1}				σ_{NB}	Plate thickness.
C_2	C_3							C_N	C_{N+1}				C_{NB}	Specific heat.
ρ_2	ρ_3							ρ_N	ρ_{N+1}				ρ_{NB}	Density.
S_2	S_3							S_N	S_{N+1}				S_{NB}	Area of plate face.

Fig. 7.

The heat flow is caused by:

1. The free air temperature at the surfaces $N=1$ and $N=NB$.
2. The radiation falling on the surfaces $N=2$ and $N=NB-1$.

3. The interchange of air between the free air at $N=1$ and the indoor room at the surface $N=NI$.

These 3 points are further explained by reference to the time-space-diagram Fig. 8, where:

N - indicates position along the space axis through wall and roof.

M - indicates position along the time axis.

V_N^M - indicates temperature at (N,M)

W_N^M - indicates radiation falling on N at time M .

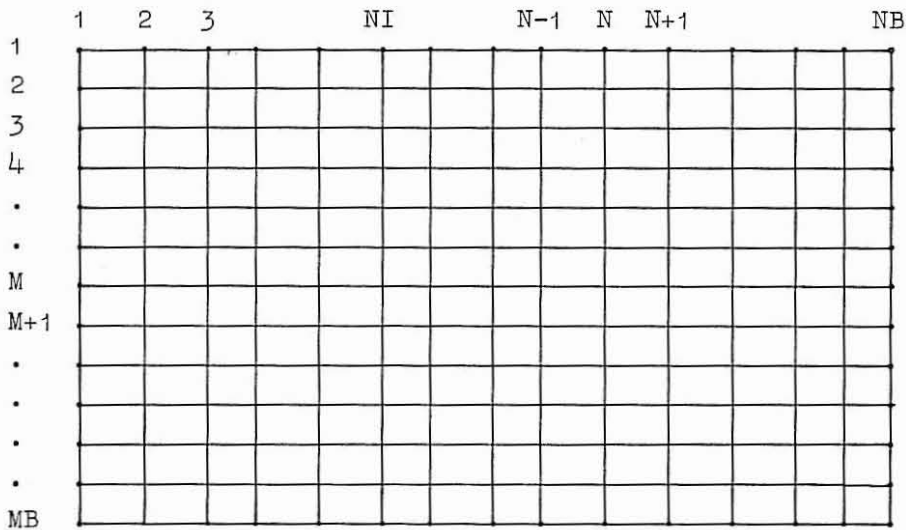


Fig. 8.

The boundary conditions of temperature at the surfaces $N=1$ and $N=NB$ and the radiation falling on the surfaces $N=2$ and $N=NB-1$ vary in a cyclic manner with period extending from $M=1$ to $M=MB$ such that the temperature at any point N in the wall will also vary periodically with time. i.e. $V_N^M = V_N^{M+MB-1}$ and in particular $V_N^1 = V_N^{MB}$ for all $N=1, NB$.

For the heat exchange caused by the airing at $N=NI$ one has:

- r - rate of air exchange.
- v - volume of indoor room.
- c - specific heat of air.
- ρ - density of air.

Let $P = r \cdot v \cdot c \cdot \rho$

The difference equation (5) may be written:

$$\begin{aligned}
 S_{N+1} \cdot \lambda_{N+1} \cdot \left[\frac{\frac{V_{N+1}^{M+1} + V_{N+1}^M}{2} - \frac{V_N^{M+1} + V_N^M}{2}}{\sigma_{N+1}} \right] &- S_N \cdot \lambda_N \cdot \left[\frac{\frac{V_N^{M+1} + V_N^M}{2} - \frac{V_{N-1}^{M+1} + V_{N-1}^M}{2}}{\sigma_N} \right] \\
 &+ \frac{W_N^{M+1} + W_N^M}{2} S_N (K_{N,2} + K_{N,NB-1}) \\
 &\left(+ \frac{V_1^{M+1} + V_1^M}{2} - \frac{V_N^{M+1} + V_N^M}{2} \right) \cdot P \cdot K_{N,NI} \\
 &= (S_N \cdot \rho_N \cdot c_N \cdot \frac{\sigma_N}{2} + S_{N+1} \cdot \rho_{N+1} \cdot c_{N+1} \cdot \frac{\sigma_{N+1}}{2}) \cdot \left(\frac{V_N^{M+1} - V_N^M}{\Delta t} \right) \text{----- (5')}
 \end{aligned}$$

Where: Δt - time interval from M to $M+1$.

$$K_{N,J} = 1 \text{ when } N = J.$$

$$0 \text{ when } N \neq J.$$

Let:

$$A_{N,1} = \frac{S_N \cdot \lambda_N}{2 \cdot \sigma_N}$$

$$\begin{aligned}
 A_{N,2} = & - \frac{S_{N+1} \cdot \lambda_{N+1}}{2 \cdot \sigma_{N+1}} - \frac{S_N \cdot \lambda_N}{2 \cdot \sigma_N} - \rho_{N+1} \cdot c_{N+1} \cdot S_{N+1} \cdot \frac{\sigma_{N+1}}{2 \cdot \Delta t} - \rho_N \cdot c_N \cdot S_N \cdot \frac{\sigma_N}{2 \cdot \Delta t} \\
 & - \frac{P}{2} \cdot K_{N,NI}
 \end{aligned}$$

$$A_{N,3} = \frac{S_{N+1} \cdot \lambda_{N+1}}{2 \cdot \sigma_{N+1}}$$

$$B_{N,1} = \frac{S_N \cdot \lambda_N}{2 \cdot \sigma_N}$$

$$B_{N,2} = \frac{S_{N+1} \cdot \lambda_{N+1}}{2 \cdot \sigma_{N+1}} + \frac{S_N \cdot \lambda_N}{2 \cdot \sigma_N} - \rho_{N+1} \cdot C_{N+1} \cdot S_{N+1} \cdot \frac{\sigma_{N+1}}{2 \cdot \Delta t} - \rho_N \cdot C_N \cdot S_N \cdot \frac{\sigma_N}{2 \cdot \Delta t}$$

then (3¹) becomes:

$$A_{N,1} \cdot V_{N-1}^{M+1} + A_{N,2} \cdot V_N^{M+1} + A_{N,3} \cdot V_{N+1}^{M+1} = -B_{N,1} \cdot V_{N-1}^M + B_{N,2} \cdot V_N^M - B_{N+1,1} \cdot V_{N+1}^M - f_N \quad (3^{11})$$

where:

$$f_N = V_1^{M+1} \cdot B_{2,1} + \left(\frac{W_2^M + W_2^{M+1}}{2} \right) \cdot S_2 \quad \text{if } N = 2$$

$$f_N = V_{NB}^{M+1} \cdot B_{NB,1} - \left(\frac{W_{NB-1}^M + W_{NB-1}^{M+1}}{2} \right) \cdot S_{NB-1} \quad \text{if } N = NB-1$$

$$f_N = \frac{V_1^{M+1} + V_1^M}{2} \cdot P \quad \text{if } N = NI$$

Equation (3¹¹) connects the temperature at 6 points of the space-time diagram, Fig. 8. This may be illustrated by the stensile, Fig. 9:

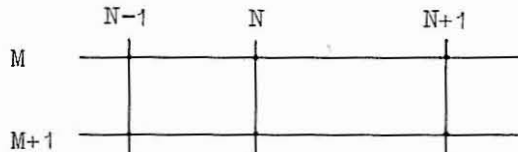
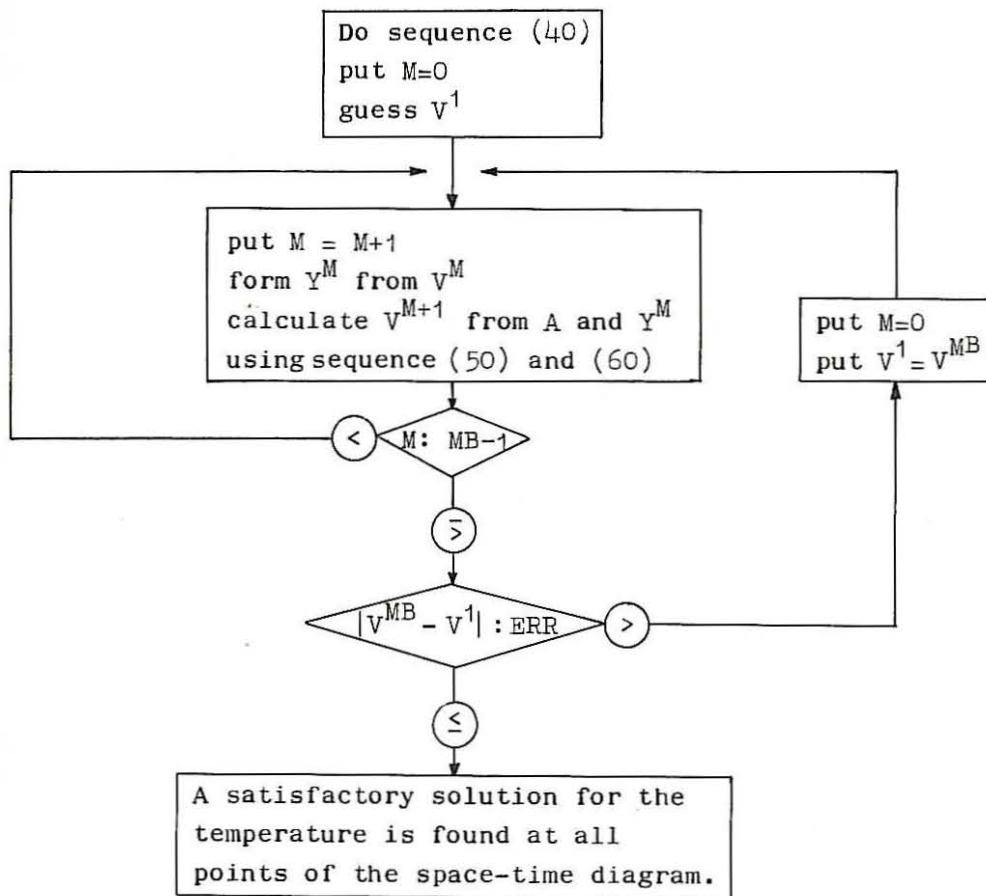


Fig. 9.

The task is to obtain the temperature $V_{(N,M)}$ for all points of the space-time diagram, Fig. 8.

The following iteration method is used:



The value ERR is input to the program

The iteration method requires less storage than the direct solution of the set $(A \cdot V^{M+1} = Y^M)$ $M=1, MB$ and makes the program applicable on any machine having a FORTRAN compiler.

Timing:

Compilation of program	11 sec.
Execution of one set	MB=13, NB=11, ERR= 10^{-6} DEG C.
Input and output	6 sec.
Calculation of one set	1 sec.

The output in this particular program is embroidered. When calculating nearly equal sets the initial guess is improved, resulting in calculation times down to 0.25 sec. for a set of MB=13, NB=11, ERR= 10^{-6} DEG C.