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Modelling minimum pressure height in short-term hydropower production planning

Frederic B. Dorn[a,*], Hossein Farahmand[a], Hans Ivar Skjelbred[a], Michael M. Belsnes[a]

[a] *SINTEF Energy Research, Trondheim, Norway*

Abstract

When planning the production for certain hydropower plants, minimum pressure is one of the major critical points. Violation of the minimum pressure causes the power plant to automatically shut down, hence violating the obligations of the plant. Automatic pressure switches and pressure constraints are difficult to model in particular when embedded in a complex water way. This problem is expected to increase when retrofitting hydro installations with new parallel units and increased exploitation of inflow resources. From a scheduling point of view, however, such switches become hard to integrate in an optimal operation plan as the constraint depends on the system state. This paper introduces a novelty in short-term production planning, namely a solution for modelling minimum pressure height in regulated watercourses when optimizing the energy production of hydropower plants. This solution is integrated in the short-term hydropower scheduling tool SHOP. The tool finds an optimal strategy to run a power station with such minimum pressure restrictions and the state dependent topological couplings within the water system. We apply the model on a complex topology, the Sira-Kvina water system, where Norway's largest hydropower station *Tonstad Kraftstajon* is operationally subject to this rigorous pressure constraint. First, in order to illustrate the concepts of the model, we apply the model on a simplified water course including one reservoir. Next, the outcome and tests are demonstrated on the final model of two reservoirs whose respective outflows are joining together above the pressure gauge, as found in the Sira-Kvina water system.

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Keywords: Hydropower scheduling; optimization; mixed-integer linear programming ;

1. Introduction

Nomenclature

h_{rsv}/h_{creek} Reservoir and creek level at period t $[m]$.

$\Delta h/H_{min}/h_O/h_H$ Head loss / Minimum pressure height / Left reservoir level (here *Ousdalsvann*) / Right reservoir level (here *Homstølvann*) at period t $[m]$.

$\alpha_1/\alpha_2/\alpha_O/\alpha_H$ Head loss coefficient for the tunnel above pressure gauge /above power plant / left above junction / right above junction $[\frac{s}{m^2}]$.

* Corresponding author. Tel.: +47-980-55-660.
E-mail address: frederic.dorn@webstep.no

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Nomenclature

$q_O/q_H/q_O^i/\ q_H^i$	Tunnel flow for left / right tunnel above junction / Segments for linearisation of q_O / q_H [$\frac{m^3}{s}$].
δ_O/δ_H	Binary variable for direction of flow for left / right tunnel above junction at period t.
$Q_{max}/Q/\ q_{cr}$	Maximum discharge through tunnels above junction/ Linearised total discharge through power station / Inflow to creek intake [$\frac{m^3}{s}$].

1.1. Background

SHOP (Short-term Hydro Operation Planning) [1] is a decision support tool for short-term hydro scheduling. The general objective of the program is to utilize the available resources taking into account the physical and technical limitation of the water system. It maximizes the profit of the hydro power company within the given short-term period by exploiting the options for buying and selling in the spot market, while fulfilling firm load obligations. The planning horizon is flexible and can be between one week to 14 days and time resolution is typically one hour.

Being a model in operational use, SHOP has a high level of detail to assess the valuation of system wide capacity allocation as well as correct pricing of energy and cost of power reserves [2]. SHOP is a part of SINTEF Energy Research's state-of-the-art models for hydropower and hydro thermal scheduling and power markets developed over more than 40 years. This includes models that are widely used by players in the Nordic power market [3] which provides SINTEF Energy Research with a broad and thorough understanding of operation research and mathematical programming together with an excellent success rate of establishing new models in the energy sector, such as long-terms models [4], mid-term models [5], short-term models [6], risk management [7]and simulation models.

The SHOP program includes all the main components in hydro production systems including reservoirs, hydro units, discharge gates, junctions, and thermal units. A firm load profile and a day ahead market are also modelled. The reservoirs are the interconnecting elements of a given water course. A reservoir may connect to one or more power station and/or one or more discharge gates downstream. Two reservoirs may supply water to one power station via two interconnected tunnels, a so-called *junction*. In this respect, the pressure balance decides the distribution of water flow between the intake tunnels. The water flow in discharge gates is a decision variable in an optimisation problem and is dependent on the relation between the volume of hydro in reservoir and the water level of the reservoir (*head*).

Topologies of a water systems and power stations are the fundamental elements for the mathematical modelling of the optimization problem in SHOP. Topological couplings within the water system allow alternative strategies on how the water can be managed in each reservoir with respect to inflow, spillage, bypass, and plant discharge. The time couplings within the scheduling period define the water balance in each time step. On the other hand, this results in a large number of equations for each reservoir in every time step during the scheduling period. Some reservoirs can be connected to the power stations through multiple penstocks, with further connections to multiple generators. Friction occurs between the water and the walls in the tunnels and penstocks. Hence head loss is also a variable function that has to be included in the modelling. This can be a complicated task since the head loss is associated with static- and effective plant head.

Head optimization for hydro plants is modelled in SHOP. This is implemented based on optimum calculated energy equivalent resulted from power-discharge curves (PQ-curves). Hence, the PQ-curve is derived from calculations regarding head-loss and efficiency. PQ-curves are an important topic in the essence of short-term hydropower scheduling. In the short-term optimisation of hydro production, the relationship between discharge and production is non-linear and non-convex, depending on the number of units in operation. It gets more complicated since the tunnel and tailrace loss is dependent on the discharge of all units in the plant, while the penstock loss is only dependent on the connected units discharge. The method for handling non-linearities and state dependencies in SHOP is successive linear programming, where the reservoir and production trajectories of the solution of one (mixed integer) linear programming (MI)LP model are used as input to the next (MI)LP call in an iterative procedure. In SHOP, MILP is

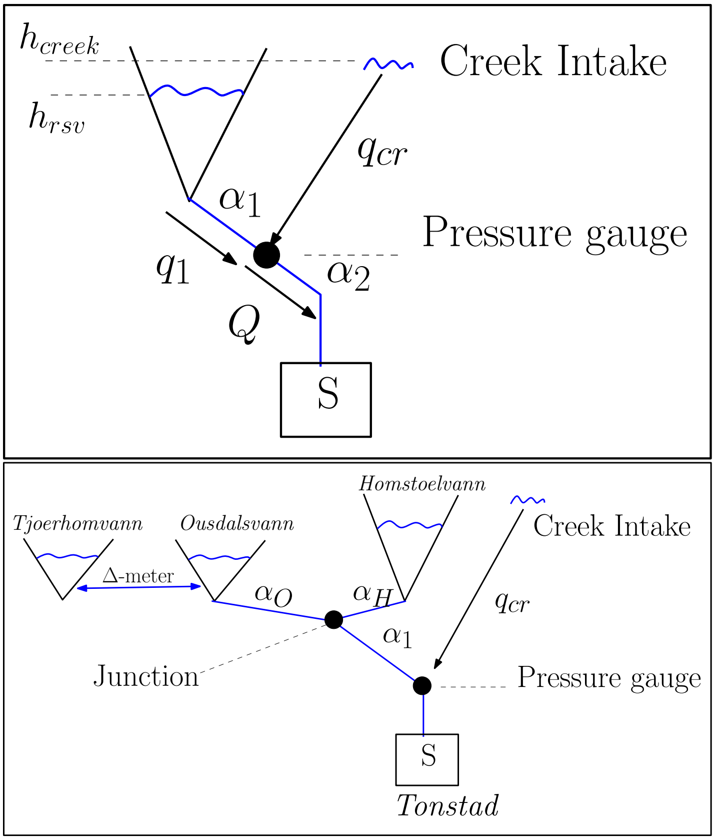

Fig. 1. Aggregated model without junction and with junction.

applied in the first run of iterations for deciding unit commitment of every generator—the so-called UC-mode or *full model*. In a second phase—the close-in mode or *incremental model*, several iterations using LP are run with a fixed commitment plan.

1.2. Minimum pressure

In this paper, we present a new functionality in the SHOP program, the modelling of the minimum pressure height. We demonstrate its outcome on the Tonstad power station in Sira-Kvina water system, Norway's largest power station in terms of annual energy production. The aggregated model is depicted on the bottom in Figure 1. The figure illustrates both the simplified and the model with junction above pressure gauge. In the model with junction, the power station uses water from both upstream reservoirs that joined together in the junction point. In the detailed studies of Tonstad power station, the so-called *Tonstad case study* is selected for the testing of SHOP program. Tonstad case study covers the physical watercourses in the area and the entire chain upper reservoirs. In this case, the left intake tunnel (Sira water course) to the junction point is connected to two small reservoirs, e.g. Tjørhomvann and Ousdalsvann each with the reservoir capacity of $3.33Mm^3$ and $12.25Mm^3$, respectively. The right intake tunnel (Kvina water course) is connected to a larger reservoir, Homstølvann with maximum capacity of $55.42Mm^3$.

Due to both the special topology and the special design of the Tonstad power station, it is necessary to maintain a minimum pressure height at the top of the penstock tunnels. This pressure height must be above the defined limit to keep air from entering the tunnel. If the pressure drops below the given treshold, the plant will shut down and the generation mismatch will be high. The company will need to buy reserve power. If the margin is too big, the company may suffer an opportunity loss when prices are good. Therefore, in the optimization problem a constraint has been added ensuring that pressure height is always above the minimum level during generation. For this purpose, all the adjustments have been implemented to reflect the impact of this constraint on the head loss coefficients. Moreover, a linearisation of the pressure balance in the junction has been applied to the LP model.

In the testing of the minimum pressure model, two cases are selected. In the first case, the junction between Ousdalsvatn and Homstølvatn is modelled as one reservoir and we assume that we have a good estimate of the head in this reservoir. In the second case, we test the model on the detailed case including the junction. We have tested the minimum pressure functionality in this way to evaluate how the new modelling behaves for borderline cases, i.e., when water values are close to market price, both with much inflow or little, high or low initial reservoir level.

The implemented functionalities resolve the targeted challenges for modelling minimum pressure height in terms of short-term production planning even for special topologies given in the Sira-Kvina water system. That is, the decision on the ratio of discharge of Sira or Kvina to the power plant Tonstad is part of the optimization routine. This paper focuses on the physical and technical particularity for the short-term scheduling of a power plant with minimum pressure height requirement. With this model one is able to calculate an optimal short-term production plan for similar hydro systems as the Sira-Kvina water system, keeping a variable minimum pressure height above certain level at Tonstad power station and takes into account the changing head-loss coefficient depending on the amount of inflow to the creek intake above power station.

2. Mathematical modelling of pressure height

The general formulation of the SHOP model has been described in [1, 2], therefore, in this section we focus on the modelling of pressure height in SHOP model.

Due to both the special topology and the special architecture of the Tonstad plant, it is necessary to maintain a minimum pressure height at the top of the penstock tunnels. Incorrect or imprecise height calculation may lead in an inoperable production plan, therefore it is crucial to model the Tonstad power station in a meaningful way. In order to understand the concepts of pressure height modelling, we first concentrate on the simplified case shown on the top in Figure 1.

In the simplified model, the junction between Ousdalsvatn and Homstølvatn is replaced by a dummy reservoir and we assume that we have a good estimate of the head in this reservoir. The pressure gauge at the top of the penstock is assumed to be coinciding with the creek intake [1]. That is possible since the only variables in the model is the reservoir level h_{rsv} expressed by the storage variable and the plant discharge Q expressed by the tunnel flow variables in SHOP. Thus, it is possible to include the head loss $\alpha_2 Q^2$ above the pressure gauge in the pressure height, Equ. 1 of the LP-model,

$$h_{rsv} - \Delta h \geq H_{\min} \tag{1}$$

and the head loss Δh will be dependent of Q and q_{cr} (Equ. 2).

$$\Delta h(Q, q_{cr}) = \alpha_1 \cdot (Q - q_{cr}) \cdot |Q - q_{cr}| \tag{2}$$

In the LP-model, Equ. 2 is linearised using a technique for calculating the tunnel flow in the penstocks [8]. For this modelling, we assume that $q_{cr}$is handled as known inflow per time-step in the model and that the head loss Δh is added as additional head loss of the plant in the busbar equation. [2]

As mentioned earlier the correct calculation of the pressure height is crucial for the plant operation. For this reason, the threshold H_{min} in Equ. 1 is implemented as a time dependent variable. That is, the user may choose for himself to adjust it for every time-step.

[1] The creek intake has a known inflow at each period t given as input data.

[2] The busbar equation controls the energy equivalent at each time step in the optimization period. It consists of total production of each plant, total energy loss, buy and sales costs, load obligations and model penalties.

2.1. Implementation of the simplified model in SHOP

In order to apply pressure height model in SHOP, we need to combine Equ. 1 and 2 to Equ. 3 as a constraint to SHOP. It is assumed that the pressure is modelled as shown on the top of the Figure 1. As mentioned earlier, the position of the pressure gauge and the creek intake is assumed to be in the same location.

$$H_{\min} \leq h_{rsv} - \alpha_1 \cdot (Q - q_{cr}) \cdot |Q - q_{cr}| \tag{3}$$

The reservoir height h_{rsv} is expressed by the reservoir storage variables together with the Δ volume/head factor, calculated in a pre-step. The second term of Equ. 3 is quadratic and needs to be linearised. This is done by introducing new tunnel flow variables for the tunnel segment above the pressure point.

2.2. Implementation of aggregated model of Tonstad

In this section we show the implementation of a junction in combination with creek intake below the junction, as found in the topology at Tonstad power plant, shown on the bottom in figure 1. In the LP model there is implemented a linearisation of the pressure balance in the junction (Equ. 4).

$$h_O - \alpha_O \cdot q_O \cdot |q_O| = h_H - \alpha_H \cdot q_H \cdot |q_H| \tag{4}$$

The values of the head loss coefficients α_O and α_H are calculated in a pre-step and are depending on the given junction flow and the creek inflow. These latter flows are part of the linearisation as described in [8].

The *pressure point* Equ. 3 is extended and duplicated, for reservoir *O* (Equ. 5) and reservoir *H* (Equ. 6).

$$H_{\min} \leq h_O - \alpha_O \cdot Q_O\,|Q_O| - \alpha_1 \cdot (Q - q_{cr}) \cdot |Q - q_{cr}| \tag{5}$$

$$H_{\min} \leq h_H - \alpha_H \cdot Q_H\,|Q_H| - \alpha_1 \cdot (Q - q_{cr}) \cdot |Q - q_{cr}| \tag{6}$$

No reservoir storage variable is required to model the level change of reservoir *H* and reservoir *O*. This is handled by the junction flow variables. We introduce two *tunnel balance* equations for linearisation $Q_O = \sum q_O^i$ and $Q_H = \sum q_H^i$, where q_O^1, q_H^1 represents the negative junction flow and $\sum q_O^{i>1}, \sum q_H^{i>1}$ the positive junction flow, respectively.

Moreover, we introduce two binary variables δ_O and δ_H for the direction of the flow and tunnel flow direction equations, for reservoir *O* (Equ. 7) and reservoir *H* (Equ. 8) [3].

$$\sum q_O^{i>1} + Q_{\max} \cdot \delta_O \leq Q_{\max} \text{and} - q_O^1 - Q_{\max} \cdot \delta_O \leq 0. \tag{7}$$

$$\sum q_H^{i>1} + Q_{\max} \cdot \delta_H \leq Q_{\max} \text{and} - q_H^1 - Q_{\max} \cdot \delta_H \leq 0. \tag{8}$$

2.3. Other topological challenges in Tonstad: Deltameter and junction

In order to represent the actual behaviour of water flow into the junction point, it is necessary to introduce the delta-meter between Tjørhomvann and Ousdalsvann. The delta-meter is a function modelling the flow balance between two reservoirs of similar height, where the flow may go both ways. The delta-meter component has been tested and concluded to work correctly for all cases in this study.

It was assumed that the water level in Tjørhomvann is at least at the same level in Ousdalsvann at all times. In order to make the delta-meter work properly, we considered different solutions and concluded that manipulating the costs of slack variables was the best strategy for this case, where the delta-meter being in series with a junction. It has been tested in different inflow and price scenarios (see Section 3). The crucial point in using the slack variables is that the LP-solver should not see any profit in using these variables. This means that the penalty of using the slack variables ranges between the possible income and the penalty for violating physical constraints (such as reservoir trajectories).

Both the deltameter slack variable and the junction slack variable can be used together with the pressure point functionality hence the reservoirs may be emptied during the optimization period.

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[3] We use one negative variable when linearizing Q_O, Q_H since it does only increase the pressure height in Equ. 7 and 6. δ_O and δ_H determine the direction of flow in the tunnel segments above the junction and equal zero if the flow is directed towards the junction and one otherwise.

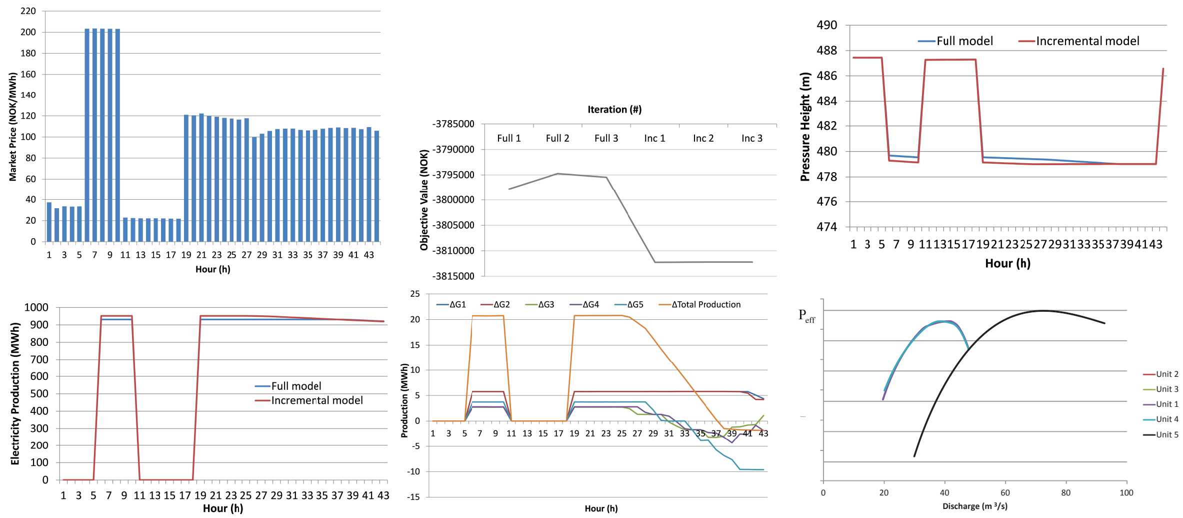

Fig. 2. Upper left: Electricity prices in the market-benchmark case. Upper middle: Objective values-benchmark case. Upper right: The pressure height at the pressure gauge point-benchmark case. Lower left: Production pattern at Tonstad power station-benchmark case. Lower middle: The difference in detailed production pattern of two models (Incremental model - the Full model) -benchmark case. Lower right: Efficiency curves of the production units-benchmark case.

3. Test results

In this section we present some results of testing the program. It is important to evaluate how the optimization behaves for borderline cases, i.e., when water values are close to market price, both with much inflow or little, high or low initial reservoir (filling at the optimization period beginning).

3.1. Case I: Fulfilling pressure constraint.

We tested the new functionality with a water value to $66\frac{NOK}{MWh}$. The electricity prices for different hours of simulation are shown on the upper left of Figure 2. In this testing process, 'Full model' with MIP is compared with 'Incremental model' without MIP. In full model with MIP the integer variables are used for the entire simulation period. In the first three simulation interactions, we use the full model with MIP and then we switch to the incremental model in the three last iterations. The graph to the upper middle of Figure 2 illustrates the objective values in the simulation iterations. A significant decrease can be observed in the objective values of the incremental model with respect to full model with MIP. A part from the slight increase from the first iteration to the second iteration, the objective value stays stable for entire optimization period and no flip-flop effect has been observed [4].

Minimum pressure height at the top of the penstock tunnel is set at $479m$. It is a crucial requirement for this simulation that the pressure height always lays above the minimum level. As shown on the upper right in Figure 2, the incremental model drives the unit harder and the pressure height level is very close to minimum level.

Tonstad power station includes five production units. The first four units are relatively small with the installed capacity of $160MW$ where the fifth unit is doubled size of the other four units with the installed capacity of $320MW$.

The illustration to the lower left of Figure 2 represents the production at every hour. The production pattern follows the electricity price variation in the market shown in the upper left of Figure 2. During the hours where the electricity prices in the market are fairly lower than the reservoir water value there is no production, i.e., in the first five hours and between hour 11 and 18.

[4] A flip-flop in the successive LP method occurs, when the solver jumps back and forth between two (or more) false optima from iteration to iteration due to ill-conditioned modelling.

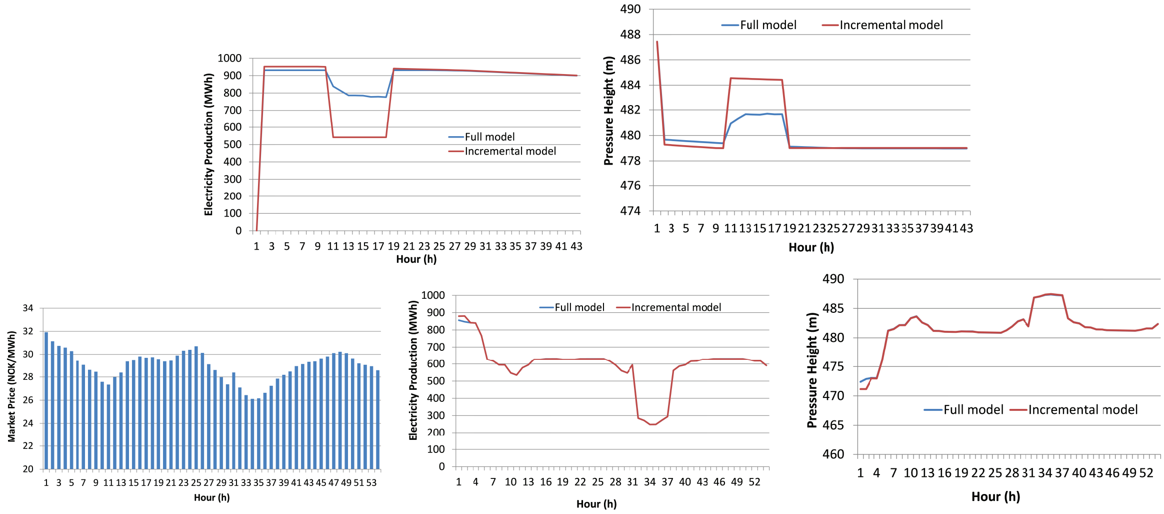

Fig. 3. Upper left: Production pattern- low water value case. Upper right:The pressure height at the pressure gauge point- low water value case.The benchmark junction case. Lower left: Electricity prices in the market. Lower middle: Production pattern at Tonstad power station. Lower right: The pressure height at the pressure gauge point.

The illustration to the lower middle in Figure 2 shows the difference in detailed production pattern of these two models, e.g. (Incremental model - the Full model). The operating sequential pattern recommends that first the small units are dispatched before the large unit will be committed.

The efficiency curves of the turbines at each production unit in Tonstad power station is depicted on the lower right in Figure 2. The efficiency curves plot the turbine efficiency versus the hydro discharge in m^3/s. As shown the first four units have almost identical turbine efficiency curves, whereas the fifth turbine efficiency is significantly different from the others.

3.2. Case II: High production.

In this case, we set the reservoir water value relatively low compared with the electricity price in the market, e.g. $20\frac{NOK}{MWh}$. This case illustrates a scenario with high production. The production pattern is depicted on the upper left in Figure 3. As shown the units are not completely dispatched to their installed capacity where the differences between the market prices and the water values are marginal, i.e. hours 11 to 18. This means that the optimization problem waits until the gap between the water values and electricity prices significantly increases in the latter hours and dispatches the production units to their maximum installed capacity. The incremental model drives the units harder in this case as well. The production in the last hours reduces because the power station is running very close to the minimum pressure height for those hours. This effect is reflected on the upper right in Figure 3.

3.3. Case III: Minimum pressure below junction.

In this section, we test the model shown on the bottom in Figure 1 including the junction between two upper reservoirs. The power station uses water from both intake upper reservoirs that joined together in the junction point. This case study covers the physical watercourses in that area and the entire upper reservoirs. In this case, the right intake tunnel (Sira river course) to the junction point is connected to two small reservoirs, e.g. Tjørhomvann and Ousdalsvann each with the reservoir capacity of $3.33Mm^3$ and $12.25Mm^3$, respectively. The right intake tunnel (Kvina river course) is connected to larger reservoir, Homstølvann with maximum capacity of $55.42Mm^3$. In the following testing procedure we focus on how the water flow through the tunnels in the junction point is influenced by the minimum pressure height $471m$ at the pressure gauge.

The illustration on the lower left in Figure 3 represents the electricity prices for different hours of simulation. We set the water values equal to $27.34\frac{NOK}{MWh}$ in all the three reservoirs, e.g. Tjørhomvann, Ousdalsvann and Homstølvann.

The lower middle in Figure 3 compares the production in Tonstad power station in both full and incremental models. As shown, apart from a very slightly difference at the beginning there is no significant difference in the production pattern of these two models. Comparing the production pattern and electricity prices reveals that since the water values are very close to electricity prices in the market, the production follows the price variations in the market. In this case study minimum pressure height at the top of the penstock tunnel is set at 471 m. The lower right of Figure 3 shows the pressure height in penstock tunnel in each simulation hour. In the beginning of the period, the unit is driven harder in incremental model and it is very close to the minimum pressure.

4. Concluding remarks & future work

The implemented functionalities resolve the above targeted challenges for modelling minimum pressure above a hydropower plant in terms of short-term production planning. With this model one is able to calculate an optimal short-term production plan for similar hydro system as the Sira-Kvina river system. In this optimal plan the decision on the ratio of discharge of Sira or Kvina to the power plant Tonstad is a part of the optimization routine. The model keeps a variable minimum pressure height above a certain level at Tonstad power station and takes into account the changing head-loss coefficient depending on the amount of inflow to the creek intake above Tonstad. Moreover, a linearisation of the pressure balance in the junction has been applied to the LP model using deltameter function. Combined, this increases feasibility and economic value of the model results.

Optimizing balancing power under minimum restriction. SHOP is used to simultaneously meet the pressure restriciton and distribute reserve obligations. A highly relevant extension of the functionality is to couple these constraints. This coupling would make sure pressure restriction is fulfilled even when the allocated reserve capacity is activated. As the optimization problem is already computationally intensive, care must be taken to find a coupling that still yields acceptable solution times. If the pressure behind capacity issue is solved, an even further step would be to look at the concept of water behind capacity. These are among the issues that will increase their impact if a larger amount of balancing power is to be provided by hydropower.

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