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Eklunds & Vasatryck, Sthlm 1959

PRINCIPLES OF POWER BALANCE CALCULATIONS FOR ECONOMIC PLANNING AND OPERATION OF INTEGRATED POWER SYSTEMS

By *Kjell Darin, Yngve Larsson, Carl-Erik Lind, Jan-Erik Ryman,* and *Bertil Sjölander*[1]

The purpose of power balance calculations is twofold, viz., first, to determine an adequate capacity to be installed when planning an extension of a power system, and second, to ensure an optimum economical utilization of an existing system. In both cases, a careful study of the utilization of the long-term storage reservoirs is necessary. This problem has been dealt with by Mr. S Stage in the Publication No. 464 of the Swedish Water Power Association. The same subject is further developed in what follows, with special reference to appropriate co-ordination of hydro-electric power and various types of thermal power.

1. Hydrological Statistical Data

11. Use of Hydrological Statistical Data for Power Balance Calculations

All calculations of draw-down schedules for storage reservoirs, etc., are based on extensive hydrological statistical data. It is therefore important that these statistical data should from the very outset be collected and presented in a practical and readily accessible manner.

In order to establish a relation with future calculations of short-term regulation, and with a view to securing an adequate accuracy in calculations, while keeping their extent within reasonable limits, it is recommended that weekly values of the discharge should serve as a point of departure for all power balance calculations. The term "week", as used in this connection, should not be referred to the calendar weeks. It should preferably be related to a division of the year into seven-day periods reckoned from the first day of the

[1] Prepared in collaboration with Torsten Bergström and Eimer Häggström.

year. New Year's Eve, and possibly also February 29th, may be disregarded without any appreciable inconvenience, and then a year will consist of a whole number of seven-day periods. The weekly values (seven-day values) of the discharge or the inflow can appropriately be expressed in terms of the total number of daily units per week.

Complete flow records for most watercourses in Sweden have been available since 1925. It is to be recommended that the calculations which can be utilized for future comparisons between different undertakings should for the present be based on the flow records covering the period of consecutive water years from 1925 to 1955. This period is subsequently extended every fifth year. Thus, the next period will comprise the years 1925 to 1960. In the calculations to be used by the undertakings for their own needs, longer periods of record should also be used, if possible.

Hydrological statistical data should represent the actual values of the discharge or the inflow, and should not be subjected to any manual operations for the purpose of cutting off values above a certain definite utilized discharge, or the like. Such operations should be performed in connection with the analysis of statistical data. Then the value of the discharge to be utilized can easily be varied or the data can be used as a reference for estimating the rate of flow in a neighbouring watercourse, etc.

Since most calculations can conveniently be made by means of data processing machines, the primary data should be recorded on punched cards, punched tape, or magnetic tape. Unfortunately, a great variety of sizes and types of punched cards and tape are in use at the present time. In some cases, this will probably cause difficulties in the interchange of information between undertakings which employ different makes of data processing machines. In order to reduce these difficulties as far as possible, it is recommended that uniform layouts of punched cards and punched tape should be used in accordance with the forms proposed in what follows.

12. Punched Card Form for Draw-Down Schedules and Power Balance Analyses

For each period of time, e.g. a day or a week, primary data, such as a water level indicated by a water level gauge, a discharge value, or an inflow value, should be recorded on a punched card, together

with the data required for identifying the time and place of observation, as well as other necessary data, e.g. a mark showing whether timber floating is assumed to take place at the time in question. Furthermore, blank spaces should be provided on the card for punching the results obtained in a computer, e.g. the requisite amount of storage, the amount of water drawn from the reservoir, etc.

In view of the design of punched-card data processing machines, it is of importance that the cards should be punched in conformity with a uniform system. This implies that the same record should always be entered in the same space on the card, in terms of the same number of digits, and with the same position of the decimal point. For checking purposes, manual punching is usually performed twice by different operators.

Fig. 1 exemplifies the form for a 40-column punched card designed for calculation of storage and draw-down on the basis of time and inflow data. This card is of the so-called seven-day type.

There are several punched-card systems, in which the number of columns varies from 21 to 130. Actually, however, each individual user has to choose this or those types of cards which his data processing machines are designed to handle.

Special converting machines are available for transferring all the data recorded, or only part of them, from punched cards to punched tape.

13. Punched Tape Form for Hydrological Statistical Records

Each punched tape should be used for recording observed values of one quantity only, e.g. the water level as indicated by a given water level gauge, the discharge at a given section, the useful inflow upstream of a given section, etc.

The observed values should be punched in chronological order so as to form an uninterrupted sequence, and should be separated only by the code symbol which is required in order that they may be distinguished by the data processing machine. All values should be reduced to the same number of decimals. In that case, it is not necessary to punch the decimal point or the corresponding code symbol. The number of decimals should be indicated at the beginning of the tape.

In order that the punching of data and the reading of input by the data processing machine may be verified, a check sum should

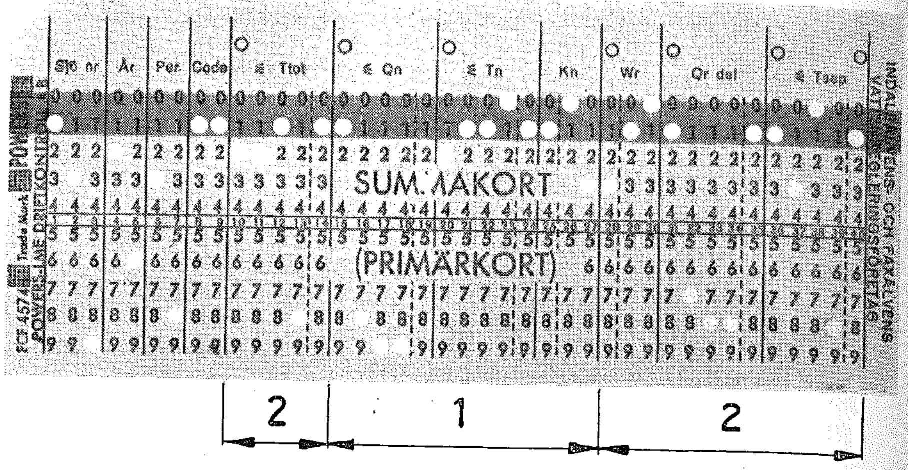


Fig. 1. Punched card designed for calculation of storage and draw-down.

1 Data obtained from manual computations or by means of data processing machines.

2 Results obtained with the aid of data processing machines.

Explanation of Column Headings

Period nr	The number of the week (seven-day period). The week No. 1 is reckoned from January 1st to January 7th.
Code	The first column indicates by various numerical symbols the period of timber floating, the period of ice covering, and other periods characterized by special draw-down conditions. The second column indicates whether the water stored in the reservoir is contained in the secondary zone, in the primary zone, or in the minimum zone, or else at the maximum or minimum permitted storage level. This indication is obtained with the help of data processing machines.
ΣT_{tot}	The total inflow, in daily units per week, obtained by summation of ΣT_{sep} for the regulated lake in question and for the regulated lakes situated upstream during the same period of time.
ΣQ_n	The natural rate of flow, in daily units per week, obtained by summation of 7 daily mean values, in cu.m. per sec.
ΣT_n	The natural inflow, in daily units per week, obtained by summation of 7 daily mean values, in cu.m. per sec.
K_n	A factor expressing the relation between the average values of the load during the week and during the year.
W_r	Regulated water level, in cm., above a certain definite zero level.
$O_{r\ def}$	Regulated discharge from the storage reservoir, in daily units per week.
ΣT_{sep}	The separate inflow, in daily units per week, calculated as the difference between ΣT_n for the lake in question and ΣQ_n for the nearest lake or lakes situated upstream.

be punched at regular intervals, and should be followed by the code symbol which enables the machine to identify this sum. After each check sum, a short interspace should be inserted on the tape, so that the tape may readily be pulled back to the extent of a check sum length in case of error in the reading of input by the machine. If the tape is used for recording daily values, then the check sum should be punched at the end of each month. If the entries are weekly values, then the check sum should be punched at the end of each year.

2. Method for Determination of Basic Rule Curve

To begin with, we shall recapitulate some of the concepts introduced by S Stage. In this connection, we shall confine ourselves to the simple case where we have to deal with a single storage reservoir and, at its outlet, a hydro-electric station which, together with a relatively small thermal station, has to carry a given load.

In order to ensure that this power system shall be operated so as to afford adequate safety against power deficiency during periods of water shortage, and so as to give optimum economic results at the same time, the storage reservoir is divided into two zones, viz., a *primary zone* or firm zone and a *minimum zone,* separated by the basic rule curve. When the water level in the reservoir is within the primary zone, the discharge drawn from the reservoir is determined by the rate of flow required in order that the hydro-electric station may be able to supply the whole load, with the deduction of the peak loads, if any, which shall be met by the thermal station. This discharge is termed *primary discharge* in what follows. When the water level in the reservoir is within the minimum zone, the thermal station is operated at full capacity, and the discharge drawn from the reservoir is confined to the rate of flow which is required for dealing with the rest of the load. This discharge is called *minimum discharge* in the present publication. The extent of the minimum zone is determined by the requirement that a certain definite degree of security shall be ensured in fulfilling the obligations imposed by the contracts for supply of power.

This section deals with the determination of that minimum zone for long-term storage reservoirs which corresponds to a given available thermal plant capacity in an existing or planned hydro-electric system carrying a given load. It is possible to obtain better economic results if one or several zones in which only cheaper thermal power, if any, shall be utilized are superposed on the minimum zone, but this question will be discussed in connection with the water value in Chapter 4. An optimum capacity of generating plant can be found by examining the costs involved in the alternative schemes under consideration. Each of these schemes shall comprise a minimum zone which is determined by means of the method described in this section, and which is applied to a period of consecutive water years.

21. General Considerations on Determination of Basic Rule Curve

The calculation of the extent of the minimum zone, i.e. that volume of water contained in the storage reservoirs at which the operation of thermal plants shall set in, is a statistical problem. This amount of storage must be determined on the basis of the inflow conditions during a large number of years so as to arrive at a low value of the probability that the reservoirs will be emptied before the end of the period of water shortage and that it will therefore be necessary to resort to power rationing. It should be pointed out that it is impossible to guard against power rationing so as to eliminate this risk altogether. All that we can do in this respect is to keep its probability low.

Since the inflow is subject to seasonal variations, the basic rule curve is a function of the season. Under the conditions prevailing in North Sweden, for instance, the storage reservoirs must be maintained more or less filled during the summer and in the early autumn, so as to minimize thermal power production. After that, the basic rule curve slopes downwards, with the result that the storage reservoirs may be allowed to be very nearly emptied towards the end of April and early in May, see *fig. 2.*

The determination of the basic rule curve at a given time can be illustrated in a simple manner by examining how the volume of water contained in a storage reservoir would vary during a number of consecutive years if the discharge drawn from the reservoir were always equal to the minimum discharge from that time on. *Fig. 3* shows as an example the family of curves which was obtained in a certain definite case by calculating the draw-down after November 15th. In order to meet the demand during all the years in the course of the 30-year period under consideration, it is necessary to resort to minimum discharge if the amount of storage at this time, i.e. on November 15th, decreases to a value correspond-

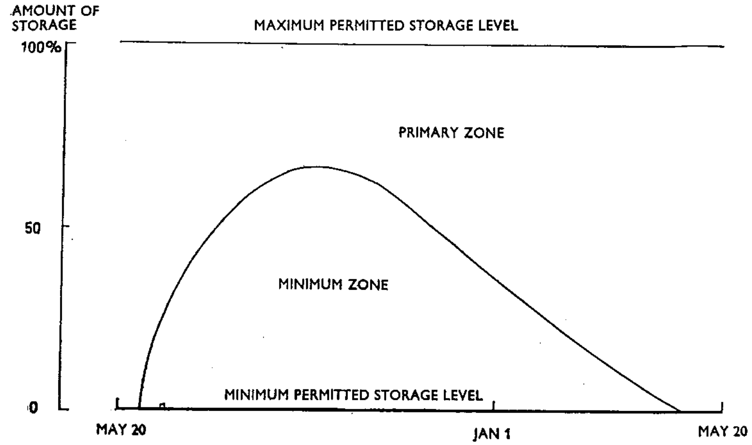


Fig. 2. Example of basic rule curve.

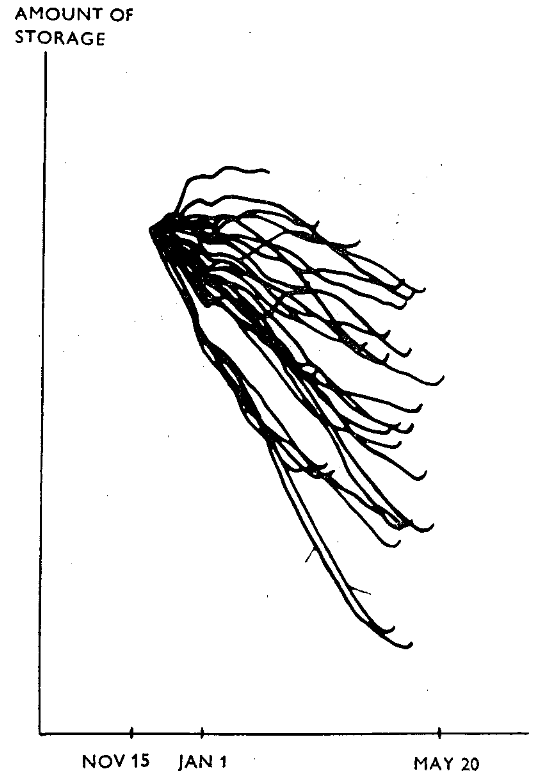


Fig. 3. Development of storage after November 15th in a calculated example.

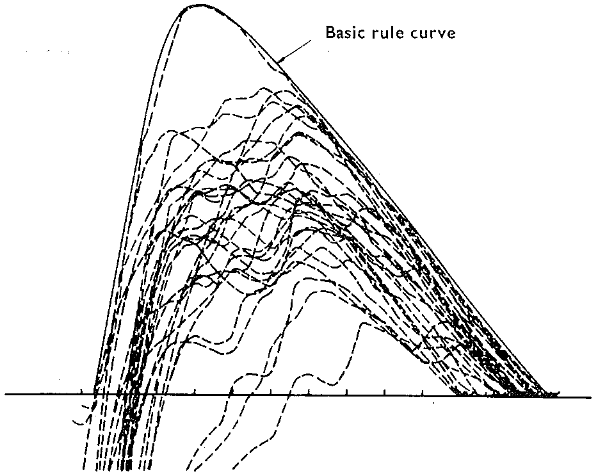


Fig. 4. The filling curves of a storage reservoir taken from fig. 3 and displaced in a vertical direction so that their minimum points lie on the minimum permitted storage level.

ing to the draw-down represented by that curve which descends to the lowest level.

This family of curves can be used to study the whole minimum zone if the curves are displaced in a vertical direction so that the minimum points lie on a horizontal line, and if the curves are extended backwards in time, see *fig. 4.* It is now the uppermost curves that represent the most difficult years. If the basic rule curve is equal to the envelope of these curves, then we can cope with the situation during all the years comprised in the period under consideration. However, this does not imply that we can also meet the demand in the course of all other years, and we know rather little about the probability that we shall be able to deal with the load. Nevertheless, by studying the whole family of curves, and by taking account of the manner in which it thins off in an upward direction, we can obtain fairly reliable results in estimating the basic rule curve corresponding to a requisite degree of availability of supply.

If it is desired to avoid subjective judgements concerning the availability of supply, then the draw-down that would be obtained before the spring flood at a continuous minimum discharge must be

studied statistically at each given time. For reasons which cannot be discussed at length in this connection, the distribution function of the above-mentioned draw-down may be assumed to be in conformity with the normal or Gaussian distribution. This assumption can in some measure be verified if the values of draw-down observed during various years are represented in the form of a distribution curve plotted on so-called normal distribution graph paper, which transforms the normal distribution into a straight line. An example of such a curve is shown in *fig. 5.* If the number of points is small, then comparatively large deviations can be tolerated, particularly at the outer edges of the graph, without justifying the abandonment of the hypothesis made in the above, cf. fig. 5. The basic rule curve corresponding to a requisite availability of supply can be read directly from graphs plotted on this paper.

An even more reliable value can be obtained if estimated parameters in the statistical distribution function of the draw-down are calculated from n observed values of the draw-down x_i. Then we get the mean value

$$\bar{x} = \frac{1}{n} \sum_{i-1}^{n} x_i$$

and the standard deviation

$$s = \sqrt{\frac{1}{n-1} \sum_{i-1}^{n} (x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1} \left(\sum x_i^2 - \frac{(\Sigma x_i)^2}{n} \right)}$$

After that, the extent of the basic rule curve is calculated by means of the t-distribution, which also takes account of the uncertainty in $\bar{x}$ and s. If it is required that the degree of availability of supply shall be p per cent, then the extent of the basic rule curve in an upward direction is given by

$$x_p = \bar{x} + t_p \cdot s \sqrt{1 + \frac{1}{n}}$$

where t_p is the p-per-cent fractile in the t-distribution having $f = n - 1$ degrees of freedom. This distribution has been tabulated, e.g. in A Hald: Statistical Tables and Formulas, New York 1952, p. 39. The basic rule curve calculated in this way is also somewhat uncertain, particularly in the region situated about

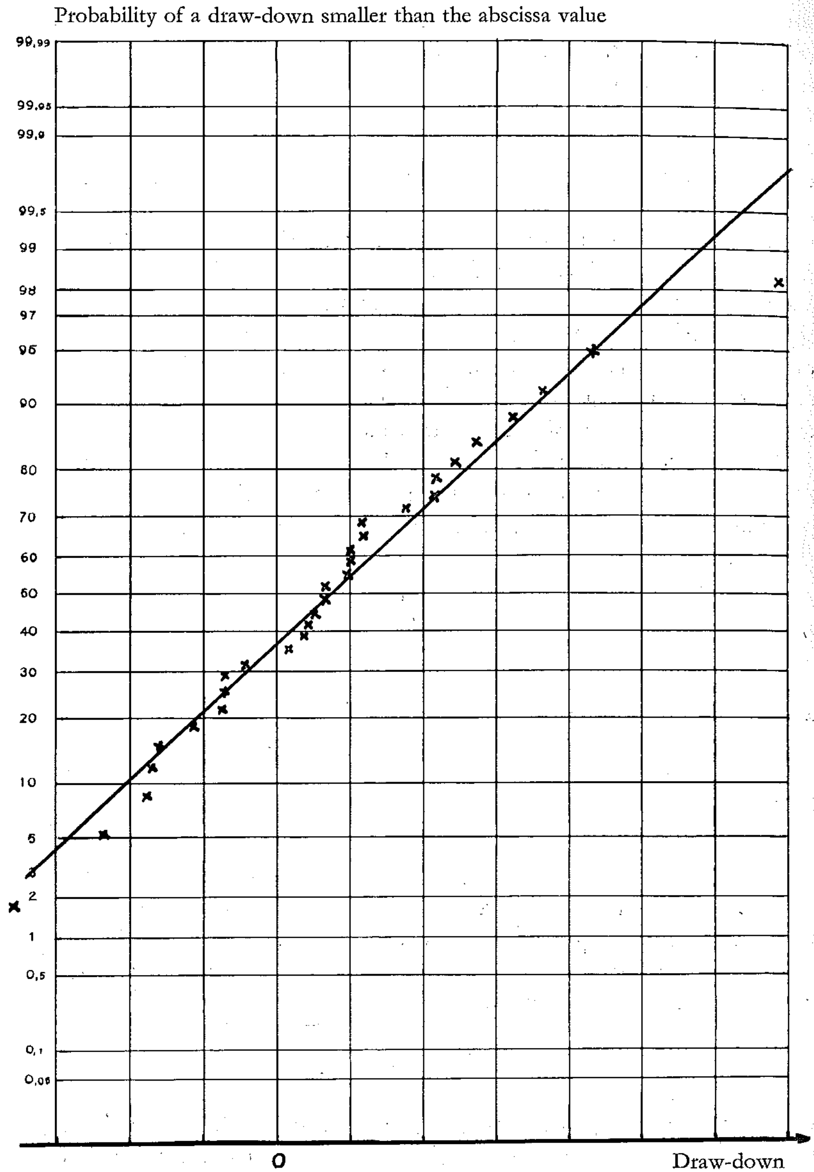


Fig. 5. Distribution curve of draw-down after a certain definite date. The horizontal axis represents the draw-down before the spring flood in daily units (DU). The vertical axis represents the probability that the draw-down will be smaller than the value indicated on the axis of abscissae.

one year earlier than the spring flood, but this is nevertheless the best result that can be obtained from the available statistical data. Besides, the accuracy in this case may be expected to be more than adequate. It is to be remembered that it is not required to maintain a given availability of supply with an accuracy of 0.01 per cent; it is sufficient if the accuracy is of the order of 1 or perhaps even 2 per cent.

During the period in the immediate neighbourhood of the spring flood, where some of the curves are missing, we have to deal with a reduced statistical material which is more difficult to handle in calculations. But since the dispersion in this case is very slight, it ought to be possible to content oneself with direct estimation.

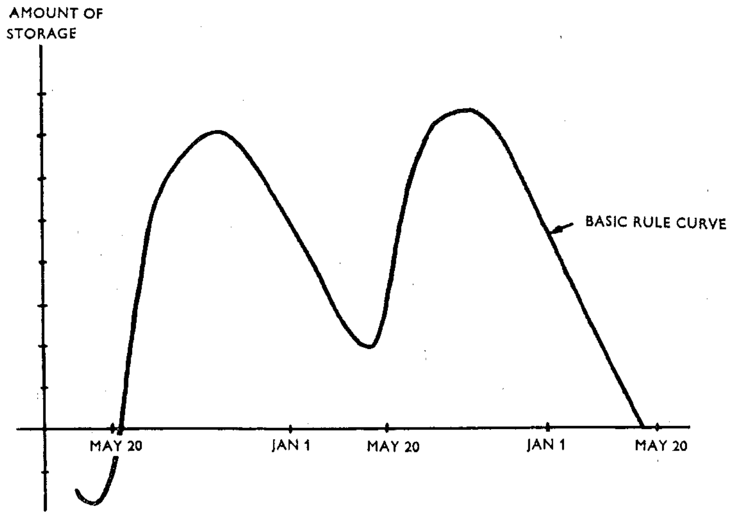
If it is found that the minimum zone extends over the maximum permitted storage level, then it is not possible to keep the availability of supply on the desired level.

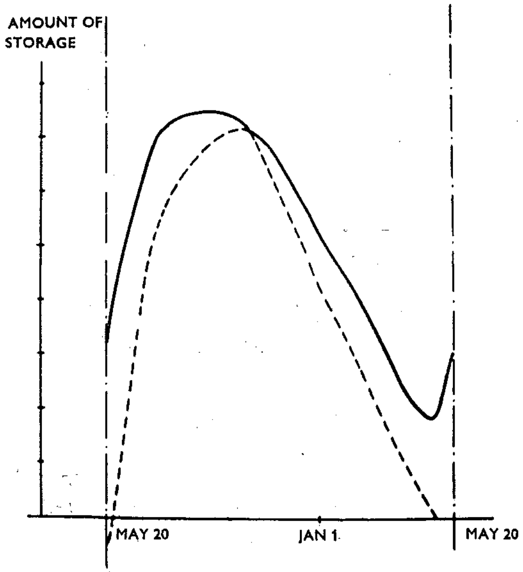
In the management of the water resources available in an actual hydro-electric system, the uncertainty in the load prediction can also be taken into account by the aid of the calculation described in the above. For this purpose, we calculate the mean value $\bar{x}$ of the draw-down and the dispersion s_1 on the basis of a mean load forecast. If the uncertainty in the load prediction is expressed in terms of the dispersion s_2 about the mean load forecast for the period extending to the normal time of spring flood, then the resultant dispersion in the draw-down is

$$s = \sqrt{s_1^2 + s_2^2}$$

which is substituted in the expression for x_p. The risk of breakdown in the thermal power stations can be taken into account in a similar manner.

In principle, the method outlined in the above can be applied not only to yearly regulation, but also to long-term regulation extending over a period of several years. If the available thermal plant capacity is small, and if the basic rule curve is calculated by means of the statistical method, then we can obtain results of the type exemplified in *fig. 6*. The point at which the basic rule curve reaches zero lies two years backwards in time. Consequently, "two-year regulation" is required in the present example. But in order to determine the discharge, we cannot actually say that this year is the year 1, and in the next year we shall draw





Figs. 6 and 7. Example of basic rule curve applied to long-term regulation (in this case "two-year regulation").

on the storage as if that year were the year 2. In fact, we have to superpose the curves corresponding to these two years, see *fig. 7*. It is then seen that the deciding factor during the period extending to October 15th is the risk of emptying the storage reservoir in the

course of the present season. After this date, it is the risk of emptying the storage during the next season that is decisive, i.e. the probability that the year will be unsatisfactory as a whole. The basic rule curve never reaches zero.

22. Approximate Methods of Calculation for Hydro-Electric Systems

In order to form a general estimate of the characteristics of the various river basins when studying the utilization of the total resources of an undertaking or an integrated power system, we can begin by examining some individual regulation schemes separately, while taking into account a certain definite portion of the load and of the available thermal plant capacity. After that, we should proceed to a study of the entire system as an integral whole, which it forms in actual fact. However, even if the computations are to be made by means of data processing machines, it is virtually impossible to start directly a perfectly correct calculation. Accordingly, we have to begin by choosing an appropriate degree of approximation. It is scarcely practicable to give any general instructions for such approximations, and they have therefore to be adapted to each individual case.

To illustrate the methods of calculation, a few examples of actual calculations are adduced in what follows.

221. *Example of Calculation of Basic Rule Curve for Single Storage Reservoir and Single Hydro-Electric Station*

This description deals with a case where it was required to determine the basic rule curve for a single storage reservoir situated immediately upstream of an isolated hydro-electric station which shall meet a given load alone but with the assistance of a certain definite amount of thermal power. No inflow was available between the hydro-electric station and the storage reservoir, and the water level in the storage reservoir did not influence the head on the power station. The calculation was made in conformity with the principles stated in Section 21. In this case, the calculation was carried out by means of a data processing machine, but the same method can also be used for manual computations. As a first approximation, only yearly regulation was dealt with in the calculation.

The basic data employed for the calculation comprised, first, the inflow values covering a period of 35 consecutive years, and second, the week-to-week values of the minimum discharge required for the hydro-electric station during a year. The inflow values, which were obtained from the observed values of the water level in, and the rate of flow from, the lake, were summed up in seven-day periods. The period No. 1 began on January 1st every year. The minimum discharge to be drawn from the storage reservoir is the rate of flow which is required, together with maximum thermal power generation, in order to produce the power needed for meeting the firm load. The week-to-week variation in the minimum

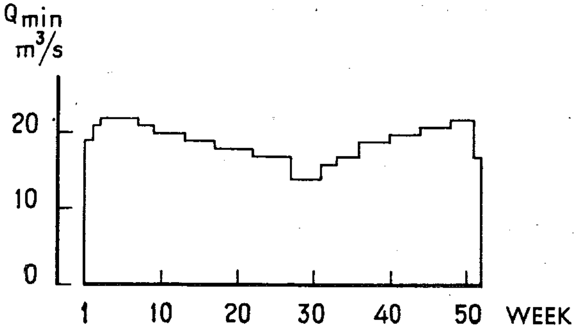


Fig. 8. Week-to-week variation in the minimum discharge from a storage reservoir during a year.

discharge during a year is shown in *fig. 8*. The minimum discharge was expressed in daily units of water per week. As has been demonstrated in Section 21, the amount of storage which is required at the end of a given week v_n of a given year is equal to the stored discharge with the deduction of the stored inflow during the time interval from the week v_n to the time when the storage is reversed, that is, the time when the inflow becomes greater than the minimum discharge,

$$M_{v_n} = \sum_{v_v}^{v_n} (Q_{min} - T)$$

where

M_{v_n} = the amount of storage required during the week v_n,

Q_{min} = the minimum discharge,

T = the natural inflow,

v_v = the week during which the storage is reversed.

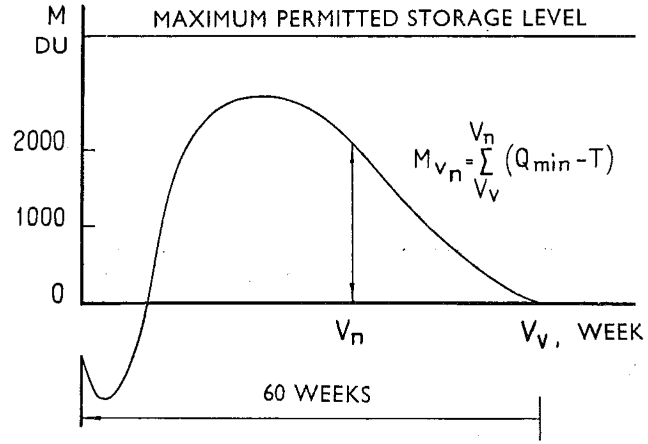
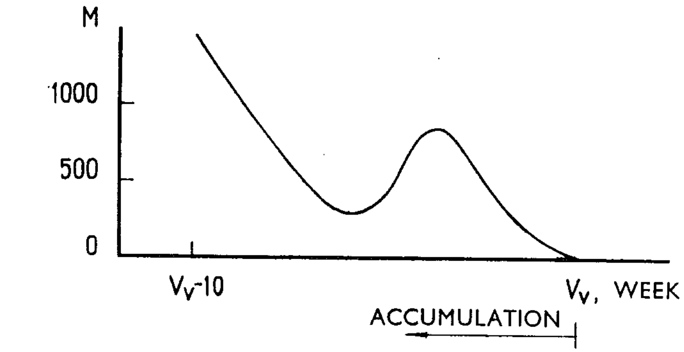
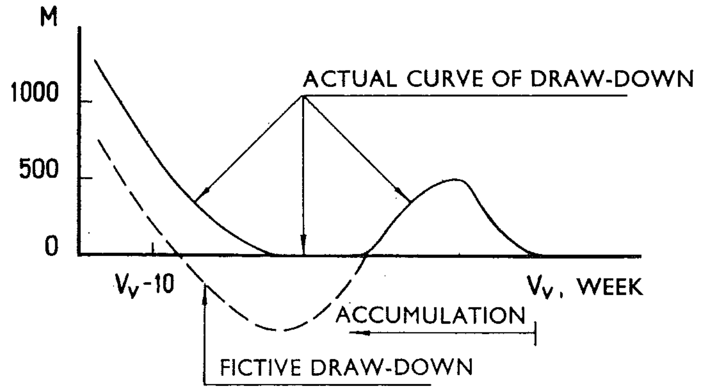


Fig. 9. The amount of storage required in the course of an individual year.

The amount of storage M_{v_n} was calculated from week to week during all 35 years by storing slightly more than one year backwards in time from the week in which the storage was reversed at the beginning of the spring flood, see *fig. 9*. During certain years, the conditions can be complicated by the circumstance that the storage is reversed several times because the spring flood "breaks down", see *figs. 10* and *11*. If the storage is reversed several times but does not become negative, cf. fig. 10, then the storage proceeds as usual. On the other hand, if the storage becomes negative, cf. fig. 11, then this implies that the storage reservoir may be allowed to be empty at the time in question, without causing any shortage of water during this year up to the spring flood time. It is therefore necessary to restore the reservoir to zero again, to find the next minimum point backwards in time, and to resume storage once more by starting from the zero amount of storage.

After that, the 35 different values of the requisite amount of storage which were obtained during a given week v_n — one value for each year — were laid off along a vertical axis, see *fig. 12*. The highest of these values represents that amount of storage which is required in order that shortage may not occur before the spring flood during any of the years comprised in the *period under investigation*, on condition that a minimum discharge is maintained from that time on. As is seen from Section 21, more adequate infor-





Figs. 10 and 11. Curves representing the amount of storage during years in which a tendency to spring flood was followed by a period of lower inflow before the spring flood proper.

mation can be furnished by a statistical analysis of the values. The mean value of, and the variance in, these values were determined so as to calculate the amount of storage which was required in order to ensure that the probability of shortage before the spring flood should not exceed a certain level, in this case 5 per cent. This calculation was made by means of the formula

$$M_{req} = \overline{x} + t_p \cdot s_s \sqrt{1 + \frac{1}{n}}$$

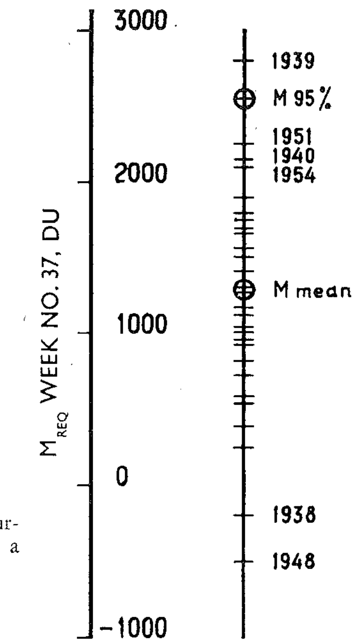


Fig. 12. The amount of storage required during a certain definite week in the course of a period of 30 years.

where

$\overline{x}$ = the mean value of all values,
s_s = the variance,
n = the number of values, in this case 35,
t_p = a constant, which is equal to 1.69 for a probability of 95 per cent,
f = 34 in accordance with the tables of the t-distribution.

This statistical calculation can be carried out for each week in the course of the year. After that, all 95 per cent points are joined together by a curve which represents the basic rule curve. However, in order to determine this curve, it is found to be fully sufficient to calculate the "95 per cent storage" for every fourth or fifth week. The results of this calculation are shown in *fig. 13*. It is obvious that the values relating to the individual years need not be plotted in the way shown in fig. 12, but if use is made of data processing

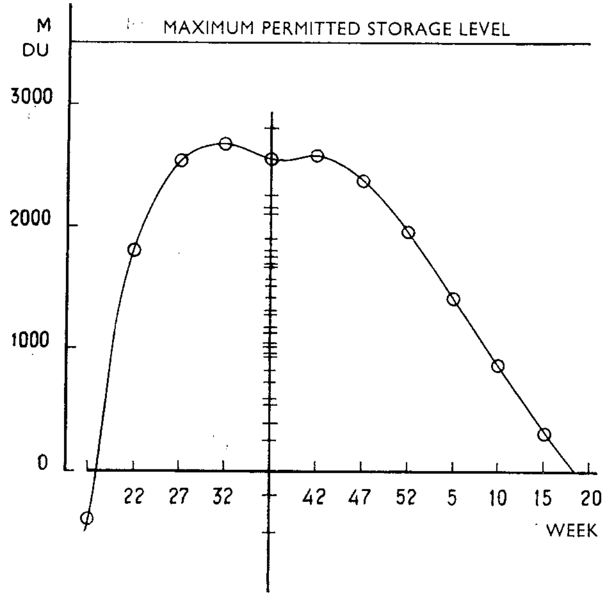


Fig. 13. Basic rule curve obtained when the probability that the amount of storage will last out a period of time extending to the spring flood is assumed to be 95 per cent.

machines, then it may be desirable to check the results from time to time in the same manner as in the case of the week No. 37 in fig. 13.

The minimum zone determined by the aid of the method outlined in the above must comply with two requirements, viz., first, that it shall not intersect the maximum permitted storage level, and second, that its ascending branch shall intersect the minimum permitted storage level. If it intersects the maximum permitted storage level, then this implies that the requisite amount of storage is not available. If it does not intersect the minimum permitted storage level, then that volume of water which is required in the course of the draw-down period cannot be guaranteed with the desired degree of security during the filling period. Accordingly, it is necessary to resort to long-term regulation. In that case, the storage must be run two or possibly several years backwards in time. Then we obtain a basic rule curve of the type shown in fig. 6, which is dealt with in accordance with Section 21 and fig. 7.

222. *Example of Calculation of Basic Rule Curve for Hydro-Electric System Utilizing Both Unregulated and Regulated River Flow. Two Dominant Storage Reservoirs Approximately Equal in Degree of Regulation*

This calculation deals with a hydro-electric system which comprises power stations on several rivers in South and North Sweden. Some of these rivers are wholly unregulated. Others are regulated in a slight degree or so that the possibilities of free disposal of water are restricted within narrow limits. However, the major part of the power generated in this system originates from two well-regulated rivers. These rivers are approximately equal in degree of regulation, and their catchment areas are similar in character. The amount of unregulated inflow to both these rivers between the storage reservoir and the hydro-electric station is quite considerable.

The generation of hydro-electric power in this system has been classified as follows:

a. Generation of power originating from the unregulated rivers and from the comparatively small regulated rivers (in which the flow is regulated in conformity with the respective normal schedules).

b. Generation of power originating from the two large rivers owing to intermediate inflow and prescribed compulsory discharge from the storage reservoirs.

c. Generation of power due to discharge from the storage reservoirs in excess of the compulsory discharge in the two large rivers.

There is nothing to be done about the generation of power in Groups a and b, and it can be calculated directly from the rate of flow. For the existing power stations or groups of power stations, this has been done on the basis of weekly values observed in practical operation, with the result that the effect of short-term regulation was automatically taken into account. For those power stations which are not yet in service, the corresponding relations have been estimated with the guidance of the available information about the possibilities of short-term regulation. These relations were represented by approximate expressions of the type

$$W = a \cdot Q - b \cdot Q^2 \qquad \text{for } Q \leq Q_{max}$$

$$W = a \cdot Q_{max} - b \cdot Q_{max}^2 \qquad \text{for } Q > Q_{max}$$

where

W = the electrical energy generated, in MkWh per week,

Q = the rate of flow, in cu.m./s.,

Q_{max} = the development rate of flow.

In many cases, however, it is possible to content oneself with a linear relation for $0 < Q < Q_{max}$.

The storage reservoirs in Group c are dealt with as if they formed a single reservoir, and this reservoir is divided into a primary zone and a minimum zone, with the intention that the discharge from the reservoir shall conform to the schedule described in what follows. When the total volume of water contained in the reservoir is comprised in the primary zone, the discharge drawn from the reservoir in excess of the compulsory discharge (for log floating, etc.) shall be so great that the corresponding amount of power generated in the hydro-electric stations on the two rivers in question, together with the power generated in Groups a and b, is sufficient to supply the whole load on the system. When the water stored in the reservoir is contained in the minimum zone, the discharge from the reservoir shall be so great that the corresponding amount of power, together with the power originating from Groups a and b as well as the power generated in the thermal plants operating at full capacity, is sufficient to meet the whole load on the system.

The basic rule curve is calculated just as in the preceding example on the basis of the week-to-week variation in the amount of storage and on the assumption that the continuous discharge from the reservoir is equal to that which would be drawn if the storage were in the minimum zone. Accordingly, we have to calculate from week to week a change in the amount of storage which is equal to

the inflow minus the compulsory discharge expressed in terms of electrical energy

plus the power generated in Group a

plus the power generated in Group b

plus the power generated in the thermal plants operating at full capacity

minus the load.

If the sum of the last four addends is found to be positive, then it must be put equal to zero, for an increase in the amount of storage can never be greater than the inflow (except in the case of pumped storage). Consequently, it is not necessary to utilize the full capacity of the thermal stations under these circumstances.

For expressing the inflow in terms of electrical energy, use can be made of an energy equivalent of water which corresponds to the amount of energy that can on an average be generated by utilizing the water drawn from the storage reservoirs when the water stored in the reservoirs is contained in the minimum zone. Now this energy equivalent of water cannot be calculated before the basic rule curve has been determined, and has been applied to a period of consecutive water years in a power balance. It is therefore necessary to begin with an estimate, which is then successively corrected. As a rule, a single correction will be found to be sufficient.

After the changes in the amount of storage have been computed in this way for each week, the calculation can proceed in the same manner as in the foregoing example, see Section 221.

23. Some Extensions of Methods of Calculation Described in Section 21

The present section deals with some possible improvements in the methods for calculating the basic rule curve.

231. *Storage Reservoirs Involving Difficulties in Discharge*

The method of calculating the extent of the basic rule curve on the basis of "draw-down curves placed on the minimum permitted storage level", which has been described in Section 21, presupposes that the discharge from the storage reservoirs does not present any difficulties. If such difficulties are encountered, then it is necessary to make a special calculation for some weeks in the neighbourhood of each spring flood so as to determine the minimum amount of storage which is required in order to ensure that the outflow of the minimum discharge shall be possible. This question is closely bound up with the management of storage reservoirs which are operated in parallel, and will be discussed at some length in Chapter 5, Section 51.

232. *Calculations Taking Account of Storage Reservoirs in Unregulated Lakes*

The method for calculating the basic rule curve described in the above does not take account of the "trends" in the inflow. Certain attempts have been made to use regression analysis in order to determine the basic rule curve as a function of the existing inflow, but these attempts did not prove very successful. A better approach seems to be to assume that the unregulated lakes are included in the water contained in the storage reservoirs. In that case, small lakes of this kind are represented by some reference lake.

In order to introduce this factor into the calculations, e. g. those described in Section 222, use is made of the following procedure. First, the change in the amount of storage in the regulated lakes is computed from week to week. Then the actual changes in the amount of storage in the unregulated lakes are added to these values. After that, the actual water levels in the unregulated lakes are taken into account in calculating the basic rule curve during the weeks immediately preceding the spring flood. In principle, this is done by the aid of the method outlined in Chapter 5, Section 51.

This procedure should also be employed in dealing with the relatively small regulated watercourses which were reckoned among unregulated rivers and therefore included in Group a, Section 222. Then the errors caused by this approximation will for the most part be eliminated.

233. *Hydro-Electric Systems Which Comprise Several Storage Reservoirs Differing in Degree of Regulation*

If the discharge drawn from the storage reservoirs can be managed in such a way that a reservoir never overflows unless the other reservoirs are filled, then it is undoubtedly possible to use the method of calculation described in Section 222. As a rule, this is the case when storage reservoirs lying close to each other do not differ too widely in the degree of regulation. However, this method of calculation is also correct when the storage reservoirs differ in the degree of regulation so long as none of these reservoirs has a degree of regulation which is so low that the reservoir overflows even in the course of the most unfavourable years. It is true that some of the water which would in reality be spilled in years of

plentiful run-off will then be included in the calculations. But this is solely an advantage because the statistical distribution of the draw-down is in this way prevented from being subjected to truncation, which would cause difficulties in the calculation of its "tail" represented by the dry years.

234. *Hydro-Electric Systems Which Comprise Some Storage Reservoirs Having Low Degrees of Regulation*

If a hydro-electric system includes some storage reservoir or reservoirs having such a low degree of regulation that they overflow even during the driest years, then these reservoirs must be specially taken into account. One method that can be used for this purpose is outlined in what follows. First, a simple preliminary draw-down schedule, which leads to large discharges during the spring, and possibly also in the summer, is constructed for each of these reservoirs. Then the operation is adjusted with a view to obtaining an empty reservoir at the time of early spring flood. After that, the watercourse in question is dealt with as if it were unregulated, and the amount of storage is taken into account in the same manner as in Section 232.

If no account is otherwise taken of the water stored in unregulated lakes, then this procedure involves quite a considerable complication. In that case, the method described below seems to be more convenient.

In principle, the problem to be solved can be stated as follows: for the storage reservoirs having a low degree of regulation, the rise in the filling curve during the spring flood must be restricted to that value which corresponds to the volume of storage. In so far as the minimum load exceeds the amount of power generated from unregulated sources, if any, in the system, this excess load should be met by means of the discharge drawn from these storage reservoirs having low degrees of regulation to the extent that is permitted by the flow utilized in the power stations situated downstream. If the system comprises several storage reservoirs of this kind, then this discharge should be distributed among them so that these reservoirs may as far as possible be integrated in conformity with the principles of drawing on storage reservoirs which are operated in parallel.

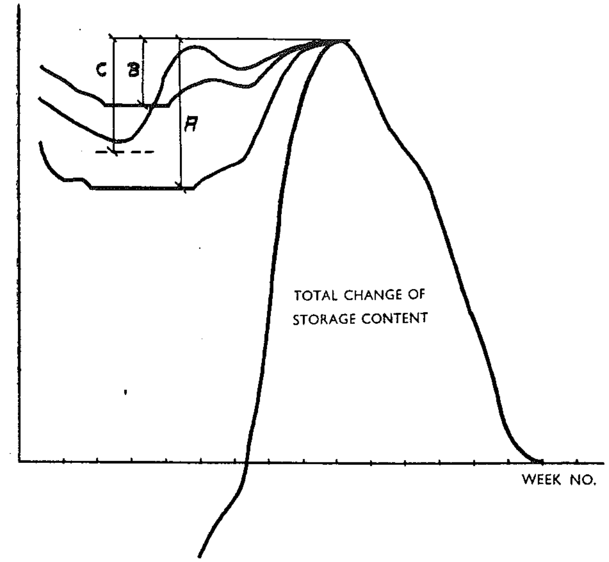
In carrying out the calculations, use can, for instance, be made of the following practical procedure. To begin with, as usual, the minimum load with the deduction of the inflow for all storage reservoirs taken together is stored backwards in time for each water year until we reach the maximum point of the filling curve. From that point on, the calculation is continued — still backwards in time — in accordance with the instructions given below.

For each one of the storage reservoirs having low degrees of regulation, it is necessary to observe the amount by which the filling curve bends downwards (seen in a backward direction). When the filling curve of any one of the storage reservoirs has reached down to a point which corresponds to the volume of storage, then this reservoir is to be regarded as non-existent, that is to say, the inflow is dealt with as if it were an intermediate inflow downstream of the reservoir in question, see *fig. 14 a.* The discharge which may possibly be drawn from these storage reservoirs is distributed so as to cause the filling curves to "run together" when they bend downwards. Most frequently, however, on account of the water required for timber floating, this discharge matters little or nothing. It is therefore fully sufficient to base the calculations on the discharge drawn from all storage reservoirs in turn according to the ratio of the deflection of the filling curve and some quantity which characterizes the magnitude of the spring flood, e.g. the mean inflow during the spring months or simply the annual mean inflow.

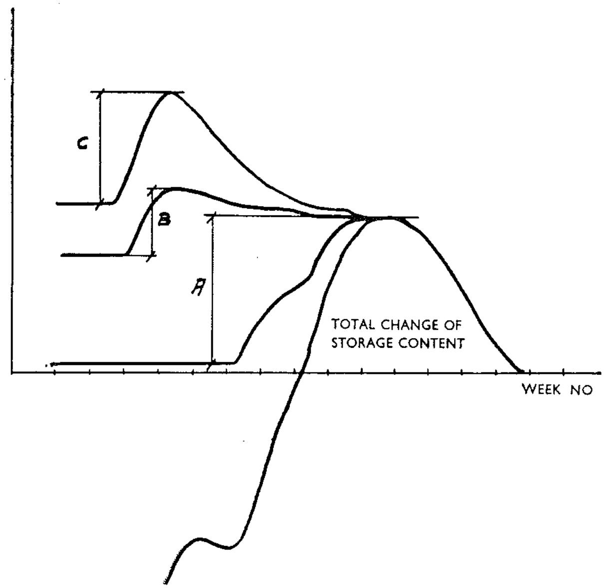
If the total filling curve reaches its maximum point late in the autumn, then the separate filling curve calculated by means of the above method for a storage reservoir from which much water is drawn for timber floating may sometimes bend upwards instead of downwards. In that case, the permissible deflection will have to be reckoned later on from the highest value which has been reached by the curve, see *fig. 14 b.*

3. Availability of Supply and Its Economic Aspects

As has been pointed out in a previous section of this report, the utilization of the long-term storage reservoirs of hydro-electric stations yields the best economic results if the start of those thermal power plants which involve the highest runnig costs is delayed until



Figs. 14a and b. Determination of the total change in the amount of storage in a power system which comprises some storage reservoirs having low degrees of regulation. (A, B and C are volumes for three storage reservoirs with low degrees of regulation.)



the lowering of the storage reservoirs reaches the basic rule curve. Now we have to distinguish between two types of thermal power. First, thermal power is called "homogeneous" if it entails a single value of the running costs. Second, thermal power is called "stratified" if it is generated in several plants which differ in running costs. In the general case, the situation of the basic rule curve is independent of whether thermal power is homogeneous or stratified. If thermal power is stratified, then the various thermal plants should pick up the load at the points determined by the passage over the limiting curves which constitute the boundaries of the respective water values. The calculation of water values is dealt with in Chapter 4.

In accordance with Chapter 2, the basic rule curve can be determined so as to ensure that the volume of water contained in the storage reservoirs, with the addition of a certain definite minimum inflow, in combination with full generation of thermal power during the remaining part of the draw-down period, shall be sufficient to secure a certain fixed value of the probability that the power system will be able to cope with a given load. Thus, for instance, if the basic rule curve has been calculated for 95 per cent, then the probability that the supply of electrical energy undertaken by contract can be effected without resorting to rationing of power is exactly equal to 95 per cent when the basic rule curve has been reached. The above-mentioned probability can serve as a measure of the *"availability of supply"* (S). This expression is preferable to the concept "risk of rationing" (R). ($R = 100 - S$ per cent.)

As has already been mentioned, a 100 per cent availability of supply in a predominantly hydro-electric power system cannot be maintained in practice. Experiences relating to earlier years of heavy drought have shown that the availability of supply in Sweden is on an average perhaps 90 to 95 per cent.

The first question that arises in this connection is what value of the availability of supply should be chosen, that is to say, what should be the basic rule curve. *Fig. 15* represents these curves as functions of the availability of supply in an arbitrary hydro-electric power system, which is supplemented with a definite but arbitrary amount of thermal power.

As is seen from fig. 15, the availability of supply corresponding

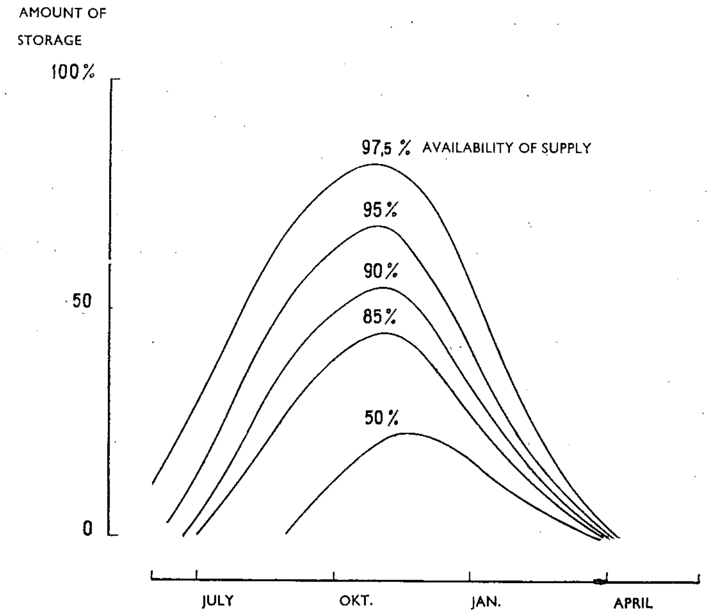


Fig. 15. Variation in the basic rule curve with the availability of supply.

to the curve which is tangent to the maximum permitted storage level is the *highest guaranteed availability of supply* that can be maintained in this power system, inclusive of supplementary thermal power. It is important that each electricity undertaking or power company should ascertain its highest guaranteed availability of supply.

As may furthermore be inferred from fig. 15, since the basic rule curve increases as the availability of supply becomes higher, a greater amount of energy must be supplied to the system from the thermal power stations, and since it was assumed here that the capacity of the thermal power stations was given, it follows that a higher availability of supply corresponds to a greater value of the total production costs in the power system. This can also be interpreted in a different manner by stating that the basic rule curve is kept unchanged, but its availability is augmented by increasing the total capacity of the thermal power plants.

31. Optimum Availability of Supply

From the point of view of over-all economy, the availability of supply should be maintained on such a level as to ensure that the cost of energy generated in the thermal power stations shall offset the costs which commercial and industrial undertakings as well as the public would have to bear on account of the power rationing that would otherwise be required. It is obvious that the inconveniences which are caused by power rationing to commercial and industrial undertakings or to the public are very difficult to evaluate in terms of money. It seems probable that moderate rationing of power can be achieved without involving any real cost simply owing to the increased tendency to economize which always manifests itself in a crisis. However, in order that the above-mentioned inconveniences might be estimated in an example, the sum of the cost of thermal power generation and the cost of energy eliminated by rationing has been calculated as a function of the guaranteed availability of supply for an ordinary power undertaking. In this calculation, it was assumed that power rationing is carried out so as to result in the least possible amount of "unnecessary rationing", i.e. that no power rationing is resorted to until the storage reservoirs have been definitively emptied. (This assumption is much too idealized, but the procedure in question can be used as a first approximation.) The value of energy eliminated by rationing was varied within the limits from 0 to 40 Swedish öre[1] per kWh on an average for the whole amount of energy eliminated by rationing. The results of this calculation are represented in principle in *figs. 16* and *17*. Fig. 16 shows the total annual cost of thermal power, with the addition of the cost of energy eliminated by rationing, and with the deduction of the revenue derived from surplus energy as a function of the capacity of the thermal power plants for several values of the availability of supply on the assumption that the energy eliminated by rationing is valued on an average at 20 Swedish öre per kWh. This diagram is used for determining that capacity of the thermal power plants which corresponds to a minimum total cost in the power system for each value of the availability of supply. The results obtained in this way are represented in fig. 17 for 20 Swedish öre per kWh as well as for the other assumed values of the cost of energy eliminated by rationing.

[1] 1 Swedish öre = 1/100 Swedish krona ~ 0.2 d ~ 2 US mills.

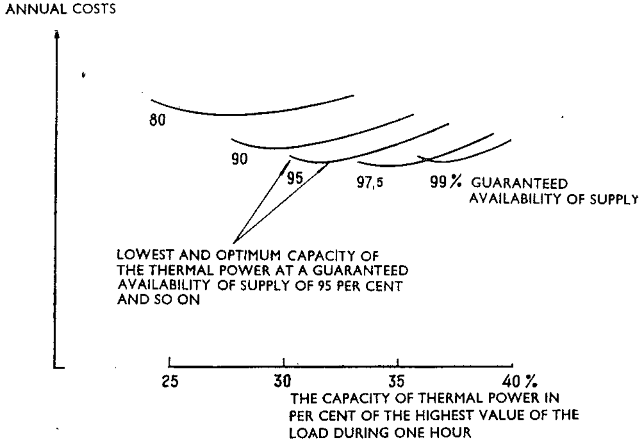


Fig. 16. The total annual cost of thermal power, with the addition of the cost of energy eliminated by rationing, and with the deduction of the revenue derived from surplus energy, as a function of the capacity of the thermal power plants for several values of the availability of supply.

Example. Thermal power: 75 Swedish kronor per kW per year + 5 Swedish öre per kWh.
Surplus energy: 0.5 Swedish öre per kWh.
Energy eliminated by rationing: 20 Swedish öre per kWh.

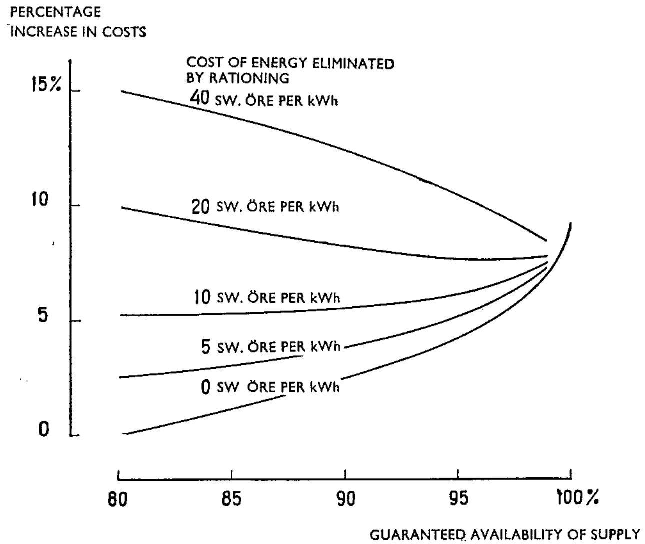


Fig. 17. The sum of the cost of power and the value of the energy eliminated by rationing represented as a function of the guaranteed availability of supply by using the price of energy eliminated by rationing as a parameter. This diagram is based on the assumption that the capacity of the thermal power stations in the power system is an optimum which is comprised in the range from 25 to 35 per cent of the one-hour maximum demand of the load.

The curve relating to the case where the energy eliminated by rationing is valued at 0 Swedish öre per kWh represents the percentual increase in cost which the electricity undertaking must incur in order to raise the availability of supply from 80 per cent to higher values extending up to 100 per cent. To indicate the order of magnitude of this increase in cost caused to the electricity undertaking, it may be mentioned that the cost increment associated with an increase in the availability of supply from 90 to about 98 per cent amounts to 4 or 5 per cent in the undertaking in question. If the value of energy eliminated by rationing is supposed to be as moderate as slightly over 10 Swedish öre per kWh on an average, then the "national economic cost" is practically constant irrespective of whether the availability of supply is as low as 80 per cent or is raised to about 98 or 99 per cent. If the energy eliminated by rationing is valued at a higher cost, then it is perfectly obvious that society as a whole will gain by maintaining the availability of supply on a level of 98 to 99 per cent. (In fig. 17, the optimum capacity of the thermal power stations, expressed in per cent of the one-hour maximum demand of the load, is comprised in the range from 25 to 35 per cent.)

A similar calculation has been carried out in another power undertaking, where the share contributed by thermal power is different, and where this share is divided in a different way between "low-cost" and "high-cost" thermal power. The results of this calculation are shown in *fig. 18,* where the curve represents the increase in the cost of power production as a function of the availability of supply, i.e. on the assumption that the value of energy eliminated by rationing is equal to 0 Swedish öre per kWh. In this case, the relative increase in the cost of power production is still smaller than in the preceding example. The increase in cost is only about 2 to 3 per cent when the availability of supply is raised from 90 to 98 per cent.

As a final result of these two investigations, it appears quite evident that, when the capacity of the generating plants is appropriately proportioned, the increase in costs incurred by the electricity undertakings is very moderate so long as the guaranteed availability of supply is not raised above a level of 98 to 99 per cent. If an attempt is made to vary the value attributed to the energy eliminated by rationing, then, as this value increases, it becomes even more

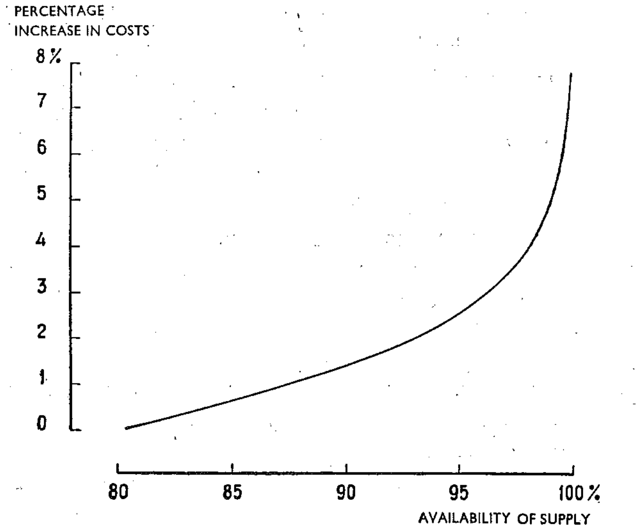


Fig. 18. Increase in the cost of power production as a function of the availability of supply.

obvious that the availability of supply in power systems should lie somewhere in the range from 95 to 99 per cent.

At the present time, the actual state of affairs in Sweden seems to be characterized by the fact that most electricity undertakings plan and operate their power systems more or less consciously so as to ensure an availability of supply (as defined in this report) ranging from 95 to 99 per cent, and run their thermal power plants in accordance with their expectations concerning optimum economic results.

32. Recommended Factor of Availability of Supply

In Section 31, it was found that the basic rule curve should be determined so as to ensure an availability of supply ranging from 95 to 99 per cent. Now we assume that the curve has been based on, say, $S = 95$ per cent. In the course of a certain definite water year, the water situation can for instance have been such that the minimum zone was not reached during the period extending from the spring flood to January inclusive. This means that no thermal power plants have been operated up to this time during the water year in question. Suppose, furthermore, that a spell of severe cold weather occurs in January and February, the rate of inflow decreases, the load increases above the level predicted by normal calculations, and the draw-down extends into the minimum zone. The thermal power plants are run to full capacity, but power has perhaps nevertheless to be rationed towards the end of March. It may be regarded as obvious that it would be difficult for the electricity undertakings in such a situation to excuse themselves in the eyes of industrial consumers and the public. On the other hand, if the minimum zone had been entered as early as in the autumn, and if the thermal power plants had been operated to full capacity during the whole winter, then the reaction against the same rationing would have been much weaker.

A method of avoiding all this trouble is to introduce a minimum zone corresponding to a value of S which is variable in the course of the year. For instance, this can be done as follows:

The basic rule curve is based on a value of $S = 95$ per cent during the period extending from the spring flood to about November 1st, which is the normal time of ice formation in the watercourses situated in the northern regions of Sweden. Starting from this date, the basic rule curve is drawn so that the curve corresponding to $S = 97.5$ per cent is definitively reached about January 1st. The latter curve is followed up to about March 1st. After this date, it is advisable to change to a curve which constitutes an ample envelope of all summation curves, see *fig. 19*.

The electricity undertakings which do not use any thermal power must also calculate their basic rule curve in this manner, and must check it so as to make sure that it does not reach the maximum permitted storage level. In such undertakings, the fact that the storage passes into the minimum zone implies merely that all supply of "secondary" power must be discontinued.

33. Availability of Supply and Rationing Conditions below Basic Rule Curve

As has been indicated in the above, the choice of power rationing method is dependent on the total costs as a function of the amount of energy to be eliminated by rationing and the time at which rationing is to be introduced. If we make the simplified assumption that the amount of energy to be eliminated by rationing should on an average be as small as possible, then the power system shall be

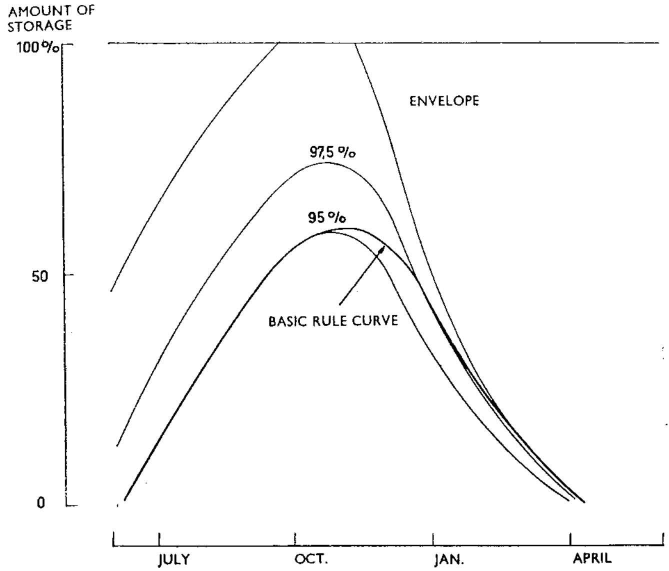


Fig. 19. Effect of the variation in the availability of supply on the situation of the basic rule curve.

operated in such a manner as to ensure that no rationing will be resorted to until the storage reservoirs have been completely emptied. If the basic rule curve is determined in conformity with the principles which have previously been outlined in the present report, then this implies that the power supply to industrial consumers will perhaps have to be interrupted for a period of about one week in 1 to 5 years per 100 water years. However, it is not a priori certain that this programme is to be regarded as an optimum for the country as a whole.

Alternative rationing procedures can be illustrated by means of *fig. 20*. This diagram shows a simplified example, in which it was decided to introduce the first stage of rationing when the instantaneous availability of supply, S_{inst}, has decreased to 40 per cent. In this case, the amount of energy to be eliminated by rationing is determined so as to increase S_{inst} to an extent at which the probability that further stages of rationing can be avoided is equal to 60 per cent, and so forth. Rationing can of course be resorted to at higher or lower values of the instantaneous availability of supply than

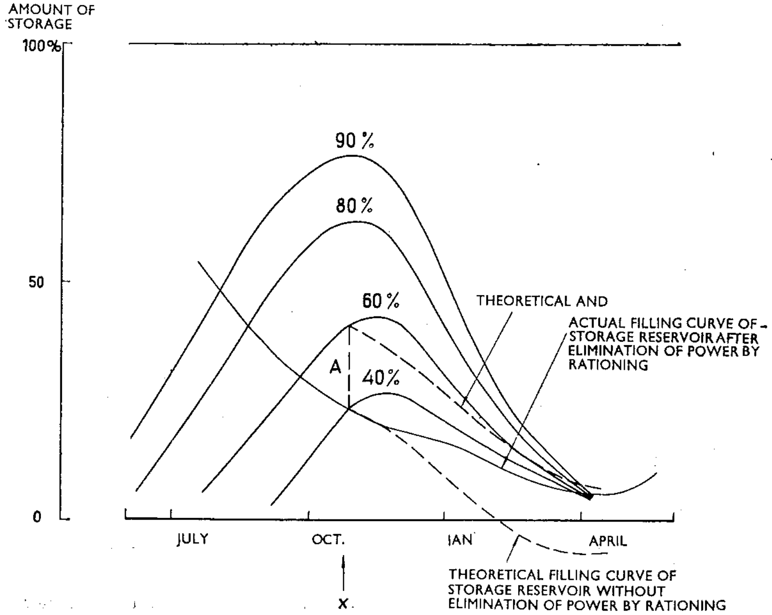


Fig. 20. Effect of power rationing on the filling curve of a storage reservoir. A reduction of the load by power rationing is carried out at "x".

that which is exemplified in the above. If rationing is introduced at an earlier date, then the amount of energy that must be eliminated by rationing can be small, and the cost that is incurred by society as a whole in abstaining from energy consumption is perhaps very low at that time, but there is a certain probability that the available power would have proved to be adequate even if it had not been rationed. On the other hand, if the probability that no further stages of rationing will be necessary after the first stage of rationing is required to be very great, then the amount of energy to be eliminated by rationing at an early date must be slightly larger. Assume, for instance, that we have to deal with a minimum zone having the extent shown in fig. 19, and that rationing is contemplated about January 1st. Suppose, furthermore, that the probability that no further stages of rationing will be needed after the first stage is required to be equal to 97.5 per cent. In that case, the amount of energy which shall be eliminated by rationing during the period extending to the spring flood is equal to the distance from the

amount of storage at the instant under consideration to the basic rule curve, and so on.

34. Average Availability of Supply

According to the above definition, the availability of supply S is determined as the probability that the volume of water available at the basic rule curve, together with the supplementary energy from the thermal power plants operated to full capacity, will be sufficient to supply the demand during the year in question. If, at the chosen value of S, the basic rule curve is not situated so that it reaches the maximum permitted storage level — because the power supplier has found for engineering and economic reasons that an optimum is reached when the capacity of the thermal power plants in the system is greater than that which corresponds to a minimum at a given availability of supply — then it is evident that the draw-down will not extend into the minimum zone every year. How often the minimum zone will have to be utilized is a question which depends on the composition of thermal power as well as on the make-up and the amount of "secondary" power supply, i.e. power exceeding contracted firm supply. For instance, thermal power can be generated in nuclear power plants, back-pressure power plants, conventional condensing steam power plants, gas turbine power plants, etc.

If the above-mentioned factors are taken into account, then practical system operation implies that those thermal power plants which entail the lowest running costs are started before the basic rule curve is reached. (For further particulars, see Chapter 4.) This partial start of the thermal power plants prior to the utilization of the minimum zone means that the availability of supply is temporarily higher than that which corresponds to the basic rule curve. It is possible to calculate an average availability of supply, which must obviously always be higher than the guaranteed availability of supply, because an early start of thermal power stations increases, just as a lowering of the storage reservoirs for the purpose of secondary power supply decreases, the temporary availability of supply, and hence also its average value.

This average availability of supply, which can be calculated theoretically, is difficult to compute in practice, since the effects produced by certain factors, especially by the "secondary" power supply, are dependent on the entire integrated power system, and can therefore scarcely be calculated in advance by each separate power undertaking. Furthermore, it is probable that in an integrated power system, which offers reasonable possibilities of power exchange between the interconnected undertakings, the storage in each individual undertaking will reach the minimum zone at approximately the same time. Consequently, it is desirable that all power undertakings should calculate their respective minimum zones on the basis of the same principles and the same guaranteed availability of supply.

4. Methods of Determining Water Value

Chapters 2 and 3 dealt primarily with the question how the available power resources should be managed in order that it may be possible to cope with the load in a situation which is met with during a dry year. On the other hand, if the storage conditions are more satisfactory, then the question to be solved is whether it may nevertheless be profitable to save water by utilizing thermal power generated in those plants which entail the lowest running costs (e.g. nuclear power stations and back-pressure plants), or whether it is perhaps more economical to increase the volume of water drawn from the storage reservoirs for occasional power supply. This question arises not only in the operation of existing power systems, but also, and not least, in power system planning. For instance, this question is of paramount importance in studying the possible utilization of nuclear power.

In each given situation, a power system should be operated with due regard to the probability of reaching the minimum zone (which involves the highest cost of thermal power) or the maximum permitted storage level (which means spillage). These probabilities determine an expected incremental value of the water such as it is stored in the reservoirs, the *"water value"*. If we start from this water value, as well as from the incremental efficiencies of the hydro-electric stations, and from the possibilities of short-term regulation, then we have a basis for calculating an absolute value of the incremental cost of generated electrical energy (the price of the last kilowatt-hour), which shall be compared, on the one hand, with the ruling price of power for occasional supply, and on the other hand, with the running cost of thermal power.

The water value in a given power system can be determined by

means of several methods. The choice of method will be dependent on the requisite accuracy in calculations and on the possibilities of representing the actual power system by an appropriate approximation. Some of these methods are exemplified in what follows.

41. Single Storage Reservoir, Homogeneous Thermal Power, Small Secondary Power Supply

We consider a power system which can approximately be represented by assuming, first, that it comprises a single storage reservoir, second, that the thermal power is homogeneous, i.e. entails a single value of the running costs, and third, that the amounts of secondary power to be supplied are small. In this case, the simplest method of calculating the water value is to start from the time under consideration and from the volume of water stored at that time, and then to investigate how the amount of storage would develop if the primary discharge continued to be drawn from the storage reservoir during different water years. If p years, out of m years, principally give rise to operation in the minimum zone, while the remaining years mostly involve spillage, then the water value can be put equal to

$$\varkappa \approx \frac{p}{m} \cdot \varkappa_s$$

where $\varkappa_s$ is the water value at the basic rule curve, and is equal to the cost of full-capacity thermal power generation divided by the corresponding reduction in the draw-down of the storage reservoirs on the assumption of normal short-term regulation. The water value can conveniently be expressed in Swedish öre per kWh of natural energy.

It may be questioned whether it is correct to use the whole water value $\varkappa_s$ for those years which bring about operation in the minimum zone. In fact, the problem under discussion is to determine the consequences of a slight change in the draw-down at the point of departure. It is therefore conceivable that the filling curve of the storage reservoir is lowered by a small amount at that point. This implies that the filling curve will reach the minimum zone at an earlier time than would otherwise be the case. In other words, the thermal power plants will be started in advance of the usual time.

The amount of this advance, and hence the increase in thermal power generation, will be dependent on the angle at which the filling curve intersects the basic rule curve. The increase in thermal power generation is frequently smaller than that which corresponds to the initial lowering of the filling curve; a certain residual lowering is produced when the filling curves passes through the basic rule curve. Part of this residual lowering can in the same way cause an increase in thermal power generation as the filling curve leaves the minimum zone, and so forth. Finally, there will perhaps remain a part of this lowering which corresponds to a reduction in spillage. Consequently, this reasoning would justify a reduction of the water value.

However, what may be objected to this reasoning is that the residual lowering of the filling curve during its passage through the basic rule curve implies a decrease in the availability of supply. One method of evaluating this effect is to reckon with the thermal power generation which would be needed in order to raise the filling curve, in spite of the fact that this is impracticable on account of the limited thermal plant capacity. In other words, the whole lowering should be evaluated in terms of thermal power if the filling curve passes into the minimum zone.

By investigating the accuracy in the calculation of the value of $\varkappa$ with the help of the method outlined in the above, it is found that this accuracy is rather low if the calculation does not cover more than 30 water years. A better result can be obtained by calculating a power balance on the basis of several values of the volume of water stored in the reservoir at the time in question, and by plotting a smoothed curve which represents the cost of supplementary power as a function of the amount of storage. The first derivative of this curve gives the water value.

42. Single Storage Reservoir, Homogeneous Thermal Power, Large Secondary Power Supply

If, in addition to the small quantities of secondary power on account of which it is required to calculate the water value, the power system has to supply an amount of secondary power which is so large that it appreciably influences the filling curve of the storage reservoir, then it is necessary to introduce a special secondary zone which shall be located in the uppermost part of the storage diagram. When the water stored in the reservoir is comprised in this zone, then the draw-down shall always be so great that it is also sufficient to cover the secondary power to be supplied.

The boundary between the primary and secondary zones may be calculated by means of successive approximations. A certain definite boundary is assumed, and then the economic result is calculated by applying the draw-down schedule to a series of water years. After that, the boundary is successively displaced by a small amount at a time, until we arrive at that zone which yields the best economic result.

The water value can now be calculated by using the same procedure as in Section 41. In the secondary zone, the filling curves are calculated on the assumption that the above-mentioned large amount of secondary power is to be supplied, but all curves are followed up to the point where spillage sets in or to the point where the thermal plants have to be started. In this case, it is not justifiable to pay any special regard to the angle at which the filling curve intersects the boundary of the secondary zone, since this boundary has been chosen as an optimum, and it is therefore quite possible that a slight displacement of the boundary in question is of no great importance.

In principle, the procedure described in the above may possibly also be used in the case where the power system comprises two (but scarcely more than two) types of thermal power plants, e.g. nuclear power stations and conventional steam power stations.

43. Single Storage Reservoir, Stratified Thermal Power, Large Secondary Power Supply in Various Price Ranges

In the present case, we have to resort to a draw-down, q, which may conveniently be regarded in general as a function of the water value, $\varkappa$, and, in view of the seasonal variations in load, also as a function of the time of the year, t, that is to say,

$$q = q\ (t, \varkappa)$$

If the effect of the short-term regulation is disregarded, then it is readily seen that q is a stepped function whose steps correspond to the prices of thermal power and secondary power in the various blocks or "strata" as well as to the thicknesses of these strata. If the short-term regulation is taken into account, then this function becomes nearly continuous. In fact, at certain definite water values, it may be profitable to operate the thermal plants in a given stratum during the day-time only, and sometimes it may be economical to sell secondary power in the night-time only, etc. In most cases, it ought to be possible to content oneself with a relatively simple expression for the function $q\ (t, \varkappa\ )$.

In this case, the water value cannot be determined separately at each point of time and each amount of storage, since the development of storage in the future will be dependent on the water values which will be passed, and these water values are unknown at the outset. Instead, we shall regard the water value, $\varkappa$, generally as a function of the time of the year, t, and the volume of water stored in the reservoir, M, that is to say,

$$\varkappa = \varkappa\ (t, M)$$

To begin with, it is only known that this function is an annual periodic function, that its value at the maximum permitted storage level is equal to zero, and that its value at the basic rule curve is $\varkappa_S$, which corresponds to the stratum where the cost of thermal power is highest.

It is possible to deduce a general functional relation between the functions $\varkappa$ at different points of time, but a numerical solution based on this relation seems to be very intricate. It is therefore recommended that numerical calculations should rather be carried out by using successive approximations and by applying the Monte Carlo Method making use of the observed values of inflow. This procedure is exemplified in what follows.

We assume a value of $\varkappa\ (t, M)$ at a number of points which are uniformly distributed in the region under consideration. We suppose now that the storage reservoir has in some way or other reached one of these points, for which it is required to determine a second approximation. We start from the point in question, and we follow the development of storage governed by the values of inflow observed during a certain definite year in the past and by the draw-down determined by the water values which are passed (these water values are calculated by interpolation in the chosen set of points). The calculation is discontinued after some twenty weeks or earlier if the storage reservoir attains the maximum permitted storage level or the basic rule curve. Then the calculation is repeated by starting from the same point again and by using the inflow

values relating to the next year, and so forth, until the whole available water year series (e.g. 30 years) has been utilized. The water value to be determined is put equal to the mean value in which the development has terminated during the various water years. As soon as a new value has been calculated, it is substituted for the old one, and is utilized in the calculation of the following points, and so on. It is advisable to go through the points backwards in time.

The character of the function $q\ (\ \varkappa\ )$ which is normally met with in practice is such that the convergence in this method is fairly rapid. Acceptable accuracy can often be obtained as early as after 3 to 4 successive approximations. This method is only approximate, and will therefore always involve a certain residual error regardless of the number of approximations. The magnitude of this error is dependent on the number and the extent of discontinuities in the function $q\ (t, \varkappa\ )$, and on some other factors.

The water value calculated in a case in hand is represented in *fig. 21* by curves of equal water value, or "equi-water-value curves", in a storage-time diagram.

It may be questioned whether it would not be possible to dispense with the minimum zone in calculations of this kind, and alternatively to put an incremental value of the water value in the case of an empty storage reservoir equal to the total cost caused by ration-

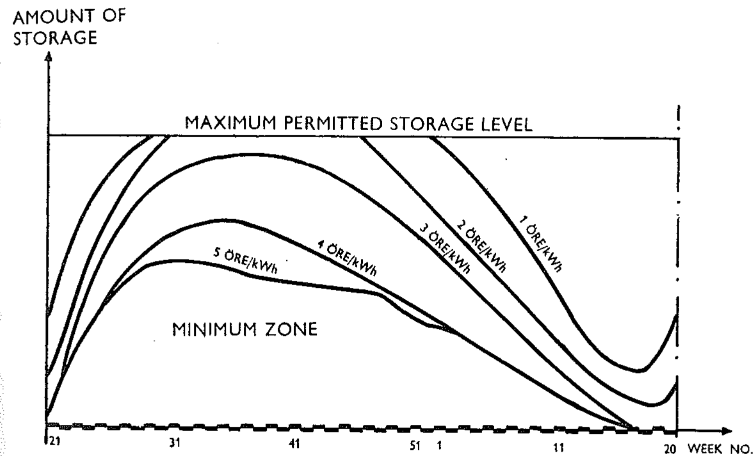


Fig. 21. Expected water value in a storage reservoir.

ing of power. However, this incremental value would be very high in comparison with the water values which are normally met with in practice, and the spacing of the points used in the calculations would therefore have to be unreasonably close in order to enable satisfactory interpolation.

44. Calculation of Water Value Taking Account of Increase in Load, etc.

In the preceding sections, it was assumed that the water value function is an annual periodic function. Strictly speaking, this is the case only if the "degree of development" does not change during the year dealt with in the calculation and during at least the next year after that. If it is desired to allow more accurately for the fact that the load increases continuously, whereas the power stations come into existence as discrete units, then use can be made of the method of calculation described in what follows. We assume that the water value is to be calculated for the year No. 1. Then, to begin with, we calculate the basic rule curve for all years ranging from the year No. 1 to, say, the year No. 4 inclusive. Next, we calculate a rough approximation of the water value for the year No. 4 in accordance with Section 41 on the assumption that the thermal power is homogeneous. After that, a slightly closer approximation for the year No. 3 can be calculated with the aid of the general method by following the development of storage in the course of a period extending into the year No. 4, while taking account of the relation between the draw-down and the water value. The calculation is carried out week by week from behind. By proceeding in this way, we can obtain a still closer approximation for the year No. 2, and finally for the year No. 1.

This supplement to Section 43 is of great importance in dealing with present-day operation of power systems, but is, as a rule, not necessary in calculations for system planning purposes.

45. Calculation of Water Value Taking Account of Inflow Trends

To begin with, the water value was assumed to be independent of the trend in the inflow. One method of making a correction for this trend is, just as was done in the calculation of the basic rule curve, to assume that the water contained in the storage reservoir includes

unregulated lakes, bogs, marshes, etc. (which are for practical reasons represented by a single or a few reference lakes). If the statistical records required for this purpose are inadequate, then the amount of storage under consideration can be corrected by adding to it a volume of water that corresponds to the deviation in the mean inflow during a few weeks to come, which is determined with the aid of regression analysis, and is represented as a function of the existing inflow.

46. Calculation of Water Value in Power System Comprising Different Storage Reservoirs

The methods described in the above are primarily applicable to the simple case where the power system is assumed to include a single watercourse and a single storage reservoir. Actually, a power system most frequently comprises several watercourses and different storage reservoirs. Strictly speaking, an individual water value is obtained for each storage reservoir. This water value is a function, not only of the water level in the storage reservoir under consideration, but also of that in all other reservoirs, and is therefore very difficult to calculate.

However, if we assume that the total amount of draw-down is distributed among the storage reservoirs so that no spillage is unnecessarily produced in any of them unless the other reservoirs are full, then the water value can approximately be expressed by a function of the time of the year and the total volume of water stored in the reservoirs. In other words, this is in principle the same assumption as that which served as a basis for the treatment of theoretical case referred to in the above. All the same, unregulated generation in the power system must specially be taken into account, but this does not present any notable difficulties.

It is obvious that the above-mentioned unnecessary spillage can never be completely avoided, but it can be limited if the total draw-down is distributed in an appropriate manner among the various storage reservoirs. This question is dealt with in Chapter 5.

5. Distribution of Draw-Down among Different Storage Reservoirs

51. Detailed Treatment of Storage Reservoirs Prior to Spring Flood, with Special Reference to Power Balance and Limitations in Discharge

One of the objects pursued in the operation of a hydro-electric power system which comprises a number of different storage reservoirs is to distribute the total amount of draw-down among them so as to ensure that the respective probabilities of spillage due to the spring and autumn floods shall be equal, or at least as nearly equal as possible, in all reservoirs. Accordingly, if the power system includes one storage reservoir having a high degree of regulation and several storage reservoirs having low degrees of regulation, then it would be desirable to have the last-mentioned reservoirs completely emptied in good time before the spring flood. All water remaining in storage, would then be concentrated in the first-mentioned reservoir, where the security against spillage during the spring flood is evidently greatest. However, if it is assumed that one or several hydro-electric stations are situated downstream of each of the storage reservoirs, then the concentration of the whole amount of water in a single storage reservoir can lead to the result that the quantity of hydro-electric energy which is available per week will be insufficient — even if the thermal power plants are operated to full capacity. The reason is that the quantity of hydro power is limited by the rated discharge through the hydro-electric stations situated downstream of the storage reservoir having a high degree of regulation. (For brevity, the rated discharge through the hydro-electric stations situated downstream will be termed "rated discharge" in what follows.) In that case, in order to maintain the power balance, particularly during the last weeks before the spring flood, it is necessary that a certain volume of water should also be kept in the storage reservoirs having lower degrees of regulation.

It may furthermore be required to keep a certain amount of water up to the spring flood in the storage reservoirs whose discharge capacity at the limit of lowering is so small that the required weekly quantity of hydro power cannot be produced.

In principle, this residual water shall be stored in those reservoirs which involve the lowest cost per MWh of electrical energy that can be generated before the spring flood. The "cost" in this case consists

of that amount of residual water which will probably be lost by spillage during the spring flood. The calculation of this cost per MWh is complicated by the fact that some storage reservoirs present limitations in discharge, and by the circumstance that several storage reservoirs can influence one another when they are situated one after another on the same river or on different tributaries of the same river. The calculation of this cost of energy is dealt with in the following sections.

This detailed control of the storage reservoirs prior to the spring flood may be said to imply that a basic rule curve in each storage reservoir is calculated for the period of time immediately preceding the spring flood.

511. *Cost of Energy. Single Storage Reservoir on River*

We consider a storage reservoir on a river which does not comprise any other storage reservoirs situated upstream. We assume that a certain definite volume, or "layer", of water, h cm. in height, is kept in this reservoir in order that it may be released during a certain definite week. In most cases, it is possible to ensure that the value of energy after the spring flood shall be equal for all storage reservoirs. Then the cost of keeping a definite layer of water in a given storage reservoir will be proportional to the probability that precisely this reservoir will overflow. Furthermore, this cost is proportional to the volume of water and to the energy equivalent of stored water for generation in downstream hydro-electric stations. Consequently, this cost can be expressed by the formula

$$K = \frac{C_1}{36} \cdot h \cdot A \cdot p \cdot s \qquad \text{Swedish kronor}$$

where C_1 = the value of energy, in Swedish kronor per MWh, in the whole power system after the spring flood,

h = the height of the boundary layer, in cm.,

A = the area of the storage reservoir, in sq.km.,

p = the energy equivalent of 3,600 cu.m. of water (1 cu.m./s. during 1 hour), in MWh for generation in hydro-electric stations,

s = the probability of overflow, in per cent.

The quantity of energy which can be generated during one week from the residual boundary layer in the storage reservoir is proportional to the volume of water and to the energy equivalent of stored water for generation in the hydro-electric stations situated downstream of the reservoir. However, this quantity of energy is limited by the rated discharges at the hydro-electric stations and possibly also by limitations in discharge from the storage reservoir. Within the limits which are determined by these factors, the utilizable quantity of energy is

$$W = \frac{100}{36} \cdot h \cdot A \cdot p \qquad \text{MWh}$$

Therefore, the cost per MWh of residual water is

$$\frac{K}{W} = C_1 \cdot s \cdot 10^{-2} \qquad \text{Swedish kronor per MWh}$$

that is to say, directly proportional to the probability of overflow of this storage reservoir.

As regards storage reservoirs presenting limitations in discharge, it is conceivable that, in order to ensure a sufficient discharge, and hence an adequate quantity of energy, during the week in question, the requisite height h of the boundary layer will be so great that the whole volume of water contained in this layer cannot be discharged in the course of this week. In that case, the expression for the cost of energy will be the same as that deduced in the above, while the utilizable quantity of energy will be limited to

$$W = 168 \cdot \triangle Q_a \cdot p \qquad \text{MWh}$$

where $\triangle Q_a$ is the increase in the mean weekly rate of flow in cu.m./s., which can be discharged.

In this case, the cost per MWh is

$$\frac{K}{W} = \frac{h \cdot A}{36 \cdot 1.68 \cdot \triangle Q_a} \cdot C_1 \cdot s \cdot 10^{-2} \quad \text{Swedish kronor per MWh}$$

($\triangle Q_a$ is limited to $\frac{h \cdot A}{36 \cdot 1.68}$ because this is the maximum mean weekly rate of flow which the boundary layer can yield.)

512. *Cost of Energy. Several Storage Reservoirs in Series on River*

We assume that the river includes two storage reservoirs, one of them being situated upstream of the other. Furthermore, we suppose

that the downstream reservoir presents limitations in discharge. In this case, it may be necessary to keep a residual boundary layer of water in the downstream reservoir in order that a sufficient quantity of water per week may be drawn from the upstream reservoir. Then the boundary layer in the downstream reservoir will remain untouched, and the water will be "transited" from the upstream reservoir through the downstream reservoir. Under these conditions, the total cost of the energy whose generation is rendered possible in this way is equal to the sum of the cost of the residual water which is kept in the downstream reservoir and the cost of the water which is drawn from the upstream reservoir. Hence we obtain the expression

$$K = \frac{C_1}{36} \cdot h_d \cdot A_d \cdot p \cdot s_d + C_1 \cdot 1.68 \cdot \triangle Q_a \cdot p \cdot s_u$$

where C_1 = the value of energy, in Swedish kronor per MWh, in the whole power system after the spring flood,

h_d = the height of the boundary layer in the downstream storage reservoir, in cm.,

A_d = the area of the downstream storage reservoir, in sq.km.,

s_d = the probability of overflow of the downstream storage reservoir, in per cent,

s_u = the probability of overflow of the upstream storage reservoir, in per cent,

$\triangle Q_a$ = the increase in the mean weekly rate of flow which can be discharged from the downstream storage reservoir,

p = the energy equivalent of stored water, in MWh per unit discharge, for generation in downstream hydro-electric stations.

The quantity of energy which can be generated per week is

$$W = 168 \cdot \triangle Q_a \cdot p \qquad \text{MWh}$$

Consequently, the cost, in Swedish kronor per MWh, is

$$\frac{K}{W} = \frac{h_d \cdot A_d}{36 \cdot 1.68 \cdot \triangle Q_a} \cdot C_1 \cdot s_d \cdot 10^{-2} + C_1 \cdot s_u \cdot 10^{-2}$$

In this case, the magnitude of $\triangle Q_a$ is limited by the margin with reference to the rated discharge.

If we have to deal with several storage reservoirs in series on the same river, and if these reservoirs do not present any limitations in discharge, then the situation does not involve any special circumstances which must be taken into account. In this case, the whole amount of residual water to be stored should simply be concentrated in that storage reservoir which is situated farthest upstream, because the probability of overflow of the water is then smallest. It is possible, however, that this rule needs to be slightly modified in the cases where power stations are situated downstream of the storage reservoirs as well as between them.

If the hydro-electric stations situated on the lower reaches of the river have a higher utilized discharge than those on the upper reaches, then it may be advantageous to keep residual water also in some intermediate storage reservoir, so that full rate of flow may be maintained at the stations on the lower reaches without giving rise to any spillage at the stations on the upper reaches. In this case, the upstream storage reservoirs and the intermediate storage reservoirs can be regarded and treated as if each of these separate groups of reservoirs were alone on the river. For the last-mentioned reservoirs, however, it is necessary to calculate the rated discharge as the difference in the utilized discharge between the downstream and upstream power stations.

513. *Cost of Energy. Several Storage Reservoirs in Parallel on River*

In this case, the various storage reservoirs can be dealt with as if they existed alone on the river. It is to be noted, however, that the sum of the amounts of water drawn from the different storage reservoirs shall not exceed the rated discharge.

514. *Calculation Procedure*

In principle, the check calculation made in order to ascertain whether the power balance can also be maintained during the weeks immediately preceding the spring flood is carried out in the same manner as the calculation of the basic rule curve. However, for reasons which have been adduced in the introduction to the present section, it is not sufficient to determine a basic rule curve for the total amount of storage during these weeks. It is necessary to calculate a basic rule curve for each one of the storage reservoirs under

consideration. The procedure to be used in these calculations is described in what follows.

For each one of the years included in the series of years under investigation, the following expression is written for each week about and immediately before the spring flood:

$$W = W_b - W_M - W_m - W_v$$

where W = the energy generated by using the water drawn from the storage reservoirs, in MWh per week,

W_b = the load, in MWh per week,

W_M = the inflow to the storage reservoirs minus the compulsory discharge from the storage reservoirs, expressed in terms of electrical energy, in MWh per week,

W_m = the inflow downstream of the storage reservoirs plus the compulsory discharge from the storage reservoirs, expressed in terms of electrical energy, in MWh per week,

W_v = the maximum amount of thermal power which can be generated in the power system, in MWh per week.

This is the same expression as that which was employed in the calculation of the basic rule curve. In the present case, however, the magnitudes of W_M and W_m must be checked for each storage reservoir and for each river. It is necessary to make sure that the magnitude of W_M does not exceed the difference between the discharge capacity and the compulsory discharge or the difference between the rated discharge and W_M.

For each one of the years under investigation, we find the latest week during which W is positive. Then it is permissible to assume that the total amount of storage is equal to zero, i.e. all reservoirs are empty at the end of this week. Accordingly, the total volume of water contained in the storage reservoirs at the beginning of this week must correspond to W MWh. Now it is required to distribute this water among the various storage reservoirs so that the total volume of water contained in the reservoirs may really be discharged in the course of the week in question. Consequently, as a first approximation, we assume that no residual water must be kept in the reservoirs during the "last" week in order to ensure a sufficient discharge in the course of this week.

The above-mentioned conditions should be satisfied at a cost that is as low as possible. Therefore, a diagram which represents the cost of energy (calculated in accordance with the preceding sections) as a function of the amount of storage, expressed in MWh, is plotted for each storage reservoir or each group of reservoirs. Furthermore, the maximum weekly quantity of energy which can be generated by using the water in each storage reservoir is represented in a diagram as a function of the amount of storage, expressed in MWh.

These diagrams are now employed in order to find out how the total amount of storage should be distributed among the storage reservoirs. To begin with, a check calculation is carried out so as to make sure that the stipulated compulsory discharge from each storage reservoir can be maintained in consideration of the inflow. Then the remaining portion of the discharge to be drawn from the storage is distributed among the reservoirs. We start with that storage reservoir which shows the lowest cost of energy at the limit of draw-down. However, from this storage reservoir, we can only obtain a quantity of energy which corresponds to the difference between the discharge capacity and the sum of the inflow to the reservoir and the compulsory discharge, or to the difference between the utilized flow and the total inflow, inclusive of the compulsory discharge. If further discharge from the reservoirs is required, then it is drawn from that storage reservoir which entails the second lowest cost of energy. This procedure is repeated by passing from one reservoir to another, until the requisite discharge from the reservoirs is ensured. If it is found, after having dealt with all storage reservoirs, that a sufficient discharge from the reservoirs has not been secured owing to the inadequate discharge capacity of the reservoirs at the limit of draw-down, then a residual amount of water must be stored in one or several of the reservoirs in order to increase the discharge capacity. For this purpose, we choose in the first place that reservoir which entails the lowest cost of obtaining a residual amount of storage. It is obvious that this analysis of the storage reservoirs yields a filling curve for each of them during the week under consideration.

After that, the same procedure is applied to the immediately preceding week, but now the calculation starts with the amounts of storage which have been computed before. The discharge which shall be distributed among the storage reservoirs during this week is given by the value of W which has been calculated for the week in question. This procedure is repeated week by week backwards in time, until it appears certain that the distribution of the total amount of storage among the various reservoirs cannot be expected to give rise to any difficulties in maintaining the power balance. Then we shall have obtained a filling curve for each one of the storage reservoirs in the course of the period immediately preceding the spring flood during the year under investigation. For the weeks before this period, it is sufficient to calculate a single filling curve which applies to the total amount of storage.

This procedure is employed in examining all the years comprised in the series of years under consideration, and the filling curves corresponding to the individual years are combined so as to obtain a basic rule curve by means of the method which has been described before. Moreover, a basic rule curve is plotted for each one of the storage reservoirs during the period immediately preceding the spring flood. The statistical records relating to the last weeks before the spring flood are very scanty—they cover only the few years during which the spring flood occurred very late. For this reason, the basic rule curves for the storage reservoirs should be constructed as envelopes of the filling curves of all reservoirs. The basic rule curve for the total amount of storage during the period immediately before the spring flood is obtained as the sum of the curves for the individual storage reservoirs. The basic rule curve for the total amount of storage during the time preceding this period is determined on the basis of a given availability of supply with the aid of the statistical method which has been described before. At the joint between these basic rule curves calculated in different ways, there will be a discontinuity in that the curve exhibits an abrupt upward bend when it is followed forward in time. This discontinuity can be smoothed out by drawing from the upper point of this upward bend a tangent to that part of the curve which refers to an earlier period of time.

52. Distribution of Discharge among Various Storage Reservoirs, with Special Reference to Risk of Overflow

If a part of the water, stored in those reservoirs which are to be regarded as most unfavourable in respect of overflow, can be drawn from them during the reservoir filling period then there is scarcely any prospect of directly calculating a probability value expressing the risk of overflow. In fact, the increase in the amount of storage will then be dependent on the magnitude of that part of the discharge required in the power system which is drawn from the storage reservoir in question. An upper limit is determined by the rated discharge. In that case, the inflow at intermediate points, if any, must also be taken into account.

Quite generally, the problem under consideration may be stated as follows: At a certain definite point of time, the volumes of water stored in the various reservoirs are given. It is required to determine the distribution of the discharge to be drawn during the next week. This should be done with due regard to the risk of overflow in each one of the storage reservoirs, so as to ensure that they shall overflow at the same time. However, this time is dependent on the subsequent operation of the storage reservoirs. Properly speaking, this time should be calculated on the assumption that the distribution of discharge will also be perfectly managed in the future. Since this is not possible, the probability of overflow has to be calculated on the assumption that the future distribution of discharge will be approximately correct.

Some approximately correct draw-down schedules for distribution of discharge are discussed in what follows.

521. *Distribution of Discharge among Various Storage Reservoirs According to Residual Degree of Regulation*

If the rule governing the distribution of discharge is only to be regarded as a first approximation, then this rule may be very simple. The simplest solution is to use a draw-down schedule in which the requisite discharge is drawn, so long as this is compatible with the development rate of flow, from the various storage reservoirs in due order of succession according to the residual degree of regulation, which is defined as the ratio of the deficiency to the mean annual inflow. If it is desired to determine the probability of overflow in any one of the storage reservoirs, then this can be done by means of the method outlined in what follows. We start with the present-day amounts of storage, and we follow the development of storage for 1½ to 2 years onwards during 30 water years with due regard to the basic rule curve for the total amount of storage calculated as usual and in accordance with the individual basic rule

curves computed in Section 51. The disharge which is required during each week is distributed among the storage reservoirs according to the residual degree of regulation. By finding out during how many of the 30 water years overflow has occurred in a given storage reservoir, we obtain a measure of the probability to be determined. Furthermore, this draw-down schedule can be refined by keeping the maximum discharge in due order of succession from those storage reservoirs in which the probability of overflow is greatest, until the load is met (with the deduction of the power generated under unregulated conditions, which is calculated as the sum of the amounts defined in Groups a and b, Section 222). Except the compulsory discharge, no water is drawn from the other storage reservoirs.

For approximate calculations, only a very simple rule is in many cases needed for the distribution of discharge among the storage reservoirs. In such cases, use can directly be made of the residual degree of regulation (however, cf. Section 525).

522. *Distribution of Discharge among Various Storage Reservoirs, with Special Reference to Simultaneous Overflow*

The distribution of discharge among various storage reservoirs solely according to the residual degree of regulation, see Section 521, does not take account of the fact that the inflow to the individual reservoirs can be distributed in different ways over the year. If it is desired that this circumstance schould be taken into consideration, then this can be done by using several methods, e.g. that which is described in what follows.

We assume an average rate of inflow, which may possibly be corrected for the trend in the inflow. Then we determine the *time of overflow* on condition that overflow occurs simultaneously in all storage reservoirs. It is obvious that this time coincides with the week v during which

$$M_{max} - M = T(v) - W(v)$$

where M_{max} = the total volume of water that can be stored in the reservoirs,

M = the actual volume of water that is stored in the reservoirs,

$T(v)$ = the accumulated mean value of the inflow to the storage reservoirs minus the compulsory discharge during the period extending from the week under consideration to the week v to be determined,

$W(v)$ = the accumulated storage load during the period extending from the week under consideration to the week v to be determined. The storage load is equal to the difference between the total load and the power generated under unregulated conditions, cf. Section 521.

After that, we determine, for each storage reservoir, a discharge $Q(v)$ such that the reservoir overflows during the week v calculated from the above formula, on condition that the assumed value of the average inflow is complied with.

For instance, if we take the storage reservoir No. 1, then we evidently obtain

$$Q_1(v) = T_1(v) - (M_{1\,max} - M_1)$$

where the subscript 1 refers to the storage reservoir No. 1.

If $Q_1(v)$ is found to be negative, then we put $Q_1(v) = 0$, and the whole calculation is repeated for the remaining storage reservoirs.

We have now obtained the approximately correct distribution of discharge which was to be determined in conformity with Section 52. However, this distribution is used only as a basis for determining the discharge during the week under consideration. The unknown discharge P_1 to be drawn during the week under consideration from the storage reservoir No. 1 is calculated from the formula

$$P_1 = \frac{Q_1(v)}{\Sigma Q(v)} \cdot P$$

where P denotes that discharge from the total storage reservoir during the week in question which corresponds to the difference between the total load and the power generated under unregulated conditions, and the sum of $Q(v)$ is taken over all storage reservoirs.

If it is found that the discharge drawn from any one of the storage reservoirs exceeds the rated discharge—with the deduction of the compulsory discharge and the inflow at intermediate points—then the discharge from this storage reservoir is maximized at the value given in the above, P is reduced by the corresponding amount of power, and the maximized discharge from the reservoir is excluded from the sum of $Q(v)$. The discharge to be drawn from the other reservoirs is determined by means of the above formula.

The change in the amount of storage during the week under consideration is determined, and the same calculation is repeated for the next week, and so on. If this calculation is only intended for use as a basis for power system design, then the distribution of discharge determined with the help of the above method may be expected in many cases to be fully adequate. To exemplify this statement, it may be mentioned that the application of the distribution of discharge proposed in the above to an actual case, which involved two storage reservoirs in parallel, gave the following results:

The number of occasions for overflow on which the storage reservoirs overflowed was

during the same week	33 times
during the same period of flow but with a difference in time of:	
one week	7 ,,
more than one week	0 ,,
during different periods of flow	1 ,,

The degree of regulation in this case was 26 per cent in the storage reservoir No. 1, and 79 per cent in the storage reservoir No. 2. The load was constant throughout the year, and was equal to 90 per cent of the average annual inflow. The calculation was extended over a series of 32 water years. The distribution of discharge during two different water years is exemplified in *figs. 22* and *23*.

523. *Another Method of Distribution of Discharge among Various Storage Reservoirs*

Instead of distributing the discharge among the storage reservoirs on the basis of the variable time of simultaneous overflow, as has been done in Section 522, it is also possible to carry out the distribution of discharge with a view to ensuring that the probabilities of overflow at certain definite moments in the course of the year shall be equal for all storage reservoirs. In consideration of the annual

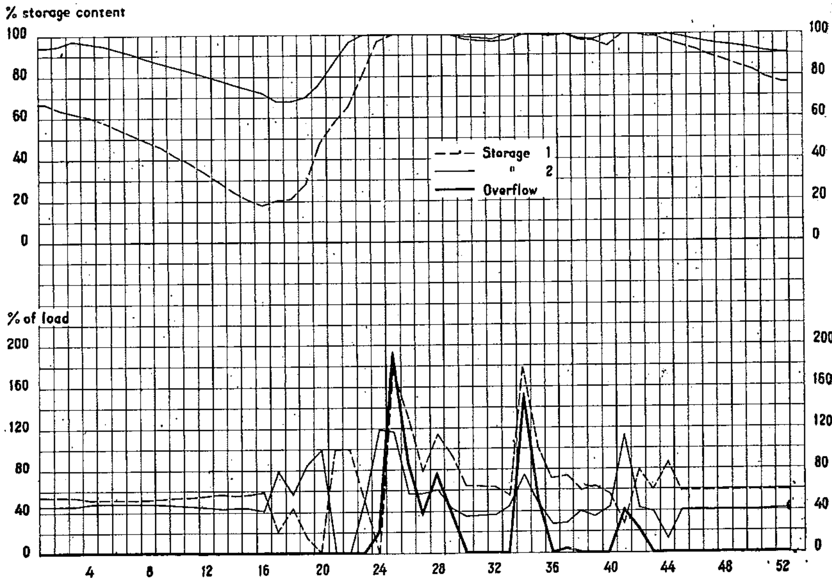


Fig. 22. Example of distribution of discharge during a water year in the presence of overflow.

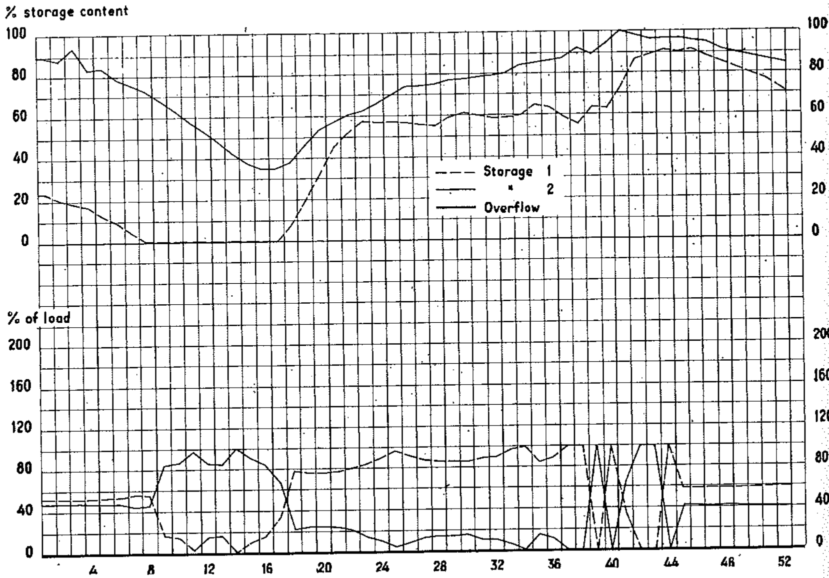


Fig. 23. Example of distribution of discharge during a water year in the absence of overflow.

variations in inflow and in load, it is advisable to divide the year into three periods, viz., the period of winter discharge, the period of spring flood, and the period of autumn flood. The period of winter discharge extends from the end of the autumn flood, normally about November 1st, to the beginning of the spring flood, normally in April. Then comes the period of spring flood, which continues to the end of the industrial holidays, about August 1st. Finally, follows the period of autumn flood.

The distribution of discharge during the periods of spring flood and autumn flood is carried out by using a procedure which is similar in principle to that described in Section 522. In the present case, however, the week v in the equation

$$M_{max} - M = T(v) - W(v)$$

is given, and the quantity to be determined is the inflow $T(v)$. Then the total inflow to the storage reservoirs calculated in this way is "distributed" among the various reservoirs on the assumption that the values of the inflow correspond to the same probability in all storage reservoirs. After that, the procedure in calculations is in principle the same as that which is employed in Section 522. In other words, we determine, for each storage reservoir, a discharge $Q(v)$ such that the reservoir overflows during the predetermined week, on condition that the value of the inflow is equal to that which has been assumed in the above.

During the period of winter discharge, the distribution of discharge among the storage reservoirs is carried out with a view to ensuring such a distribution of the residual amount of storage at the beginning of the spring flood that the probabilities of overflow at the end of the period of spring flood are equal in all storage reservoirs. The probable magnitude of the residual amount of storage is calculated for each week during the period of winter discharge from the formula

$$M_{res} = M + T - W$$

where M_{res} = the total residual amount of storage at the beginning of the period of spring flood,

M = the actual volume of water that is stored in the reservoirs,

T = the accumulated most probable value of the inflow to the storage reservoirs during the period extending from the week under consideration to the beginning of the period of spring flood,

W = the accumulated most probable value of the storage load during the period extending from the week under consideration to the beginning of the period of spring flood.

After that, the distribution of the residual amount of storage among the individual reservoirs with a view to securing equal probabilities of overflow is carried out on the basis of curves which represent the total inflow to each storage reservoir during the period of spring flood at different values of the probability of overflow. For this purpose, it is furthermore required to know the compulsory discharges, the discharges necessitated by timber floating, and the most probable storage load during the period of spring flood. With the help of these data, we can plot a curve for each storage reservoir which represents the probability of overflow at the end of the period of spring flood as a function of the amount of residual storage in this reservoir. Moreover, we plot a summation curve, i.e. a curve obtained by adding together, for each value of the probability of overflow, the corresponding values of the amount of residual storage in all reservoirs. This summation curve represents the probability of overflow as a function of the total residual amount of storage. From this summation curve, and from the value of the total residual amount of storage, which is calculated by means of the procedure outlined in the above, we find the probability of overflow, which, in its turn, determines the amount of residual storage in each one of the reservoirs. Then the discharge to be drawn from a given storage reservoir during the period extending from the week under consideration to the beginning of the period of spring flood, Q_1, is calculated from the formula

$$Q_1 = T_1 + M_1 - M_{1\,res}$$

where the subscript 1 refers to the storage reservoir No. 1.

After that, the unknown discharge from the storage reservoir during the week under consideration is calculated with the aid of the same method as in Section 522.

524. *Generalization of Draw-Down Schedules Described in Sections 522 and 523*

The principle stated in Section 522 implies that the probabilities of overflow in all storage reservoirs are equal at that time of overflow which corresponds to an inflow of 50 per cent. On the other hand, the principle established in Section 523 involves the determination of a discharge which leads to equal but varying probabilities of overflow in all storage reservoirs at a certain time that is given in advance. A drawback of the former method is that it would sometimes be desirable to determine the time of overflow at a value of inflow other and higher than 50 per cent, whereas a drawback of the latter method is that it may sometimes be desirable to replace the predetermined time of overflow by the most probable time of overflow. In what follows, we shall discuss the question how the draw-down schedules described in Sections 522 and 523 can be modified and combined so as to obtain a draw-down schedule which may be expected largely to eliminate the abovementioned drawbacks.

In the first place, attention is directed to the following considerations:

1. The most probable time of overflow can have been preceded by a period during which the storage reservoirs have nearly been filled, even at an average inflow, and then were lowered again. In other words, the filling curve of the storage reservoirs has a maximum before it reaches the maximum permitted storage level. This can be exemplified by examining the conditions which are met with in the autumn. If the inflow during this period is large, then it may cause the storage reservoirs to become almost 100 per cent full. After that, the level of the water stored in the reservoirs sinks during the period of winter discharge, and it is possible that the reservoirs do not overflow until the next spring flood. Under such conditions, it is evidently desirable to guard in the first place against overflow at the time when the filling curve of the storage reservoirs reaches its maximum, and to put the time of the maximum filling of the reservoirs which occurred in the autumn equal to the time of overflow.

2. Even if overflow does not take place at the most probable inflow during a period of filling which is followed by a protracted period of discharge, an attempt should be made to guard against

overflow caused by high values of inflow whose probability is lower than 50 per cent. It is true that — in consideration of the subsequent time of overflow — the distribution of the amount of storage among the reservoirs at the beginning of the period of discharge is normally somewhat incorrect, but this error is automatically corrected in the course of this period.

In this case, too, the conditions in the autumn can be adduced as an example. We assume that the inflow to some of the storage reservoirs during the autumn has a markedly skew distribution involving a certain, not negligible probability of relatively high values of the inflow. If the most probable inflow is supposed to be lower than the load, then, in conformity with the principles stated in Section 522, no overflow will occur in the autumn, and, in accordance with Point 1 in the above, the filling curve of the storage reservoirs will not reach a maximum, but it is nevertheless desirable to guard against overflow in case the inflow becomes unusually high.

The above-mentioned circumstances can be taken into account by using the method described in what follows.

The following notations will be employed:

$T_i(v)$ = the difference between the accumulated inflow to the storage reservoirs and the compulsory discharge in the course of a certain definite water year, denoted by i, during a period extending from an arbitrarily chosen week v_0 to the week v,

$W_i(v)$ = the accumulated storage load during the same period and the same water year,

$T_a(v)$ = the accumulated inflow to the storage reservoirs whose probability does not exceed a per cent during the period extending from the week v_0 to the week v.

For each one of the years comprised in the series of water years, we shall now investigate once for all a function

$$F_i(v) = T_i(v) - W_i(v)$$

Normally, this investigation needs to cover only the period $30 < v < 50$, and it is convenient to put $v_0 = 30$, cf. *fig. 24*. The power balance calculations described in Chapter 4 can readily be modified so that the function to be determined is obtained directly.

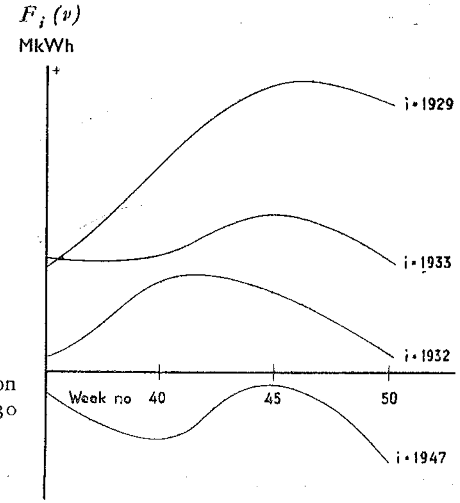


Fig. 24. Example of the function $F_i(v) = T_i(v) - W_i(v)$ when $V_0 = 30$

For a few selected weeks, e.g. 35, 40, 45, and 50, we determine the statistical distribution of $F_i(v)$, cf. *fig. 25*. On the basis of these distribution curves, the function $F(v)$ is plotted for a few different values of the probability a which are lower than 50 per cent, cf. *fig. 26*. It is assumed that these probability curves for $F(v)$ have their maxima at the times v_a. The earliest of these maximum points is denoted by $v_{a'}$, and the corresponding value of the probability is designated by a'. After that, just as in Section 522, we examine that moment v at which the function

$$M_{max} - M - T_{50}(v) + W(v)$$

becomes equal to zero. (It is to be observed that $T_{50}(v)$ is the

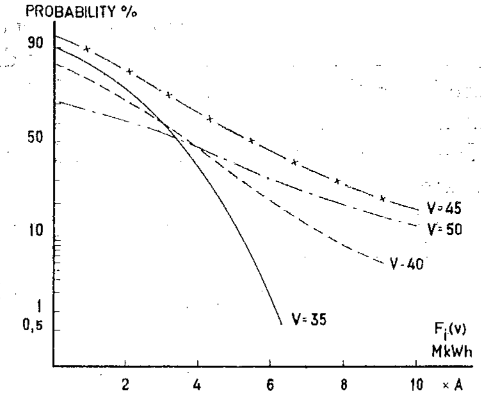


Fig. 25. Example of distribution curves of the function $F(v)$ for $V_0 = 30$.

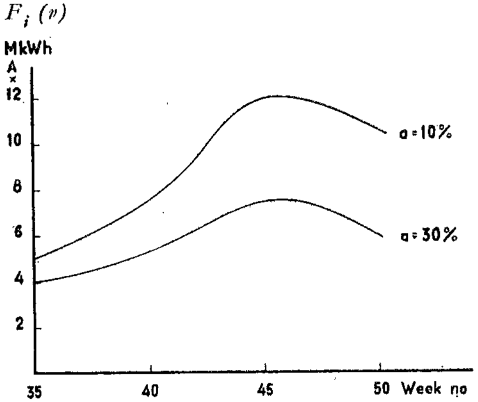


Fig. 26. Example of the function $F(v)$ for two values of the probability a.

mean value of the accumulated inflow to the storage reservoirs, and has previously been denoted by $T(v)$ alone.) If the passage of the above function through zero takes place after the time v_a, then the time v in this section is replaced by v_a' and the probability 50 per cent is replaced by a' per cent.

As a rule, the determination of the time v_a' and the probability a' carried out in the above may be simplified by investigating only the difference between the total inflow and the total load. For this purpose, let $T_{tot,\,a}\,(v)$ denote the total inflow whose probability does not exceed a per cent and which is accumulated during the period extending from the week v_0 to the week v, and let $W_{tot}\,(v)$ designate the accumulated total storage load during the same period.

We investigate once for all the function

$$T_{tot,\,a}\,(v) - W_{tot}\,(v)$$

so as to determine its maxima at a few different values of a which are lower than 50 per cent, e.g. 40, 30, 20, and 10 per cent. It is assumed that the corresponding maxima occur during the weeks v_a. The earliest of these weeks, v_a', and the corresponding probability a' are the values to be determined, cf. *fig. 27.*

After that, the discharge to be drawn from the storage reservoir No. 1 is fixed at the respective values

$$Q_1\,(v) = T_{1,50}\,(v) - (M_{1\,max} - M_1)$$

and

$$Q_1\,(v) = T_{1,\,a'}\,(v_a') - (M_{1\,max} - M_1)$$

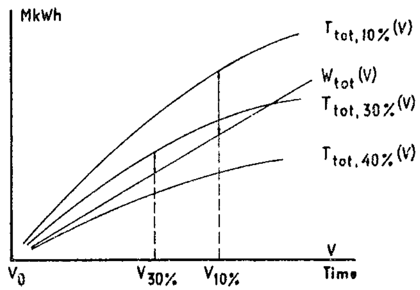


Fig. 27. Simplified method of determining the time of overflow.

where the lowest value of $Q_1(v)$ is zero, cf. Section 522. In the first of the above formulae, the subscript 1,50 refers to the inflow to the storage reservoir No. 1 whose probability is 50 per cent. This inflow has previously been denoted by $T_1\,(v)$ alone. Furthermore, the subscript 1, a' in the second of these formulae refers to the inflow to the storage reservoir whose probability is a' per cent.

If the week v_a precedes the week v, and if all values of the inflow to the storage reservoirs may with sufficient accuracy be regarded as equally distributed, then it is convenient first to determine a constant

$$k = \frac{T_{tot,\,a'}\,(v)}{T_{50}\,(v)}$$

After that, the discharge to be drawn from any one of the storage reservoirs can be calculated from the formula

$$Q_i\,(v) = k\,T_{i,\,50}\,(v) - (M_{i\,max} - M_i)$$

Finally, the discharge under consideration is obtained from the formula

$$P_1 = \frac{Q_1\,(v)}{\Sigma Q_1\,(v)} \cdot P$$

where P_1 is maximized at a value which is equal to the difference between the utilized flow and the compulsory discharge, as has been shown in Section 522.

If several storage reservoirs which differ in downstream head are operated in parallel, then the values of the inflow, the volume of the water stored in the reservoir, the compulsory discharge, and the utilized flow should preferably be expressed in terms of electrical energy units.

525. *Discussion of Various Methods Used for Distribution of Discharge among Several Storage Reservoirs*

The methods described in Sections 522, 523, and 524 make it possible to ensure a uniform distribution of discharge among the storage reservoirs as well as a smooth and relatively slow-acting change in this distribution in case of alteration in the distribution of the inflow among the reservoirs. This is an advantage in operation planning, e.g. with respect to short-term regulation. However, if sudden and violent changes occur in the inflow to some individual storage reservoir, then this may cause the discharge to react too slowly, with the result that there will be unnecessary spillage.

The probability of sudden and violent changes in the inflow is small during the period of winter discharge in Sweden. Moreover, the volume available in the storage reservoirs during this period is in general sufficient to take care of such changes. The methods described in Sections 522, 523 and 524 are therefore best adapted to this period. On the other hand, the use of the method outlined in Section 521 may be preferable for shorter periods during spring flood and autumn flood. This method should perhaps be slightly modified, first, in order to take account of the fact that the distribution of the inflow during the year can vary from one storage reservoir to another, and second, in order to equalize the highly non-uniform distribution of discharge which this method is liable to cause. Varying inflow conditions can be taken into account by putting the residual degree of regulation equal to the ratio of the deficiency to the average inflow during the remaining part of the period. Abrupt changes can be smoothed out if the rate of flow during the discharge from the storage reservoirs is caused to be dependent on the volumes of water stored in the respective reservoirs, so that the rate of flow increases as the volume of water becomes greater.

For all that, the question which of these methods is most suitable for use during each one of the periods referred to in the above cannot be answered definitively until they have been applied in practice to the operation of relevant power systems. Inflow and storage conditions vary within wide limits from one individual case to another. Therefore, it is not certain that a method which has proved to be best in one particular case will also be found to be most appropriate in all other cases.

526. *Special Considerations on Distribution of Discharge*

In Sections 521 to 525, the point of departure for distribution of discharge was exclusively the requirement that all storage reservoirs should overflow at the same time. It may perhaps be questioned whether the requirement that the *water values* in all storage reservoirs should be equal would not be a more correct starting point. In that case, the water value would be calculated by means of a method which is analogous to the procedure described in Chapter 4, that is to say, not only the probability of overflow of the reservoir, but also the probability that the amount of storage reaches the minimum zone would be taken into account. However, such a calculation would probably be so laborious that it could not be performed within a reasonable time even if use were made of the most rapid data processing machines which are available at present. Furthermore, it is not to be expected that a calculation of this kind will be necessary because, from the point of view of energy balance, it does not matter whether the amount of storage in an individual reservoir—or in several reservoirs—reaches the minimum zone, or even whether this or these reservoirs are emptied. The reason is that, in this situation, it is the *total* amount of storage that is a decisive factor in determining the water value in the power system as a whole, and hence also the generation of thermal power, the sales of secondary power, etc. On the other hand, the water value is influenced by the manner of co-ordinating the probabilities of overflow of the individual storage reservoirs, since inadequate co-ordination leads to an increase of the total spillage in the power system, and hence to a decrease of the water value. It is therefore sufficient, at any rate in the great majority of cases, to consider solely the probability of overflow when determining the distribution of discharge among the various storage reservoirs.

All the same, in some special cases, it is moreover necessary to make a particular check in connection with the emptying of storage reservoirs. One of these cases is met with when thermal power stations are to be operated in a system which comprises storage reservoirs having a very high degree of regulation. The other case presents itself when check is required to make sure that the distribution of discharge among the storage reservoirs is such as to secure the possibility of generating the requisite quantities of day-time energy in the hydro-electric stations during each week in the course of the period of discharge.

In order to illustrate the first case, we can imagine a power system which comprises a storage reservoir having a very high degree of regulation, where the amount of storage is comparatively large, and several reservoirs having a low degree of regulation, where the amount of storage is relatively small. Furthermore, it is assumed that this system reaches the minimum zone, and that thermal power generation is therefore started at full capacity. Then it is obvious that the hydro power generation will be reduced in a corresponding measure, and that the whole of this reduction will be achieved by diminishing the quantity of power which is generated by the water drawn from the storage reservoir having a very high degree of regulation. The reason is that an optimum probability of simultaneous overflow of all storage reservoirs after the spring flood can be obtained in this way. However, owing to the high degree of regulation of, and the large amount of storage in, the last-mentioned reservoir, it contains so much water that the time available will not be sufficient for discharging the whole content of this reservoir before the spring flood, even if the reservoir in question is drawn down at a rate corresponding to the rated discharge. The reduction in discharge caused by the thermal power generation will therefore diminish the quantity of energy which can be taken from the reservoir[1] by an amount which corresponds to the thermal power generation. Consequently, the thermal power generation will be entirely lost so far as the purpose in view is concerned.

Accordingly, in power systems which comprise storage reservoirs having a high degree of regulation, it is necessary to keep a special check on the discharges from the reservoirs each time it is required to resort to thermal power generation. In making this check, it is necessary to ensure that the discharge from the storage reservoirs having a high degree of regulation is large enough to enable the whole quantity of water contained in these reservoirs to be discharged before the spring flood, even if the inflow to the reservoirs in question is great. If the power system comprises several storage reservoirs having a high degree of regulation, then a situation may arise in which the generated power would exceed the load if the discharge from these reservoirs were adjusted so as to make sure that the reservoirs will be emptied before the spring flood. In such a case, the discharge is evidently reduced to such an extent that the power generation balances the load, and the discharge is distributed in such a manner that the probabilities of emptying before the spring flood are equal for all reservoirs. The above-mentioned problems can also be met with in the operation of reservoirs having a moderate degree of regulation if the rated discharge is low in comparison with the storage capacity.

[1] Before the spring flood.

The second of the cases referred to in the above is analogous to the problem which has been discussed in Section 51, i.e. the distribution of discharge among the reservoirs during the period immediately preceding the spring flood. In solving this problem, a basic rule curve was calculated for each one of the reservoirs. In certain cases, it may be desirable to extend these individual basic rule curves over the whole period of discharge. The point of departure used in calculating such a curve is a given probability, e.g. 99 per cent, that a certain definite minimum discharge from the reservoir in question can be maintained up to the spring flood. This calculation is carried out by means of the method which has been described in Section 21, and the above-mentioned minimum discharge from each reservoir is chosen in such a manner that the requisite quantity of day-time energy can be generated during each week in the course of the period of discharge. These individual basic rule curves are utilized only for checking the amounts of storage in the reservoirs. So long as these amounts of storage are situated above the respective basic rule curves, the distribution of discharge is carried out by the aid of some of the methods which have been outlined in Sections 521 to 525.

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KJELL DARIN, YNGVE LARSSON, CARL-ERIK LIND, JAN-ERIK RYMAN and BERTIL SJÖLANDER: *Principles of Power Balance Calculations for Economic Planning and Operation of Integrated Power Systems.* Svenska Vattenkraftfören. publ. 476 (1959:11). Storing of hydrological data. Method for determination and calculation of basic rule curve for different hydro-electric systems. Availability of supply. Rationing conditions. Determination of water value with discussion of different kinds of thermal power, secondary power supply and storage reservoirs. Distribution of draw-down among various storage reservoirs. Calculation of risk of overflow.