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# Comparing Bidding Methods for Hydropower

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**Abstract**

In this paper we compare several methods used by hydropower producers for the determination of bids to the day-ahead market. The methods currently used by the hydropower industry are based on heuristics and expert assessments, but we also include a new method for formal optimization of the bid decisions in our study. The optimization of bids is based on stochastic programming with explicit mathematical representation of the bid decisions, which none of the heuristic methods accommodate. We illustrate the potential benefit from using increasingly complex bidding methods in a case study representing real operating conditions for a hydropower producer on three specific dates.



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*Keywords:* Bidding problem; Hydro unit-commitment; Successive linear programming;

## 1. Introduction

In deregulated power systems, participation in various markets for selling electricity is a paramount daily task for generation companies. As this task is undertaken every day with the goal of profit maximization in mind, good strategies for trading in the power markets is a competitive advantage. This paper investigates optimal determination of bids to the day-ahead market by comparing the load committments and the revenues obtained by a price-taking hydropower producer when using different bidding methods.

Optimal scheduling of hydropower has traditionally been divided into long, medium and short time horizons due to the complexity of the problem as explained in [1]. Short-term scheduling covers the day-to-day operation of the hydropower production assets and can be divided into two sub-problems: determination of market bids and determination of optimal water release to cover load commitments. The first task is undertaken prior to market clearing and must hence be determined under uncertainty of prices and inflow. The latter problem is performed after market clearing when prices and load commitments for the next operating day are known. Inflows and the prices for the rest of the week are still unknown at this stage, so both problems must be solved under uncertainty. Fig. 1 gives an illustration of the daily scheduling process for hydropower producers participating in the day-ahead Elspot market at Nord Pool which is the main market place for Nordic electricity. Similar market set-ups can be found in other countries with deregulated power markets. Market participants submit bids for the expected supply or demand for

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Fig. 1. An illustration of the time line for the daily scheduling of hydropower. Until 12:00, producers must determine what volumes to produce and bid to the market for the next operating day. Price and inflow for the next day and the rest of the short-term horizon are uncertain at this stage. From 12:00 to 13:00, the market is cleared and the producers are notified of their load commitments. From 13:00 and until midnight when the operating day begins, the producers continuously monitor and reschedule their production decisions. The prices for the coming operating day are now known, but prices and inflow for the rest of the week is still uncertain.

power the next day in the form of a matrix of price-quantity pairs for each hour of the following operating day. The bids are valid for individual hours the next day, starting at midnight. The bids are hence submitted 12-36 hours before the delivery hour. The market is cleared once every day, and producers are notified of their obligations. With known market commitments, producers have the option of re-optimizing the production schedules in order to cover the load with minimum costs.

Several approaches for finding the cost minimizing schedule with known loads or prices can be found in the literature as the hydro unit commitment or self-scheduling problem, see for instance [2] or [3] for a discussion. Formal decision support tools based on deterministic optimization as described in [4] are in daily operational use by most large Nordic hydropower producers for finding the optimal production schedule.

The market commitments for the individual producers are available only after the market has been cleared. Decisions taken at the bidding stage limit the opportunity for making a good schedule in the after-spot stage. Even though producers have the option of buying or selling unbalances in the intra-day and balancing markets, the strategy for bidding into the day-ahead market should give production commitments close to physically feasible production schedules. This will reduce the risk of having to buy (sell) power at a price higher (lower) than the spot price in less liquid markets to cover commitments. In 2013, the total traded volume on the Elspot day-ahead market was 349 TWh, while the volume traded on the intra-day market Elbas was 4.2 TWh [5], demonstrating that the day-ahead market is the most important market place for power. The decisions taken here are hence important for the profitability of the hydropower companies. The bidding decisions should also reflect the long-term strategy for operating the hydropower resources, as this is the only stage in the hydro scheduling hierarchy where strategies are carried out in physical trading.

To the authors' best knowledge the bidding decisions of most Nordic producers are currently determined by expert judgment and heuristic methods. Multi-scenario runs of deterministic models as presented in [4], [6] and [7] is a common method, but also other methods based on results from the after-spot problem are used. A drawback of these methods is that the uncertainty of future market prices are not explicitly represented in the decision support tool; hence the need for manual methods in order to adapt the solutions from the deterministic model to yield coherent bidding strategies.

In recent years, several studies have investigated the use of formal optimization tools for solving the bidding problem and the field of stochastic programming has proved promising. A model for hydropower scheduling based on stochastic programming is presented in [8] and has later been expanded to include bidding functionality based on the formulation in [9]. Evaluating the results and identifying possible improvements from using this new model is the main objective of this paper.

Bidding models for hydropower are presented in [9]-[12] and a recent literature survey on the subject is given in [13]. The potential gain by formally solving the bidding problem varied within 0-16% in [12], but this added value is very dependent on what the stochastic model is compared to. Compared to results from bidding the optimal volumes from the expected value problem, [9] show a benefit of the stochastic model of 7.75-9.33%, while [7] show no improvement compared to the multi-scenario method. A fair benchmark would be to compare the results of the stochastic bid model to the methods currently used for bid determination in the industry, as this would quantify any gains obtainable to producers by switching to the new model. This is done in [6], but the model presented there has a less general description of the hydropower system. The contribution of the current paper is the comparison of the stochastic bid model to methods currently used in the hydropower industry.

A few notes regarding the scope of this paper are in place. First, we assume that producers are price-takers, and hence we use the term *bidding problem* in the same manner as in [13], where also the *strategic bidding problem* is defined as the determination of bids when a single producer's bid may affect the market prices and the producer hence have market power. Second, we are only concerned with day-ahead bidding in this paper, as opposed to developing bidding strategies for sequential electricity markets. The electricity market scheme varies across countries, but there are typically markets for day-ahead supply, intra-day trades, real time balancing and procurement of reserves, see [14] for a discussion. Expected generation for the next day is traded on the day-ahead market. Since there is some time lag between the clearing of the day-ahead market and the delivery hour, new information or unforeseen events may occur and producers might want to reschedule and adjust their commitments in the intra-day market, where power can be bought or sold closer to real-time. In the balancing power market, producers having additional capacity might offer this as regulating power in order to maintain the instantaneous balance between demand and supply. The other markets may offer additional sources of revenue to producers, but complicate the bidding process and calls for coordinated bidding strategies. With an expected development towards larger shares of power from intermittent renewable resources, both volatility and liquidity in the balancing markets will increase and power companies should seek to maximize profits across all available markets. Interesting takes on multi-market bidding is found in [15]-[17], and such strategies will be important in the near future. The day-ahead market is still the largest and most liquid market for physical power in the Nordic region, and hence the interaction with this market is of paramount importance to producers.

The rest of this paper is organized as follows: Section 2 describes the bidding problem in detail. Section 3 presents the implementation of the stochastic bid model, whereas Section 4 presents heuristic methods used by the industry. Section 5 explains the evaluation method used in the case study in Section 6. Final conclusions are given in section 7.

## 2. Bidding hydropower

Producers aim to generate bid curves that reflect the marginal uploading of generation capacity as the prices increase. For hydropower, the marginal costs are given by the opportunity cost of production. Since precipitation is free, the water resources are only limited by their availability and storability. The marginal costs of hydropower are thus interlinked in time. Production in one hour may diminish the resources for production in later hours, but saving water for later increases the risk of spillage. Short-term scheduling balances these two objectives in a perspective of up to 2 weeks, after which a boundary condition in the form of a value for stored water or target reservoir level is set. Computing the future value of water is a complex task as it depends on time, expectations of future prices and inflow and the topology of the reservoir system. Models such as the long- or medium-term model described in [1] may be used to evaluate the value of water, which is the resource cost of water in the short-term perspective. Evaluation of water values is beyond the scope of this paper, and the value of water is seen as an input to the bidding process. This is consistent with the current industry practice.

Returning to the short-term horizon, for a single generation unit and no other binding constraints in the system, it is optimal to produce at best-point efficiency as soon as the price is higher than the cost of water given by the water value. As the price further increase beyond the water value, increased production will be a trade-off between getting a higher price and producing at decreasing efficiency above best point. This trade-off depends on the energy conversion function, i.e. the water input to energy output ratio, of the particular generating unit, which again is dependent on the three-dimensional relationship between turbine efficiency, plant head and discharge. We do not consider the impact of head effects in this paper, but the optimization models will handle these effects if it is relevant. The constraints of hydropower scheduling are not elaborated here, but include reservoir balances, unit-commitment decisions, production functions, topology constraints and end-valuation of water. The mathematical formulation of the stochastic model can be found in [8]-[9] and the general modelling principle for all our model is presented in [4] which also gives the foundation of the heuristic bidding methods. This model is currently in operational use at Nordic hydropower companies.

## 3. Stochastic bidding

The stochastic model is based on the deterministic method implemented in the operationally used SHOP software [4], except that it allows for a stochastic representation of inflow to the reservoirs and day-ahead market prices. Inflow uncertainty is important in the general case, but can be neglected if the size of the reservoirs are very large compared to the inflow. For the case study in section 6, inflow uncertainty is important as there is a small reservoir acting as a bottleneck in the river cascade. However, in this paper only price uncertainty is considered as it is the effect of stochastic prices that is investigated. The stochastic approach includes explicit equations for determining bids based on the formulation in [9] where the bid curve is formulated as the piecewise linear curve determined by interpolation between the price-volume points in the optimal bid matrix found by the model. Solving the stochastic model gives a bid matrix which will be sent to the market operator prior to market clearing.

## 4. Heuristic bidding

We present the three methods for generating bids based on the deterministic model: a) bidding the expected volume, b) bids based on the water value and c) bids based on results from multi-scenario runs of the deterministic model. These methods are presented in detail below. The heuristic methods each give a bid matrix which will be sent to the market operator prior to market clearing.

### *4.1. Bid the expected volume*

The bid volumes are found by deterministic optimization against the forecasted market price. This method is very simplistic and is included mostly for comparison purposes. If price uncertainty is small or negligible, the expected value bids may perform well, similarly if the prices could be perfectly predicted. However, these assumptions do not hold in the general case. For each hour, this method will bid a certain volume regardless of the actual price in this hour and the bid curve will be a vertical line.

### *4.2. Bid according to the water value*

The bids should reflect the marginal cost of production, which in principle equals the water value. A strategy could hence be to bid the best-point production as soon as the price exceeds the water value. This would produce a bid curve with only one break point. A more elaborate strategy would be to bid the water value adjusted for the decreased efficiency of production above best point. For price points higher than the water value, the bid volume should be determined by the water value corrected for increased water use per unit power above best-point. This will produce a bid curve with several breakpoints above best-point, and is hence a step towards price-dependent bidding.

Such a curve can be approximated by the deterministic model as a post-calculation based on the results from the optimization, and is often referred to as the marginal cost curve for users of the SHOP model. This method will account for price uncertainty to a larger degree than bidding the expected volumes, but still only consider information from a single price scenario.

### *4.3. Multi-scenario deterministic bidding*

In lack of a model that formally takes into account the stochastic nature of prices, producers generate a set of production schedules by using the deterministic model for a set of forecasted prices. This multi-scenario method has the potential of considering information from more than one price scenario. The resulting production schedules for each individual price scenario are combined into a bid matrix in the required form of price-volume pairs for each hour of the next operating day. For each hour, the production volumes are sorted according to the price they are optimized for. The bids have to be increasing for increasing prices. Any instances where a lower price in one scenario yields a larger volume than a scenario with lower price for a particular hour is resolved by using the higher volume in both scenarios. This not a formal optimization procedure, but due to skilled operators with detailled system knowledge the method gives good results in practice. The method also considers price uncertainty to a greater extent than just optimizing for the expected price and using the results directly or doing some post-calculations as in the water value approach. However, the volumes have still been obtained by deterministic optimization of each individual scenario separately, and thus in general suffer from look-ahead bias.

## 5. Evaluation method

For comparison of the four bidding methods, we look at two measures: a) the market commitments and b) the revenues obtained by the producer.

### *5.1. Load commitments*

As a first criterion, we look at how the competing bid methods compare in terms of the commitments from the market clearing. Load commitments should be close to physically feasible production schedules which are characterized by few start and stops of generation units and reservoir levels within their bounds. In the case study of Section 6, we obtain the market commitments by using historical realized market prices to calculate the volumes for each bidding method if the resulting bid matrix had been sent to the power exchange prior to the operating day. The method for comparison can be seen as four price-taking producers with identical reservoir systems that participate in the same power market on the same day with the only difference being the bidding method. Producer 1 bids the expected volumes, Producer 2 bids according to the water values, Producer 3 uses the multi-scenario method and finally Producer 4 uses the stochastic method. Each producer must then produce according to their obligations after market clearing, and we try to assess which producer has the best commitment profile.

### *5.2. Comparing revenues*

With known commitments, the production schedules are re-optimized in the after-spot problem. This is a chance to correct any undesired consequences of the bidding decisions, but limited to the now known load. The revenues obtained from producing according to the cost minimizing schedule are used as a comparison measure for the four bidding methods. The market clearing price and the model for rescheduling production according to commitments are the same for all producers, so any differences in revenue is solely due to the bid method used.

## 6. Case study

The four bid methods are used for a reservoir system consisting of two reservoirs and two plants in series. The upstream reservoir is quite large and can hold water for up t 18 weeks of production at full capacity at the belonging plant. This plant releases water into the downstream reservoir which is so small that it will be filled up with about a day's production from the upstream plant. The reservoir can be emptied with 13 hours of production from the downstream plant. For the case study, we bid for 3 specific, independent dates: 30 June 2014, 14 July 2014 and 22 January 2015. These dates were chosen because the hydropower company supplying the data thought that handling uncertainty would be of particular importance on these dates, due to combinations of forecasted values for prices and initial conditions in the watercourse. Our results may thus overestimate the gains from the bid methods that address uncertainty, and do not represent average results. Data for forecasted prices and inflow and the system state has been made available to us by a Norwegian hydropower producer and corresponds to the exact same information that was available to operators at the hydropower company at the time of bidding. The forecast for price and inflow were generated by the in-house market and hydrology department at the hydropower company. The realized prices used for market clearing are public information found on Nord Pool's website. Depending on the date, 9 or 13 scenarios for price were used in the bid optimization in the multi-scenario and stochastic method. Inflow is considered deterministic throughout our analysis.

### 6.1. Load commitments

The load commitments for each of the three test days are shown in Fig. 2, where also the realized prices are plotted (black, solid line). The committed volumes should roughly follow the profile of the prices, as it is desired to increase (decrease) production in the hours where prices are at their highest (lowest).

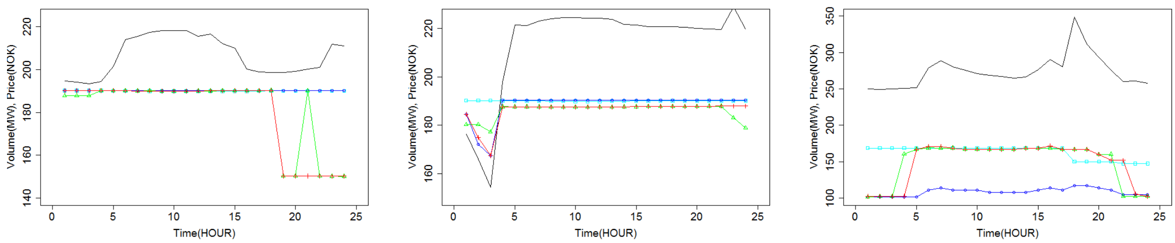


Fig. 2. The load commitments for Day 1 (left), Day 2 (middle) and Day 3 (right) after market clearing when using the expected value method (light blue, squares), the water value method (dark blue, circles), the multi-scenario deterministic method (green, triangles) and the stochastic method (red, cross).

Looking at the load commitments it is evident that all four bidding strategies result in loads that to some extent follow the price profile. For Day 1, all of the bidding methods produce at full capacity for the high prices in hours 6-15. The stochastic and multi-scenario approach decreases production in some of the lower priced hours towards the end of the day, but the multi-scenario method starts an additional generator in hour 21. Starting up a turbine for only a single hour of production is not wanted, unless there are some really high price spikes. Start-ups are costly due to wear on equipment. These costs are included in the optimization model by using binary variables, but less attractive start-ups are often manually adjusted by the planner after analyzing the results from the optimization. From the plot of the load commitment for Day 1 in Fig. 2, it is evident that the multi-scenario deterministic method has an unfortunate start in hour 21. As there is no particularly high price in this hour to make up for the extra cost, this start is likely to be rescheduled or adjusted in the after-spot stage. The stochastic approach avoids this additional start-up.

For Day 2, all bidding methods except the expected value follow the price dip at the beginning of the day and then follow the rather flat prices. All methods suggest flat production at full capacity most hours of the day. For Day 1 and 2, which are in the summer, the value of water is low compared to the price and production close to maximum capacity was expected. For Day 3 in the winter, the water value is more or less the same as the price, which could result in more dynamic production volumes depending on the price. For Day 3, the stochastic model somewhat better matches the high prices in hours 5-22 than the other methods, by starting up more production in the same hours as the prices start to increase - not slightly before or after as the deterministic and water value methods do. For the price peak around hour 18 of Day 3, both the stochastic and deterministic model run at full capacity, so no more production can be started even if the price is really high. The expected value method seems to not match the price, and in fact decreases production for the peak price in hour 18. This is due to the fact that the forecasted/expected price has a quite different development over the day than the realized prices on this date, as can be seen from Fig. 3.

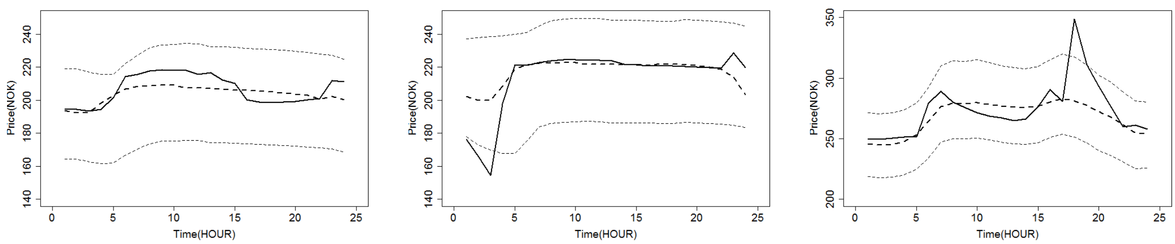


Fig. 3. The price profile for the realized prices (full drawn line) and the expected prices (dotted line) for Day 1 (left), Day 2 (middle) and Day 3 (right). The minumun and maximum scenarios are also plotted (thin, dotted lines).

### 6.2. Comparing revenues

Regardless of the profile of the load commitments, it is the revenue obtained by utilizing the different bidding strategies that is the true benchmark for producers. After market clearing when load commitments are known, producers optimize the production decisions in order to find the cost minimizing generation schedule. The objective function values from the after spot problem are a fair comparison measure for the different bidding strategies as this is an indication of the profits obtained by the producer. The results for each of the four bidding strategies are shown in Table 1.

Table 1. A summary of the results for objective function value obtained by using each of the four bidding methods. The improvement over the expected value method is calculated for each of the other methods in both values and percent.

| Day 1, 30 June 2014 | Expected Value | Water Value | Multi-Scenario | Stochastic |
|---|---|---|---|---|
| Objective function value (NOK) | 407 993 | 408 659 | 413 667 | 415 560 |
| Improvement from expected (NOK) | - | 666 | 5 674 | 7 567 |
| Improvement from expected (%) | - | 0,16 | 1,37 | 1,82 |

| Day 2, 14 July 2014 | Expected Value | Water Value | Multi-Scenario | Stochastic |
|---|---|---|---|---|
| Objective function value (NOK) | 509 753 | 510 975 | 511 234 | 511 362 |
| Improvement from expected (NOK) | - | 1222 | 1 481 | 1 609 |
| Improvement from expected (%) | - | 0,24 | 0,29 | 0,32 |

| Day 3, 22 January 2015 | Expected Value | Water Value | Multi-Scenario | Stochastic |
|---|---|---|---|---|
| Objective function value (NOK) | 336 431 | 340 671 | 343 372 | 348 601 |
| Improvement from expected (NOK) | - | 4 239 | 6 941 | 12 170 |
| Improvement from expected (%) | - | 1,24 | 2,02 | 3,49 |

In order to analyze the results, it may be beneficial to consider the uncertainty of prices as consisting of two components: the magnitude or level of the prices and the shape of the price profile. Some days may have a higher or lower general price level. At the same time it is uncertain which hours will have the lowest and highest prices within the day, i.e. the price profile is also uncertain. Fig. 3 shows differences between the forecasted and the realized price profiles for all three days. A good bidding method should consider both the uncertainty of the price level and the price profile. The uncertainty of the price profile is important due to the fact that marginal costs for hydropower are linked in time, as production in one hour affects the resources avilable for production in other hours.

The expected value only uses information from a single price scenario and hence does not consider uncertainty at all, neither in level or profile. Bidding according to the expected prices thus may result in production patterns that in hindsight do not seem to follow the price profile, as seen from the load commitments from bidding the expected value for Day 3 in Fig. 2. In general, the expected value method will only give good results if the realized prices could be seen with perfect foresight.

The water value method considers price level uncertainty by offering a bid curve that reflects the increasing cost of production due to lower efficiency as the production is increased above best-point. For the three days tested here, this gives an improvement over bidding the expected volumes. The bids are however still only based on information from a single deterministic optimization against the expected price. Any dynamics in the bids regarding an uncertain price profile is still lost with this method. For water course topologies where time constraints are less important, the water value method may give good results.

With the multi-scenario method, several price scenarios are considered and thus both the level and profile uncertainty of the price may be addressed. For each run of the deterministic model the results will be optimally adapted to the input prices. Hence the scenarios will have different production patterns depending on their input price profile. However, when the results are sorted into the form a bid matrix, the connections in time within the scenarios may be lost since the production volumes are sorted independently for each hour. The price profile uncertainty is hence not fully considered throughout this method. For our three test days, the multi-scenario method performs better than both the expected value and water value method.

We get the highest revenues when using the stochastic model for bid determination. This method is able to consider both the uncertainty in price level and profile and is the only method that formally solves the bidding problem. These results were obtained using the same information as is already available to the hydropower company when determining bids, and so the costs of advancing to the stochastic method seems low. However, calculation time is a major issue for the stochastic method. For the three test days, the stochastic optimization takes roughly 2.5 times as long as solving all the deterministic scenarios in sequence. For producers already pressed for time when using the deterministic model the long calculation time is the main challenge in the future development of the stochastic method.

## 7. Conclusions

This paper has investigated the use of a formal optimization model for determining bids to the day-head electricity market based on stochastic programming. The bids generated from the stochastic model have been compared to other methods of determining bids, all of which are different heuristic methods based on results from a deterministic model.

In a case study representing real operating conditions and using the same information as was available to production planners in a hydropower company at the time of bidding, the stochastic model outperforms the heuristic methods in terms of load commitments and higher obtained revenues. It must be stressed that the dates used in the case study were chosen because the producer thought that considering uncertainty would be of particular importance on these dates. The gains obtained may thus be overestimated, and cannot be generalized to all days of the year or all water course topologies. Our results are however consistent with other finds in the literature. The multi-scenario deterministic method seems like the most successful alternative as the calculation time for the stochastic model is currently a limiting feature.

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