MATHEMATICAL MODELLING OF A NORDIC HYDROLOGICAL SYSTEM, AND THE USE OF A SIMPLIFIED RUN-OFF MODEL IN THE STOCHASTIC OPTIMAL CONTROL OF A HYDROELECTRICAL POWER SYSTEM

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1. INTRODUCTION

In the planning of long-term hydroelectric power production, a kind of stochastic optimization where some particular assumptions on the boundary conditions are implicitely present, are widely used in Norway ("The water value method", based on the incremental cost principle).

Usually the calculation is also based on the assumption of no time-correlation in the stochastic part of the run-off, i.e. the white noise assumption. To get an idea of the effect of such a simplification, it is of great interest to investigate the importance of coloured noise in the run-off, i.e. the effect of dynamical states in the system which governs the run-off to the primary controlled hydroelectric water reservoirs to be controlled.

The first stage in such a project is the hydrological model-building. Such a model may have several purposes, as:

a. An aid in the simulation and better understanding of the dynamics of hydrological systems.

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*) The main part of this work was done while the authors were with the Division of Automatic Control at The Norwegian Institute of Technology.

b. River flow prediction.

c. Simulations for sub-optimal hydroelectric power systems planning and production.

d. In the computation of stochastic optimal control laws of power production.

In the case a it is obviously preferable to have a model which is physically based, while this is not necessary for instance in the case d. In the latter case, a simple abstract model which posesses the main dynamics is appropriate, partly because of unavoidable uncertainty in the long range all the same, and partly because of the difficulties encountered when applying too complex models in optimization.

2. HYDROLOGICAL MODELLING

2.1. Process characteristics and the multilevel approach.

Three kinds of models of an IHD-representative basin are presented, where different degrees of complexity are suggested. All of them has a multilevel structure. The first level consists of lumped, interconnected nonlinear reservoirs, where the water contents are the dynamical state variables. The second level changes some parameters in the model when the states exceed certain definite values, and in dependence of some parameters governed on the third level. Finally, the third level governs some of the parameters according to the temperature history. This is necessary in Norway because of the alternating climatic conditions.

Consider a hydrological basin, as shown in figure 1. The hydraulic inputs/outputs are precipitation (v2, not shown), channel flow (q_s), groundwater flow (q_g) and evapotranspiration (q_e, not shown). The non-hydraulic inputs or disturbances as temperature, wind and sun radiation are also influencing the hydrological system to a greater or smaller extent.

It is difficult to make a reasonably simple and general model of such a distributed-parameter system like a hydrological basin.

A widely used approach in flow systems, for instance in chemical engineering, is to apply physical lumping of the system. Hence, we subdivide the basin into partial basins where the water storage parts of the model are considered as stirred tanks. In this way, the sub-basins can more easily be adapted to general, physically based, mathematical models. It is assumed that the lumping is done such that an acceptable accuracy in the description is obtained for the application in question.

A typical partial basin is shown in figure 2. The components v_i, which together with q_s and q_g are considered to be the main inputs/outputs (inflows/outflows) of the system, are measured. $q_{e(out)}$ is the total evapotranspiration [m^3/day]. The vector $\underline{y}$ is the measurement vector ($\underline{y} = \underline{y}(\underline{x})$), while $\underline{q}_{(out)}$ is the outputs (outflows) from the model.

Observe that the flows $q_{s(in)}$ and $q_{g(in)}$ in general may consist of several contributions. Firstly, we assume that the partial basin is sufficiently homogeneous such that mean values characterizing the disturbances, the surface and the soil (precipitation, evapotranspiration, temperature etc.) are good approximations. Secondly, we assume that the basin is an uncontrolled, natural basin with soil, i.e. urban basins, glaciers and areas with naked mountains only are not considered. This forms the basis of the physical lumping in the model-building. The idea is of course not new in hydrological model building; physical approximation and representation of underground reservoirs by tank models have been used with success [2].

The crust of frozen earth and the snow during the winter season complicate a Nordic model, since the temperature and its history (the temperature is in fact a state variable in a possibly enlarged model of nature in this respect) is of importance for the discharge from the basin. Another problem is how the infiltration progresses, because infiltration is not measured systematically by the hydrologists.

Considering the time aspect, we are interested in a model encompassing the most important long-term properties, since its potential use is for economical dispatch of hydroelectric power at long sight. However, it ought to have a certain degree of accuracy with respect to estimated run-off, such that prediction errors important to the economical dispatch are reasonably well minimized. Expressions like this, and "degree of accuracy" will be given special attention elsewhere [5].

It is seen that the nature may be considered to function like a multilevel system. The complete structure is illustrated in figure 3.

In this paper the 1st level will be represented by a dynamical water balance system, which is assumed to be nonlinear and lumped.

Its simplified mathematical representation in continuous form is the vector differential equation

$$\dot{\underline{x}} = \underline{f}(\underline{x},\ v2(k), \overline{v3},\ \underline{q}_{(in)},\ \underline{p}_1(k),\ \underline{p}_2, \underline{\alpha}) \tag{1}$$

and

$$\underline{q}_{(out)} = \underline{q}(\underline{x},\ \underline{q}_{(in)},\ \underline{p}_1(k),\ \underline{p}_2,\ \underline{\alpha}) \tag{2}$$

$$\underline{y} = \underline{g}(\underline{x},\ \underline{q}_{(in)},\ \underline{p}_1(k),\ \underline{p}_2,\ \underline{\alpha}) \tag{3}$$

Here $\overline{v3}$ is the mean evaporation during the spring and the summer, v2 is precipitation, $\underline{q}_{(in)}$ is the inflow vector and $\underline{p}_1(k)$, $\underline{p}_2$ are parameter vectors steered from the higher levels of the model. $\underline{p}_1$ is piecewise constant in time, and is changed discretely in time with fixed intervals. $\underline{\alpha}$ is the unknown parameter vector (to be determined), and finally, $\underline{x}$ is the state vector, comprising the volumes of water in the tanks of the model. $\underline{q}_{(out)}$ is the outflow vector, being a direct function of the parameters, inflow and states, and $\underline{y}$ is the measurement vector.

The second level consists of a system governing state-dependent parameters $\underline{p}_2$,

$$\underline{p}_2 = \underline{p}_2(\underline{x},\ \underline{p}_1(k)) \tag{4}$$

On the third level, the "seasons" are used as "states", and these are governed by the temperature (v1) history, the latter being an input to the model. On this level, certain temperature-dependent parameters $\underline{p}_1$ are directly given by the season vector $\underline{p}_0$,

$$\underline{p}_1(k) = \underline{p}_1(\underline{p}_0(k)) \tag{5}$$

whereas the transitions of $\underline{p}_0$ are given by a Huffman table, which formally may be written as

$$\underline{p}_0(k+1) = \underline{h}(v1(k),\ \underline{p}_0(k)) \tag{6}$$

The components of $\underline{p}_1$ and $\underline{p}_2$ are of "on-off" type (zero and one). A diagram illustrating the possible transitions of "seasons" is given in figure 4. The Huffman table approximates the dynamics and hysteresis of the seasonal transitions. The components of $\underline{p}_0$ are the "season", a counting parameter to registrate the TMEAN-days period and the integrated temperature (in order to calculate its mean v1MEAN over TMEAN days).

2.2. Parameter observability.

All parameters of a practical hydrological model cannot be determined from simple observations and selective measurements of specific physical parameters. It is also clear that since a hydrological model is a simplified one of a distributed process, even exact knowledge of physical parameters is less valuable, since such parameters in greater or smaller extent will loose their physical interpretation in the approximate model. Hence, many parameters of the model have to be adjusted on the basis of measured input/output time series for the basin. The output measurements will normally be relatively few in number compared to the number of unknown parameters, and the question of state and parameter observability [10] of nonlinear models comes heavily into the problem of sensible model building. This question has been neglected in hydrological model building.

Of course a yes/no answer to the observability question is valuable. However, for practical design of a model, information about how observable the model is, is equally important. Information about this may for instance be obtained from the covariance of the parameter estimation error of an estimation algorithm [4], [5]. This problem will not be treated in this paper.

2.3. Model A.

For the first level, this version is shown in figure 5. (Level 1A.) State variables and parameters can as a rule be given a hydrological explanation, but this will not be done in detail here. However, in brief we have as states:

- x1: Land-surface water storage (water, ice, snow),
- x3: Reservoir storage (lakes), referred to the discharge threshold level
- x4: Accessible soil moisture
- x5/x6: The part of the groundwater volume which does not/does interact with the reservoir storage.

The parameters K_i ($i = 1,2,\ldots$) multiplied by the volumes x_i contribute to the rate of change of the volumes. Hence, a K_i is in principle the inverse of a time-constant. These parameters depend on a number of physical parameters like area, crust in the soil, the specific yield of the soil, the specific hydraulic conductivity, hydraulic inclination, depth to bedrock and the roughness and vegetation of the surface. The dimensionless parameters G_i ($i = 1,2,\ldots,7$) are difficult to determine a priori, but they are mainly dependent on area. The parameters A_i can be determined directly from a topographical map, since they depend on area only. The Q_i-parameters are dimensionless distribution parameters.

As is clear from figure 2, the measurements in this system are the groundwater level, water stage in the reservoir and the downstream flow rate from the reservoir. However, the latter is partly related to the water stage. The model on level 1A is thus given by 5 non-linear differential equations, 3 output flows given as functions of 5 states and 3 inputs, and finally 3 measurements.

On the second level (Level 2A) the value of the parameter vector $\underline{p}_2^T = (B1,B3,B4,B6)$ is dependent of the state vector $\underline{x}$ and the parameter vector $\underline{p}_1^T = (F1,F2,F3)$. The components of $\underline{p}_2$ change their values when the components of $\underline{x}$ exceed certain treshold values, the "D"-parameters.

On the third level (Level 3A), possible transition of the "season" is done every TMEAN days. We found that the representation of eq. (6) by a Huffman table was more convenient for the problem at hand than a cumbersome formulation with discrete-time equations containing logical expressions. The motivation for this level of the model, is the inertia in the temperature-dependent "parameters". Rapid temperature variations affect the hydrological system very little: The specific heat, melting and evaporation heat of water are large, and snow is a good insulator, too. This also means that the value and the duration of a positive temperature gradient must be larger to get the system switch from "winter" to "spring", than those required for a switch from "autumn" to winter". These phenomena are represented by hysteresis functions. The evapotranspiration is larger in the "spring" than in the "autumn", because of the increasing temperature and since larger areas are covered by water in the spring.

In this way, level 3A represents approximately the complex dynamics of freezing and melting in the nature. A first order differential equation describes approximately the melting (decay of x1).

Parameter observability of model A.

If the Schoenwandt criterion for local observability [10] is used, observability can easily be tested for the model, since the model is piecewise analytic in the states. A test can be made for each of the situations occurring with respect to reservoir levels versus the threshold values. It is then not surprising that the model A is not observable. There are 14 completely unknown parameters and 5 state variables to be estimated. In addition, it is to be noted that we have assumed that all the parameters on the 3rd level can be fairly well rated, and that the unknown "reference value" H1 (which is that part of the groundwater reservoir assumed not to influence the discharge from it, see figure 5) can be rated a priori.

The conclusion is that the model has to be simplified in order to get a model of a complexity which matches the amount of information got in this basin.

It may also be observed that model 1A is simpler than the now well-known Stanford Watershed Model [2].

2.4. Model B.

For this version, the levels 2B and 3B are the same as 2A and 3A respectively.

The 1st level, level 1B, is shown in figure 6, and is a simplified version of level 1A. The parameters and states of this model can however to a less extent than for model A be given a physical interpretation, apart from the fact that $\underline{x}$ still contains the "available" water resource in the basin. In particular, it is to be noted that the infiltration is not described by a differential equation in model B. G5 (= 1 - G6) encomprises in one constant the specific hydraulic conductivity, surface roughness and hydraulic inclination. Assume now that x1 can be estimated from measurements of v2, or by a measurement y4 using snow pillows. Assume also that as many of the parameters as possible are rated a priori with good accuracy, this includes all parameters on level 2-3. It then turns out that the following states and parameters must be estimated:

x2, x3, K4 (or K5), G3 (= 1 - G4), G5 (= 1 - G6) and G8.

If v2 $\neq$ 0 or x1 $\neq$ 0 one can prove by applying the Schoenwandt observability criterion that model B is locally observable in any state, provided the winter season is not present. This also applies if AL2 = 0 such that $\underline{y}^T = (y_1, y_2)$. During the winter, it turns out that G5 is not observable.

Such peculiarities of a hydrological model must be taken into account if a sequential state/parameter estimator is constructed, since non-observable parameters within certain time intervals should not be adjusted. This will not cause any trouble to us, since batch estimation is used, such that the best constant-valued set of parameters is found.

2.5. Model C.

In order to compare model B with a simpler version with respect to the 3rd level, model C contains Level 1B and Level 2A. On the third level, the Huffman table is not included, and "seasons" are made directly dependent on v1MEAN.

Under the same conditions as put on model B, this model is observable.

2.6. Adaption of the parameters.

In order to get some feeling of the problems encountered in this first investigation, a simple batch estimation of the parameters and states was tried. Although it is obvious that some of the parameters depend on the climatic conditions in a much more subtle way than in the models here, it is of interest to get an idea of how well such lumped models could be fitted to the measurement data. Since model A is not observable, the unknown parameters and states of the models B and C were adapted to measurements from a part of the IHD-representative basin "Sagelva". This part of the basin, which is illustrated in figure 7, is a small basin, but unfortunately not very homogeneous.

The well-known principle of many parameter estimation schemes is shown in figure 8, where $\underline{\alpha}$ represents the four unknown parameters (of model B) to be estimated. As adjustment strategy a simple hill-climbing method has been applied ("one-at-a-time") over a data interval of 2 years with very changing climatic conditions. (In a later work [5], a SIMPLEX search method included in a batch estimation program for the UNIVAC 1108 [4] was used, being considerably more efficient.) The loss functional to be minimized for optimal parameter values was taken as

$$S = \int_{t_1}^{t_2} (|y_{1m}(t) - y_1(t)| + 8 \cdot |y_{2m}(t) - y_2(t)|)dt \qquad (7)$$

Results from a "ballistic" simulation forcing the model B with the input data over 1 year, are shown in figure 9. $\bar{T}$ is the mean temperature during 15 days, and vl is precipitation per day. xi, i = 1,2,3, are simulated water storages in the basin, respectively land-surface water storage, groundwater storage and reservoir storage. yl is simulated groundwater level, while y2 is simulated reservoir water storage level. $y_m i$, i = 1,2, are the corresponding measured levels.

With the parameters obtained from the estimation, so-called recession ("dry weather"-) curves were simulated. These are shown in figure 10. Here q_s is surface discharge from the groundwater storage. They are both simulated according to the temperature history shown. In addition parts of recession curves being characteristic of each season are plotted: q_{ss} denotes pure summer surface discharge, q_{sa} pure autumn surface discharge, and q_{sw} correspondingly for the winter season.

Similarly, estimation and simulations were performed for model C, but the results were less reliable than for model B under unnormal winter conditions.

The conclusion is that for a Nordic hydrological model it seems necessary with some kind of sequential control of temperature-dependent parameters, which also in an approximate way takes care of the dynamics of melting and freezing under different conditions. It seems worth while to make further investigations on the basis of a model having a structure like model B.

3. STOCHASTIC OPTIMIZATION OF HYDROELECTRIC POWER DISPATCH

3.1. System description.

In the long term planning for the economical dispatch of hydroelectric power, the optimization interval over which the given performance functional is to be minimized (or maximized), usually is in the range of a few months to about one year. Because of uncertainty in the future run-off into the reservoirs, a reasonable goal is to minimize the expected value of the functional. Hence, we will have to consider a system model where the environmental model representing the run-off contains stochastic state variables. See figure 11, where we have

a. a mathematical process model for the production system, with control vector $\underline{u}$ and states (volumes) $\underline{x}_1$,

b. a lumped state variable model for the environment (state vector $\underline{x}_2$), yielding the run-off $\underline{r}(\underline{x}_2)$ to the reservoirs. The input to this model is an expected mean function v_o plus a white noise sequence Δv with a given distribution (the precipitation $v = v_o + \Delta v$).

In addition, there are given data for the power demand, which possibly also may be decomposed like the precipitation, in a mean value function plus a stochastic term.

In Norway it is usual to divide the optimization interval into subintervals of one week, and use the so-called "water value method" based on the incremental cost principle. (A description of the basic principle may be found in [9].) An analysis of this approach will show that the run-off is considered as pure stochastic (white noise) around a deterministic function of time. Considering for instance figure 10, it is observed - especially during the winter season - that such an approximation is less accurate relative to the fineness of the time discretization the smaller this discretization interval is. There is considerable dynamics in the run-off, which may be expressed by the autocorrelation function (in the linear case), or more generally, by a set of 1st order differential equations.

The dynamics will show up in the evolution of the probability distribution, as sketched in figure 12, which shows the "stationary" probability distribution of Δr as a function of time. In the linear, Gaussian case, the evolution of the probability density is uniquely given by the differential equation for the covariance $E\{\Delta r^2(t)\}$.

To be more specific, the complete system may be formulated as

$$\dot{\underline{x}}_1(t) = \underline{f}_1(\underline{x}_1(t), \underline{r}(\underline{x}_2(t)), \underline{u}(t), t) \tag{8}$$

$$\dot{\underline{x}}_2(t) = \underline{f}_2(\underline{x}_2(t), \underline{v}(t), t) \tag{9}$$

$$\underline{x}(t) \in X \quad (\underline{x}_1(t) \in X_1), \quad \underline{u}(t) \in U.$$

3.2. Discussion of the run-off model.

For long-term optimization problems of the kind discussed here it is obvious that uncertainty is very pronounced, as observed from figure 12. There seems to be no practical reason - at least for reasonably homogeneous or small basins - to work with higher order run-off models.

An abstract, 1st order linear model with a time-variable parameter ("time-constant") established, say, on the basis of initial condition responses ("recession curves") of a more complex model like the responses of figure 10, has the form

$$\dot{x}_2(t) = -a(t)x_2(t) + v(t) \tag{10}$$

where

$$v(t) = v_0(t) + \Delta v(t)$$

We may then assume a linear relationship between the environmental state x_2 of eq. (10) and the run-off r,

$$r(t) = k \cdot x_2(t) = r_0(t) + \Delta r(t) \tag{11}$$

Substituting into eq. (10), we have

$$\dot{r}(t) = -a(t)r(t) + k(v_0(t) + \Delta v(t)) \tag{12}$$

The recession function is given by the unforced solution of eq. (12),

$$r(t) = r(0) \cdot e^{-\int_0^t a(\theta)d\theta} \tag{13}$$

By letting a(t) be a function of time, it is possible to take into account the expected main seasonal changes in the climatic conditions. A sensible approximation is to apply three different values for a, these values respectively referring to the winter season, the snow-melting period and the period without snow, snow-melting and frost. The time constant $\frac{1}{a}$ is dependent on the basin, and is typically between 10 and 90 days, having its largest value during the winter.

During the snowmelting period, the water from the melted snow will usually be a dominating part of the run-off. A main part of this flow will be discharged into the reservoirs from the surface.

In this work, no attempt is done to make use of an optimal adaption of a(t) to the behaviour of the basin in question.

It is quite obvious that inertia in the run-off dynamics is of greater and greater importance the smaller the ratio between reservoir volume and integrated run-off to the reservoir through one year is. For instance, if a reservoir can accumulate on an average the run-off through 2-3 years (without discharge from the reservoir), it is obvious that a dynamical run-off model, characterized by a time-constant of about a month, will have almost no effect on the economical dispatch of such a system.

3.3. The optimization problem.

A dynamical description of the stochastic part of the run-off implies two essential distinctions for the economical dispatch problem, compared to a run-off which is not correlated in time.

a. Instead of using the "stationary" distribution of the run-off and possibly consider it as white noise, the dynamical evolution of the run-off and its probability density from a given initial condition, is taken care of (possibly with a given uncertainty in the initial condition).

b. Since we work with the expected evolution of the environmental states, these functions and their associated density functions are per definition given for the whole optimization interval. As is well known, this will in a control problem result in a realizable "feedforward" coupling from the environmental states to the control vector. Further, there will be a coupling from the reservoir volumes to the control vector, which is the "feedback part" of the control law. (Of course, in a nonlinear problem, these parts cannot be separated, but the principle is still there.) See figure 13.

To apply solution by Stochastic Dynamic Programming (S.D.P.), the system equations are used in their time-discrete form. With a discretization interval T, we have for a single reservoir,

$$x_1((k+1)T) = x_1(kT) - u(kT) + kx_2(kT) \tag{14}$$

and for the environmental model

$$x_2((k+1)T) = e^{-aT}\, x_2(kT) + \int_{kT}^{(k+1)T} e^{-a\left((k+1)T-\tau\right)}(v_o(\tau)+\Delta v(\tau)) \tag{15}$$

If v(t) is considered constant within the interval $\left(kT, (k+1)T\right)$, and Δv is taken as a discrete-time white noise sequence, the latter equation simplifies to

$$x_2((k+1)T) = e^{-aT}x_2(kT) + \frac{1}{a}(1-e^{-aT})(v_0(kT) + \Delta v(kT)) \tag{16}$$

To simplify the notation, we will in the sequel use $x_i(k)$ for $x_i(kT)$ etc.

The objective function for the optimal control of the system is as follows. In Norway it is commonly assumed that the marginal incomes/ expenditures dependent on the dispatch are a given function

$PF(u_p(k) - u(k))$, where PF is price per energy unit (öre/kWh). $u_p(k)$ is power as ordered by contract from customers within the optimization interval, and $u(k)$ is the actual power production. (GWh/month.) This function is often given as a staircase function like the one in figure 14. There is however uncertainty in the future power prices, so it might have been sensible to take this uncertainty into consideration. In S.D.P. this can be done without any difficulties, but with an increase in computation time. In the example here, however, the smooth curve as shown on figure 14 has been used without undertainty on it.

The expenditure within an interval $[k, k+1]$ is

$$W_k = \int_0^{u_p(k)-u(k)} PF(\mu)\,d\mu \tag{17}$$

The optimal criterion is to minimize the expected expenditures during the optimization interval (0,N),

$$E\{J\} = E\{\sum_{k=0}^{N-1} W_k(u_p(k) - u(k))\} \tag{18}$$

As data, the functions $u_p(\cdot)$ and $v_o(\cdot)$ and the probability density distribution $p(\Delta v)$ of Δv are given.

Since the main purpose here is to obtain a feeling of the importance of dynamical modelling of the environment of a hydroelectric power system for the economical dispatch, straightforward S.D.P. [1] is applied without any subtleties. The basis of the method can be studied in the textbook of Aoki [1]. An advantage in such applications as this using D.P., is that the state space is constrained because of maximum and minimum reservoir volumes. Also, maximum/minimum values for the run-off states may be rated fairly well. Complicated optimization criteria imply no difficulties. The most serious draw-backs are the well-known dimensionality problem and long computation time. The storage requirements for reasonably low-order systems (max. 4-5) may be solved by applying a mixture of different kinds of extensions of ordinary D.P. techniques [7], [8].

3.4. Example.

Computation of optimal controls for the first month in an optimization interval of five months in a certain year has been done using data for a small power station in the middle of Norway, named "Julskaret". The data of the production system are:

Power station.

Maximum storage capacity: 60 mill. m^3
Mean height difference between power station and the reservoir: 100 m
Mean energy conversion: 4.17 mill. $m^3 \rightarrow 1$ GWh
Machine installation: 8 MW.

This gives the constraints

$$0 \leqslant x_1(k) \leqslant 14.4 \quad \text{(GWh)}$$
$$0 \leqslant u_1(k) \leqslant 5.6 \quad \text{(GWh/month)}$$

$u_p(k)$ is given in the following table (dim u_p = GWh/month):

Month:	1	2	3	4	5
u_p:	2.7	1.9	4.6	4.4	3.9

The run-off system.

The total precipitation basin for the station is $A = 149.5\ \text{Km}^2 = 149.5 \times 10^6\ \text{m}^2$. The time-constant for the run-off is estimated to $T_1 = \frac{1}{a} = 1.2$ months on the basis of a recession curve. For simplicity, a^{-1} is assumed constant. We assume $r = k \cdot x_2 = x_2$. The run-off equation with $\dim[x_2] = m^3$, $\dim[v]$ = m/month, is

$$x_2(k+1) = e^{-\frac{T}{T_1}} x_2(k) + \frac{AT_1}{4.17 \cdot 10^6}(1 - e^{-\frac{T}{T_1}})(v_o(k) + \Delta v(k))$$

or

$$x_2(k+1) = 0.434\ x_2(k) + 24.8(v_o(k) + \Delta v(k))$$

which is assumed valid throughout the optimization interval. Realistic values of $x_2(k)$ are assumed to be within $0 \leqslant x_2(k) \leqslant 10$. The density function $p(\Delta v)$ is estimated on the basis of precipitation through 40 years. The data are not given here, but to get an impression of the spread, the variance $\sigma^2_{\Delta v}$ is given in the following table, where also $v_o(k)$ is tabulated:

Month k	1	2	3	4	5
$10^3 \cdot v_o(k)$	43	39	41	34	37
$\sigma_{\Delta v}(k)^2$	90	94	87	57	61

Performance criterion.

For the objective function the smooth curve $PF(u_p(k) - u(k))$ in figure 14 is used.

The results would be rather uninteresting in practice if the terminal state $x_1(N)$ is not considered in the optimization problem, since this would imply a policy which aims at emptying the reservoir towards the end of the optimization interval. Many kinds of criteria taking the expected final state into account could be thought of. For instance, an analysis of the principle of the procedure used in [9], shows that within the assumption of linearity in the process equations, the policy is to aim at reproducing the reservoir volume after one year [3]. A reasonable policy might be to let the expected final state $x_1(N)$ have a sensible value based on experience for that month of the season. A more direct, and in fact an equivalent approach, is to include a weighting on the final state in J, with such a weighting that the expected final state has a reasonable value. Hence, we use as an optimal criterion

$$E\{J'\} = E\{J + dx_1(N)\} \tag{19}$$

where J is given by eq. (17) - (18).

Results.

It is interesting to find the variation in the optimal power production $u_{opt}(0)$ of the first month as a function of the initial condition $x_2(0)$ in the run-off model. The results are shown in figure 15 for three different initial storages $x_1(0)$ in the power station reservoir and $\underline{d = 3}$. As expected, the initial state $x_2(0)$ has a considerable effect on the optimal policy. The expected final state $E\{x_1(N)\}$ (applying the expected run-off and picking the control from the computed tables of optimum stochastic controls) is 7.4 GWh at d = 3, and 8.2 GWh at d = 6. The two different values of d gave no difference in the optimum control for the first stage. However, at d = 0, $u_{opt}(0) = 3.2$ at $x_1(0) = 100\%$ (14.4 GWh). The control policy for the first stage is rather insensitive to the weighting factor on $x_1(N)$, as long as the expected final state has a reasonable value for the month in question. This is mainly an effect of the uncertainty of the future, and also indicates that it should not be necessary to use larger optimization intervals than, say, half a year, in order to compute the optimal control for the first month.

An interesting comparison is to compute the optimum control if Δr is pure stochastic (white) with approximately the same probability density as that one which can be estimated from the run-off observations. It is not surprising that the computed value in this case, u_{opt} = 4 GWh/month, at $x_1(0)$ = 100% corresponds to a value (see figure 15) which is close to the mean in the run-off for that month.

Of course, the numerical values obtained here should not be used in a general discussion of the goodness of approximation by using a non-dynamic run-off description in the computation of the economical dispatch for any hydroelectric power system. However, the example clearly shows that the problem should be given attention.

4. CONCLUSIONS

Results on simple batch parameter estimation of a hydrological system have been presented in the first part. The number and kind of measurements justify the synthesis of a rather crude model only. This conclusion has been drawn on the basis of observability analysis. Hence, it is not surprising that the goodness of fit will vary somewhat dependent on the season, and that the simple model has deficiencies like inaccurate reservoir level during the winter and the spring, and too low groundwater level during the late autumn. However, it should be kept in mind that the errors in the fitting will distribute on each variable according to the weighting factors in the loss functional [5].

In the last section, with respect to the application of a hydrological model in the stochastic optimization of a hydrological power system, it has been demonstrated that the use of a dynamical run-off model may be necessary in the computation of the optimal control. Although it is open for discussion how complex such a model should be, it is likely that significant improvements in the control policy can be attained by representing the most important dynamics of the environmental system in a simple first-order, stochastic model with time-varying parameters.

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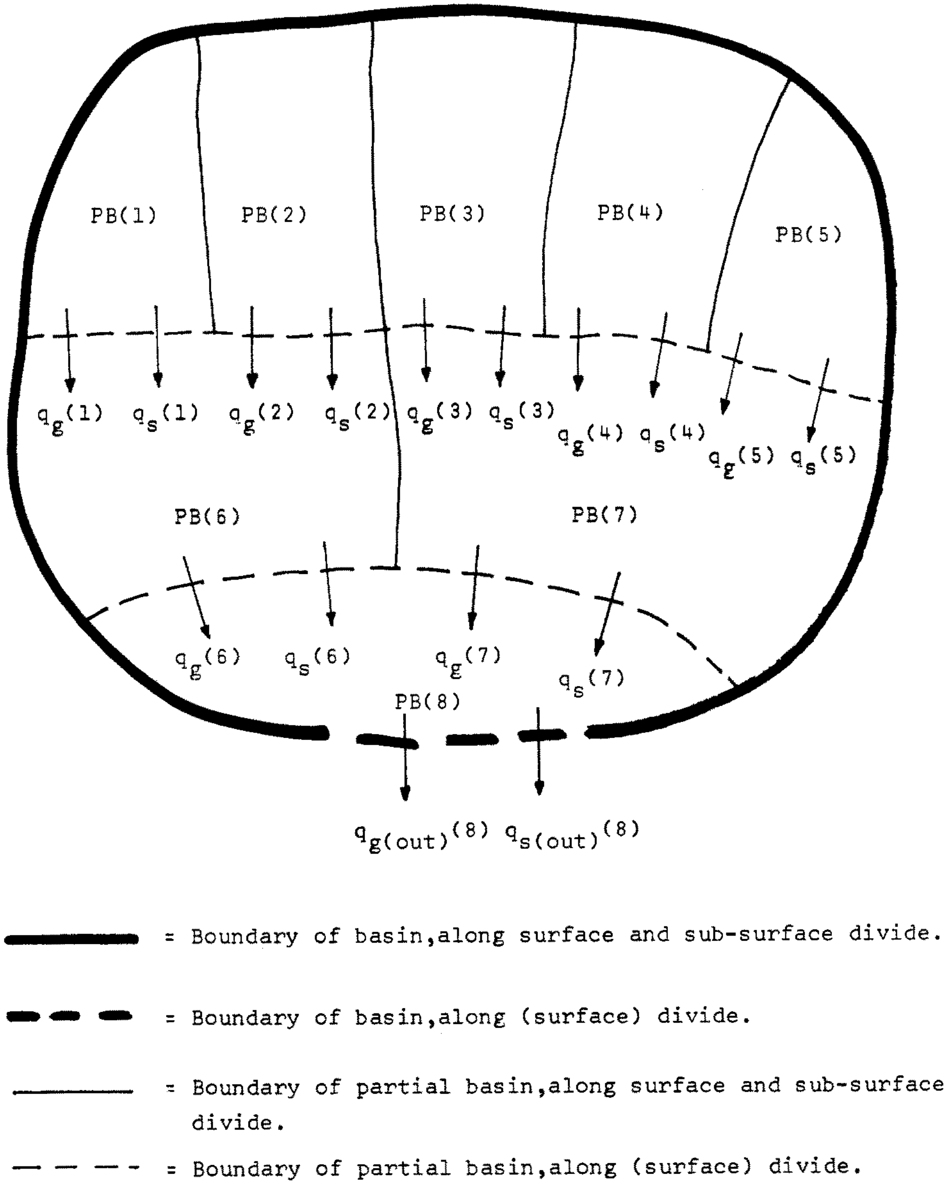
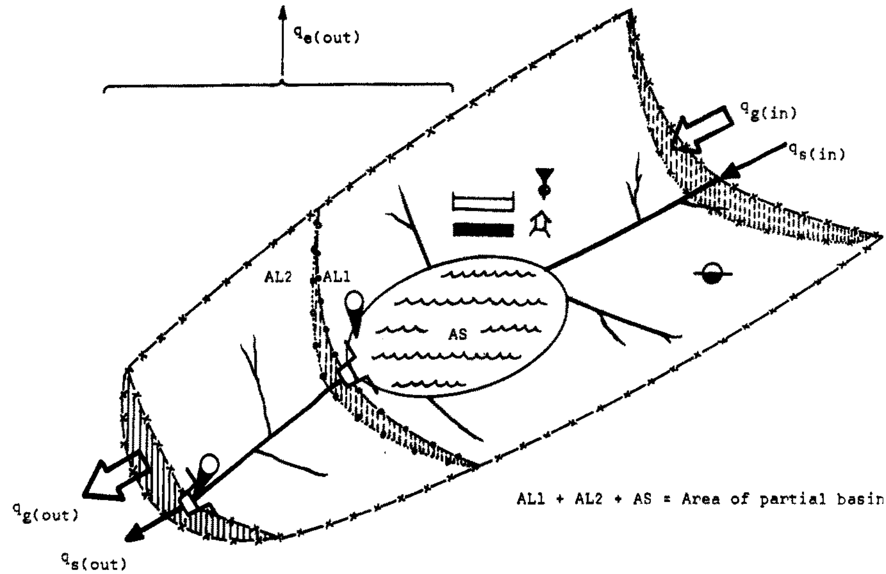


Fig. 1. A large (hydrological) basin.



AL1 + AL2 + AS = Area of partial basin (PB).

—x—x— = External boundary of basin,along divide.

—•—•— = Internal boundary of basin,along divide.

= Channel flow.

= Vertical section through soil moisture- and groundwater-zone.Only drawn where the divide is not also a sub-surface divide.

AS = Area of reservoir.

AL1 = Area of land,from where overland flow runs into reservoir.

AL2 = Area of land,from where overland flow runs into channel downstream reservoir.

= Meteorological station,with temperature recorder (v1).

= Recording precipitation gauge (v2).

= Evaporation pan
= Evapotranspirometer
are measuring evaporation ($v3_{AS}$) and average evapotranspiration coefficient (EL = $\left[\frac{\text{evapotranspiration}}{\text{evaporation}}\right]$).

= Recording groundwater level (y1).

= Recording water stage gauge,in reservoir (y2) or downstream reservoir (y3).

= Outlet or measuring weir,where the function q(y2) or q(y3) is known.

Fig. 2. A typical partial basin.

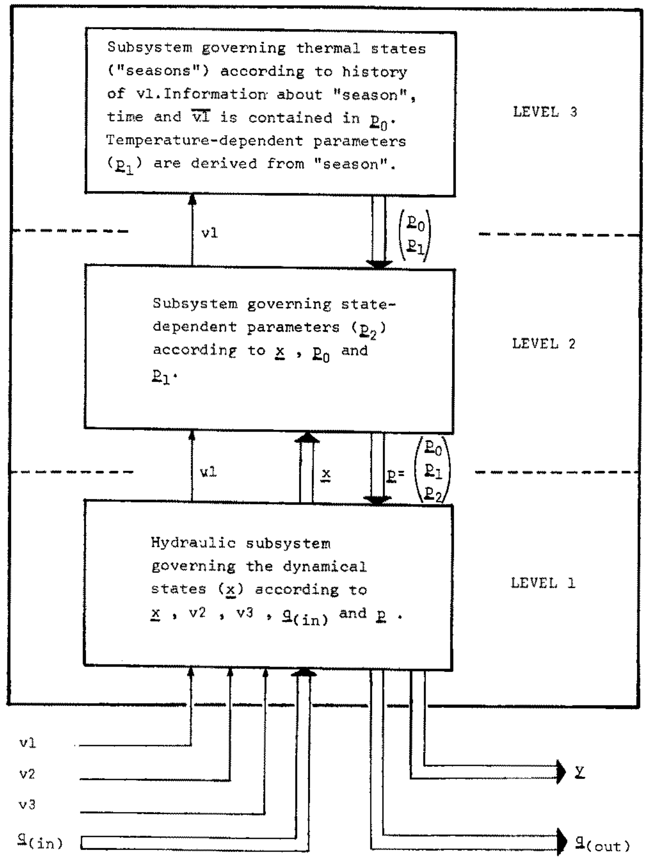


Fig. 3. The basin sketched as a hierarchical system.

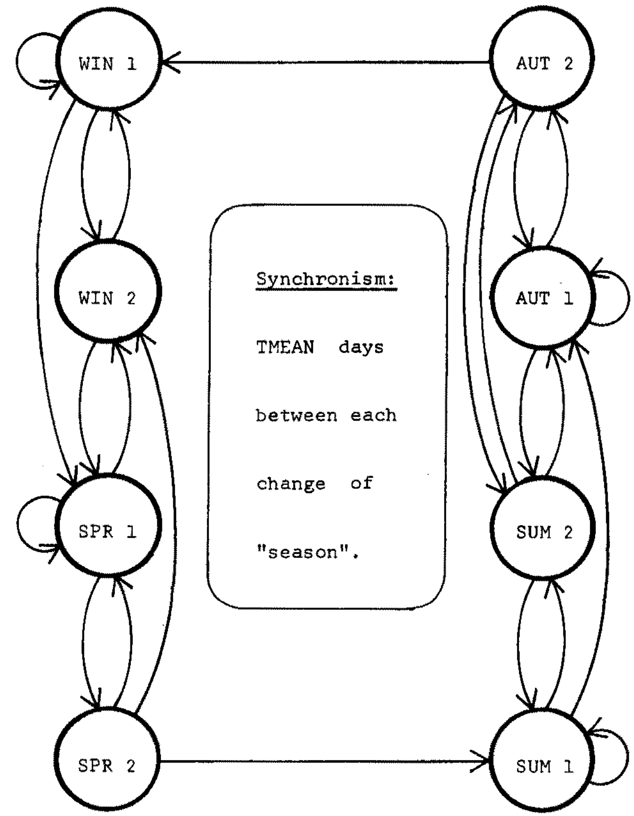


Fig. 4. "Season" diagram .

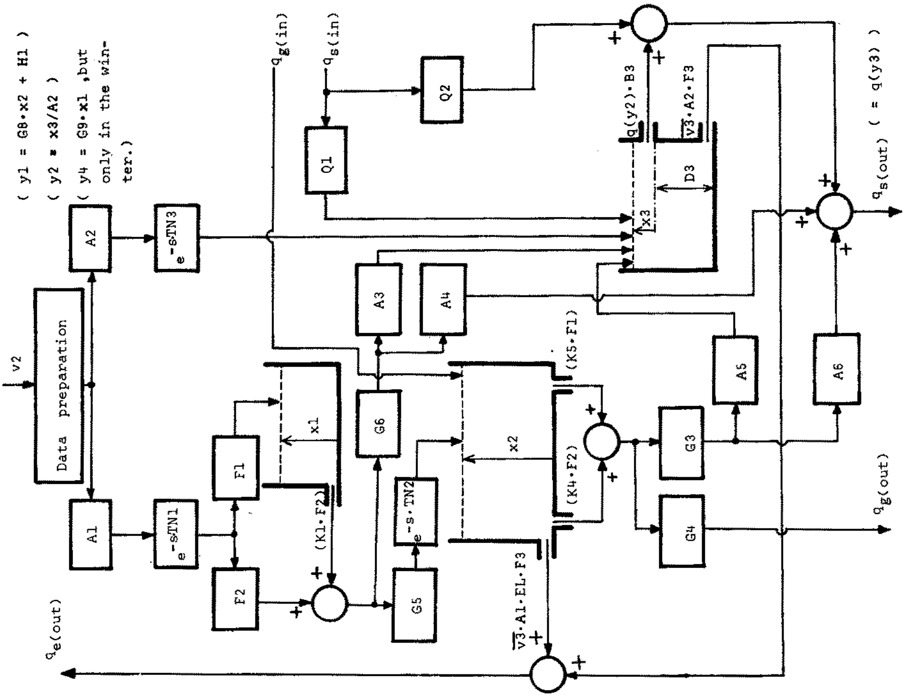


Fig. 6. LEVEL 1B .

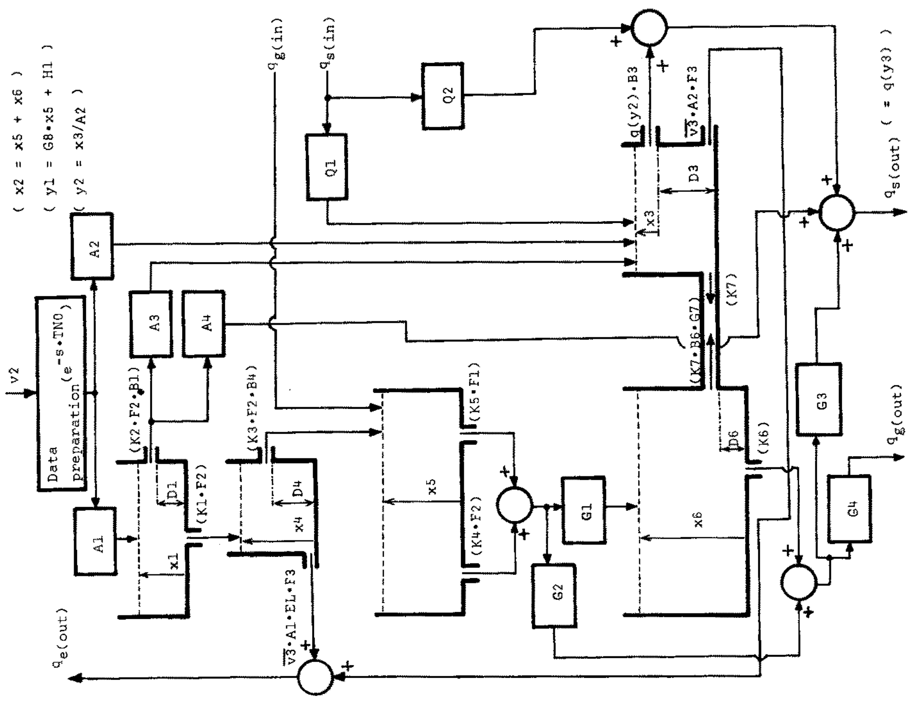


Fig. 5. LEVEL 1A .

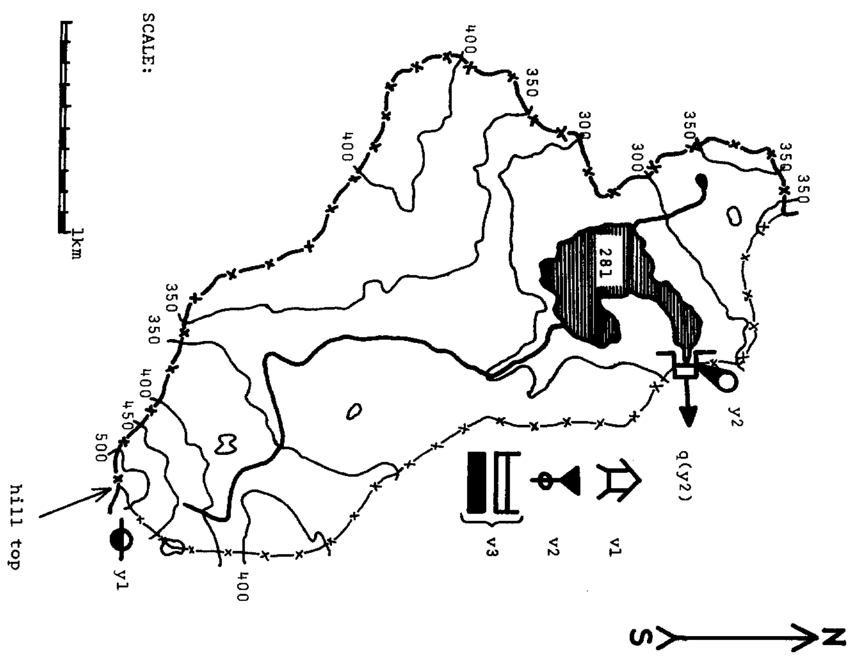


Fig. 7. A part of Sagelva IHD Representative Basin,Norway.

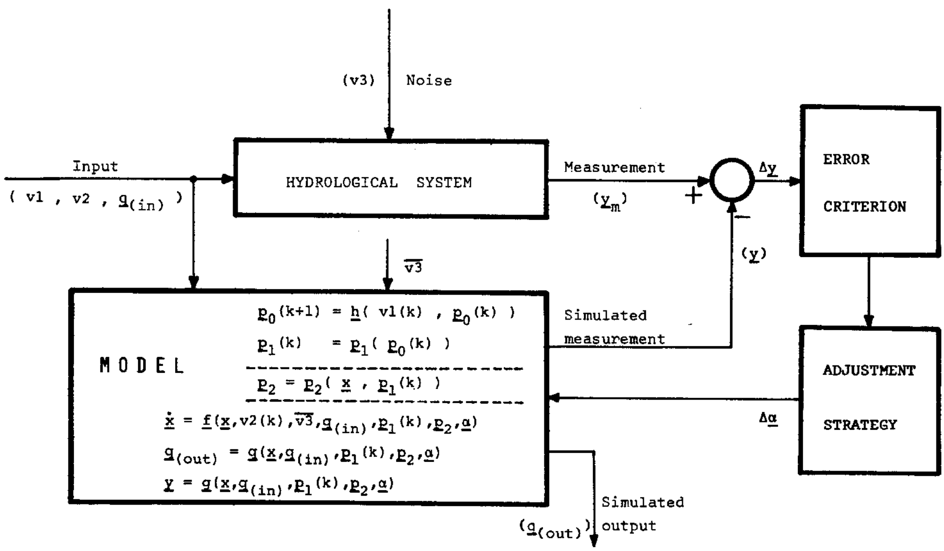


Fig. 8. Simulation and adjustment plan.

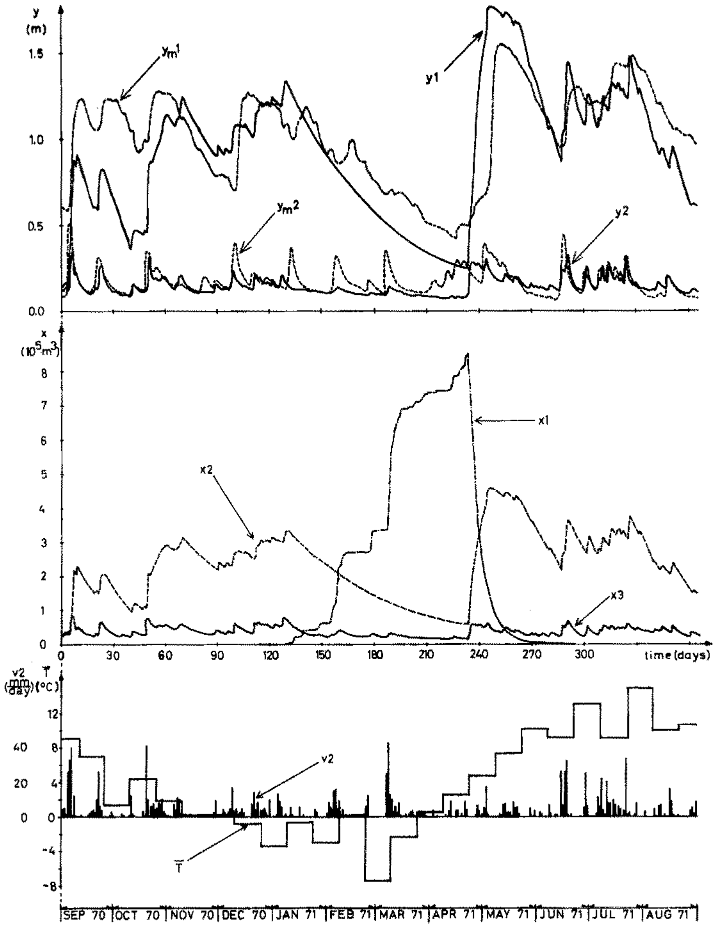


Fig. 9. Simulation and measurement.

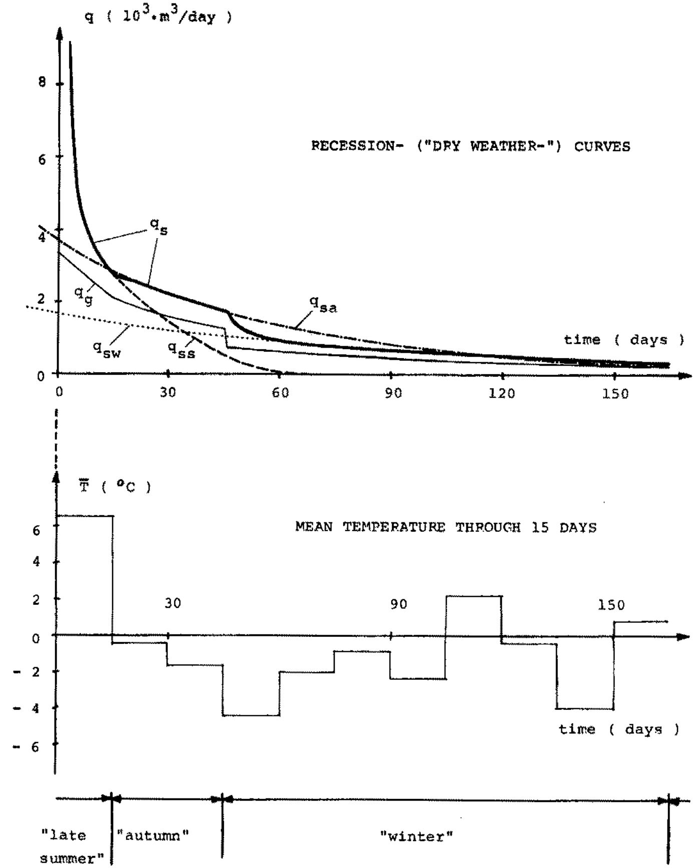


Fig. 10. Simulation with precipitation = constant = 0.

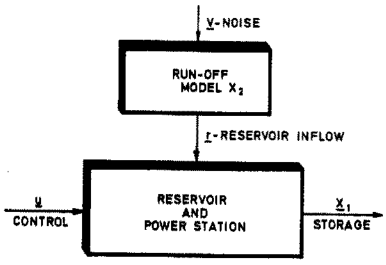


Fig. 11. Process and environmental model.

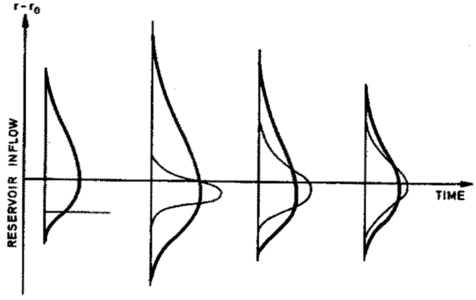


Fig. 12. "Stationary" and conditional evolution of the probability density.

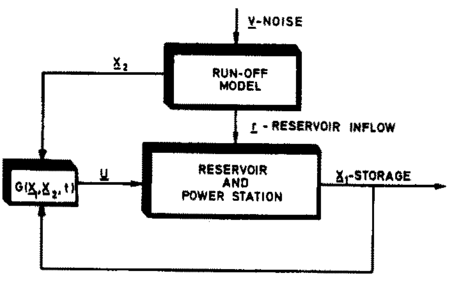


Fig. 13. Principle of control system solution.

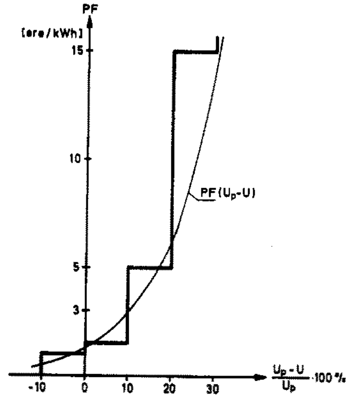


Fig. 14. Cost per energy unit.

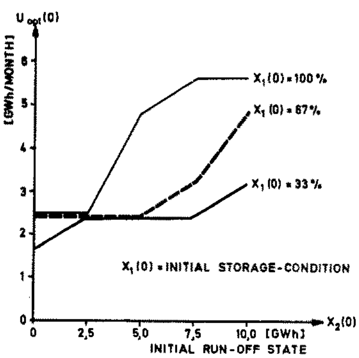


Fig. 15. Optimum control for the 1st month as a function of the initial values in the states.