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TITLE

A Survey of Heuristics for the Vehicle Routing Problem
Part I: Basic Problems and Supply Side Extensions

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CLIENT(S)

The Research Council of Norway

REPORT NO.	CLASSIFICATION	CLIENTS REF.	
SINTEF A8361	Open	140689/I30	
CLASS. THIS PAGE	ISBN	PROJECT NO.	NO. OF PAGES/APPENDICES
Open	978-82-14-04406-5	90A206	29/0
ELECTRONIC FILE CODE		PROJECT MANAGER (NAME, SIGN.)	CHECKED BY (NAME, SIGN.)
richsurv-I-SINTEF-TR-final.doc		Geir Hasle	Martin Stølevik
FILE CODE	DATE	APPROVED BY (NAME, POSITION, SIGN.)	
	2008-10-24	Roger Bjørgan	

ABSTRACT

This survey paper reviews the recent heuristic and metaheuristic solution methods for the well-known capacitated vehicle routing problem and arc routing problem as well as several extensions of the basic problems related to the supply side. Among the discussed extensions are time dependent travel times, multiple use of vehicles, tactical fleet size and mix problem and location-allocation routing. An introduction is provided for each topic and recent heuristic and metaheuristic solution techniques are briefly discussed. For earlier approaches, we refer to previous survey articles. The Vehicle Routing Problem (VRP) is one of the most well-known combinatorial optimization problems, and holds a central place in distribution management and logistics. The objective of the VRP is to deliver or supply a set of customers with known demands on minimum-cost vehicle routes originating and terminating at a central depot. Motivated by significant practical importance as well as considerable computational difficulty, there has been a huge amount of research on VRP and its different practical extensions. The purpose of this two-part survey is to review the recent heuristic solution methods for different multi-vehicle variants of the VRP. We focus on papers written in 1995 or after that. For earlier methods, we refer to previous survey papers. This first part reviews the methods for the basic capacitated vehicle routing problem and arc routing problem, as well as different supply side related extensions such as the fleet size and mix determination and the location of the support facilities. Extensions related to the demand side are discussed in the second part of this survey

KEYWORDS	ENGLISH	NORWEGIAN
GROUP 1	Transportation	Transport
GROUP 2	Optimization	Optimering
SELECTED BY AUTHOR	Vehicle Routing	Ruteplanlegging
	Rich VRP	Rike VRP
	Survey	Oversikt

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TABLE OF CONTENTS

1 Introduction **4**

2 The Basic Routing Problems **5**

2.1 The Capacitated Vehicle Routing Problem and Open Vehicle routing Problem 6

2.2 Arc routing 10

3 Supply side extensions **12**

3.1 Heterogeneous fleet of vehicles and usage of trailers 12

3.2 Time-dependent travel times 14

3.3 Multiple use of vehicles 16

3.4 Location Routing and Location-Arc Routing 16

4 Acknowledgements **20**

5 References **20**

1 Introduction

In Europe, the last decade has seen wide-ranging political and regulatory changes in several markets like telecommunications and transportation. These changes led to stronger interactions between the economies of the different European countries. On the other hand, demand has increased rapidly over the last decades, leading to an increase in transportation volume. Furthermore, according to forecasts, demand will continue to increase faster than the transportation network. Thus, improvements in the efficiency of the transportation services are desirable from both a macroeconomic and a microeconomic single company level. Freight transportation, in particular, is one of today's most important activities, not only measured by the yardstick of its own share of a nation's gross domestic product (GDP), but also by the increasing influence that the transportation and distribution of goods have on the performance of virtually all other economic sectors (Crainic and Laporte [1]).

Freight distribution operations are typically very complex, involving the interaction of many loads with different origins and destinations, a geographically distributed network of terminals with different facilities and capacities, a number of vehicles with different characteristics, drivers with their work rules and personal preferences, and customer requirements such as time windows and storage and handling requirements. All these factors change rapidly in a dynamic and stochastic environment. Managing these distribution operations is therefore a challenging task, and sophisticated decision support tools are needed to improve efficiency.

Effective distribution management presents a variety of decision-making problems at three levels. These levels are strategic, tactical and operational planning. Decisions relating to the location of facilities, e.g. plants and depots are viewed as strategic, while the problems of fleet size and mix determination are termed tactical. Finally on the operational level one has to make various decisions concerning routing and scheduling of vehicles. Vehicle Routing Problems (VRPs) are all around us in the sense that many consumer products such as soft drinks, beer, bread, snack foods, gasoline and pharmaceuticals are delivered to retail outlets by a fleet of trucks whose operation fits the vehicle routing model.

A typical VRP can be described as the problem of designing least cost routes from one depot to a set of geographically scattered points (cities, stores, warehouses, schools, customers etc). The routes must be designed in such a way that each point is visited only once by exactly one vehicle, all routes start and end at the depot, and the total demands of all points on one particular route cannot exceed the capacity of the vehicle. In practice, however, this basic model can be extended with various side-constraints, such as mixed pick-ups and deliveries, hard and soft time windows, route duration constraints etc.

The complexity of the side constraints dealt with in the literature to a large extent determines the applicability of academically developed methods and concepts to real-world vehicle routing problems. The aim of this two-part survey paper is to give an overview of the basic vehicle and arc routing problems, and some types of extended vehicle routing problems that are important to industry. An introduction is given for each problem type, and the recent solution techniques are briefly discussed. Given the high computational complexity of the routing problems, we focus on heuristic and metaheuristic solution methods. This first part focuses on the basic problems, i.e., the capacitated vehicle routing problem and arc routing problem. Some relevant supply side related extensions are also briefly discussed. Demand side related extensions are considered in the second part of the paper.

This article is organized as follows. In the next section we briefly introduce the basic routing problems, i.e., the vehicle routing problem and the arc routing problem, where the customers are modeled as arcs of the network instead of nodes. Section 3 is devoted to extensions related to supply side such as heterogeneous vehicle fleet, time-dependent travel times, multiple use of vehicles and location-routing models.

2 The Basic Routing Problems

One of the best known routing problems is the Traveling Salesman Problem (TSP). In TSP a number of cities have to be visited by a salesman who must return to the same city where he started. In solving the problem one tries to construct the route such that the total distance traveled is minimized. In the m-TSP problem, the m-salesmen have to cover the given cities and each city must be visited by exactly one salesman. Every salesman starts from the same city, called depot, and must return at the end of his journey to this city again. The vehicle routing problem is the m-TSP, where a demand is associated with each city or customer, and each vehicle has a certain capacity that cannot be exceeded. It can be said that vehicle routing problems are all around us in the sense that many consumer products such as soft drinks, beer, bread, snack foods, gasoline and pharmaceuticals are delivered to retail outlets by a fleet of trucks whose operation fits the vehicle routing model. In this section we describe the two basic vehicle routing problems: the capacitated vehicle routing problem and the arc routing problem. Most practical applications can be considered as extensions of these basic problems. Therefore, a lot of research has been directed to solve these problems effectively.

2.1 The Capacitated Vehicle Routing Problem and Open Vehicle routing Problem

The capacitated Vehicle Routing Problem (VRP) is defined on an undirected graph $G=(V,E)$ where $V=\{0,\ldots,n\}$ is a vertex set. $E=\{(i,j):i,j\in V,i<j\}$ is the edge set. Vertex 0 is a depot while the remaining vertices are customers. With each vertex of $V/\{0\}$ is associated a non-negative demand q_i and with edge (i,j) is associated a non-negative cost or length c_{ij}. The VRP consists of designing m vehicle routes of least total cost, each starting and ending at depot, such that each customer is visited exactly once, the total demand of any route does not exceed the vehicle capacity Q, and the length of any route does not exceed a preset bound L. The VRP was first formulated by Dantzig and Ramser [2]. Lenstra and Rinnooy Kan [3] showed that the VRP with or without side-constraints is an NP-hard combinatorial problem. Hence, exact algorithms are useful only for tiny problems. For real-life problems, heuristics are much more appropriate. Heuristics are often much faster than exact methods, but there is no guarantee of optimality in the solutions found.

Several families of heuristics have been proposed for the VRP. These can be broadly classified into two main classes: *classical heuristics* developed mostly between 1960 and 1990, and metaheuristics whose growth has occurred in the last decade. Most standard construction and improvement procedures in use today belong to the first class. These methods perform a relatively limited exploration of the search space and generally produce solutions of reasonable quality within modest computing times. Moreover, most of them can be easily extended to account for the diversity of constraints encountered in real-life contexts. Therefore they are still widely used in commercial packages. In metaheuristics, the emphasis is on performing a deep exploration of the most promising regions of the solution space. These methods typically combine sophisticated neighborhood search rules, memory structures, and recombination of solutions. The quality of solutions produced by these methods is usually much higher than that obtained by classical heuristics, at the cost of longer computing time.

Basically, two types of heuristics can be distinguished: initial and improvement heuristics. Initial heuristics generate a feasible solution to the VRP, given the data on customers, depot, vehicles and side-constraints. Principally three main groups of initial heuristics can be distinguished: route-construction heuristics, two-phase heuristics and heuristics based on exact algorithms. The most famous and used initial heuristics are those of Clarke and Wright savings [4], Solomon's cheapest insertion [5] and the sweep mechanism of Gillett and Miller [6]. Recently, Altinel and Öncan [7] suggested a new enhancement of the classical Clarke and Wright savings heuristic that considers customer demands in addition to distances. The authors report 2–5% relative improvements, although at the cost of higher computing time. Campbell and

Savelsbergh [8] demonstrate implementation strategies for insertion heuristics to handle complicating constraints, often faced in real-life problems, efficiently. The considered constraints include time windows, shift time limits, variable delivery quantities, fixed and variable delivery times and multiple routes per vehicle. An initial solution to the VRP can be enhanced through the application of an improvement heuristic. These procedures try to improve a feasible solution by relocating and/or exchanging stops within or between routes. For details, we refer to r-opt exchanges of Lin [9], Or-opt exchanges of Or [10], 4-opt* exchanges of Renaud et al. [11], string relocation schemes of Van Breedam [12], edge exchanges of Kindervater and Savelsbergh [13], λ-interchange mechanism of Osman [14], 2-opt* exchanges of Potvin and Rousseau [15], ejection chains of Rego [16], compounded large neighborhoods of Ergun et al. [17] and Agarwal et al. [18], and the surveys by Laporte et al. [19], Beasley et al. [20] and Funke et al. [21]. Funke et al. [21] present also an analysis of different neighborhood structures that shows how the properties of the partial moves and the constraints of the VRP influence the choice of an appropriate search technique.

Global optimization heuristics, on the contrary, succeed in leaving the local optimum by temporarily accepting moves that cause a worsening of the objective function value. These heuristics are often called metaheuristics because the procedure used to generate a new solution out of the current one is embedded in a heuristic that determines the search strategy. As far as we are aware, six main types of metaheuristics have been applied to the VRP: 1) simulated annealing, 2) deterministic annealing, 3) tabu search, 4) genetic algorithm, 5) ant systems and 6) neural networks. For details, see excellent surveys by Laporte et al. [19], Gendreau et al. [22], Cordeau et al. [23, 24] and Tarantilis et al. [25]. So far tabu search heuristics have proved to be amongst the most successful. See for example Gendreau et al. [26] and adaptive memory programming methodology of Rochat and Taillard [27]. The adaptive memory concept is applied also in Tarantilis and Kiranoudis [28] and Tarantilis [29] where the search is based on memorizing and attempting to combine attractive node sequences that have appeared frequently during the search. The created partial solutions are then completed with a savings heuristics and improved with a tabu search and 2-opt and vertex swap and relocation heuristics. Barbarosoglu and Ozgur [30] used the tabu search to solve real-life distribution of electronic household commodities. The applied algorithm is fairly simple, employing small standard neighborhoods and no diversification. Toth and Vigo's [31] efficient granular tabu search is based on retaining edges whose length does not exceed a given granularity threshold. Golden et al. [32] suggest a new metaheuristic that combines a similar granularity principle as of Toth and Vigo [31] with the record-to-record principle of Dueck [33] and savings and relocate and exchange heuristics. Li et al. [34] continue the work and introduce a new set of very large-scale problems up to 1200 customers. In Li et al. [34] also 2-opt neighborhood is used. The idea of granularity is applied also in Park et al. [35] who describe a fast path-exchange-type local search that uses edge lengths to restrict the search to promising moves. The unified tabu search of Cordeau et al. [36] has shown good performance in a number of different vehicle routing problems through allowing intermediate infeasible solutions. The neighborhood used is the well-known GENI-heuristic of Gendreau et al. [37].

The genetic algorithm of Prins [38] is based on standard construction heuristics for creating the initial population, and well-known order crossover and arc and node exchange local search heuristics. The results are concluded to be within 0.08% from the best known. Baker and Ayechew [39] apply also well-known local searches during the search, but a part of the initial population of the GA is created randomly, and the classical 2-point crossover is used for recombination. The authors report results within 0.5% from the best-known with reasonable computation times. In the hybrid genetic algorithm of Berger and Barkaoui [40] the search is performed through combinations of large neighborhood search principle, edge and vertice exchanges, and Solomon's [5] cheapest insertion heuristic. The authors report average results within 0.5% from the best-known using competitive computing times. Kubiak [41] and Jaszkiewicz and Kominek [42] present a genetic local search method with distance preserving recombination operators with respect to similarity measures of solutions. The operators are based on preserving edges, vehicle assignments or clusters of customers common in both parents. The local search part makes use of the standard savings and 2-opt heuristics and a sector relocation procedure. The key element in the proposed approaches is the use of global convexity tests that allow finding the solution features that are essential for solution quality. In the evolution strategies of Mester et al. [43] the search is mainly driven by mutation based on a remove-insert mechanism and a composite of standard improvement heuristics. In addition, two embedded decomposition schemes are proposed to speed up the search. The authors report solutions within 0.1% of best-known solutions to a set of CVRP benchmarks. Mester and Bräysy [44] present an improved version of the method of Mester et al. [43]. The improved method combines the evolution strategies with guided local search metaheuristic [45], resulting in iterative two-stage procedure. The suggested method yields the best-known solution to 70 out of 76 tested benchmark instances.

Reimann and Doerner [46] report also excellent results by applying an Ant System, based on the savings heuristic of Clarke and Wright [4], decomposition of the solutions to smaller subproblems, and on the application of 2-opt local search to ants' solution. The best results are concluded to be within 0.06% from the best known. For a parallel implementation of the algorithm, we refer to Doerner et al. [47]. Bell and McMullen [48] suggest also an efficient ant system. Its key components are usage of multiple ant colonies, candidate list strategies for customers considered for routes and 2-opt improvement heuristic. The downside of the heuristic is high variability of results: in some cases results are over 10% worse than the best-known. Another very simple ant colony algorithm is described in Mazzeo and Loiseau [49].

For the other recent approaches, see Renaud et al. [50] who proposed a quick heuristic based on creating 1- or 2-route petals to service all customers within a given sector, and combining the created routes by solving a set partitioning problem. Girard et al. [51] describe a simple perturbation heuristic that applies only suggested new weighted Clarke and Wright savings heuristic and 2-opt and 3-opt improvement heuristics. The solutions are perturbed by randomly splitting routes. The authors report solutions within 0.97% of the best-known solutions. Baker and Sheasby [52] extended the generalized assignment heuristic by simple local search and by adjusting the seed positions to reduce the optimal objective value for the generalized assignment problem. In Campos and Mota [53] a generalized assignment heuristic or savings heuristic (Clarke and Wright [4]) is used to create an initial solution that is subsequently improved with relocations or exchanges of single customers, guided by tabu search. A variant that uses the information produced by a branch-and-cut scheme to produce subsets of customers served by the same vehicle is proposed as well. Zeng et al. [54] suggest a simulated annealing metaheuristic with ruin and recreate local search. On each iteration, a certain number of nodes are removed from the solution. The removed nodes are then inserted back by solving an assignment problem with the well-known Hungarian algorithm. The method appears to be quite fast and the reported results lie within 1.1% from the best-known on the average.

Tarantilis and Kiranoudis [55] propose a modification of the deterministic annealing based on well-known local search neighborhoods, and combine it with a spatial decision support system to tackle a real-word VRP in the Athens area. The role of the decision support system is to check addresses, guide the development of vehicle routes, and analyze and represent the solution. Baker and Carreto [56] introduce a visual interactive approach where the user is allowed control of the routes and GRASP heuristics used for the search. The authors report results within 1% from the best-known solutions, resulting from interactive sessions of 15 minutes. Irnich et al. [57] introduce an efficient technique, called sequential search, for scanning neighborhoods within local search algorithms. The key idea is to systematically decompose moves, which allows pruning within local search based on associated partial gains. The authors report substantial speedups. Pisinger and Röpke [58] present a unified heuristic capable of solving several variants of vehicle routing problem. The search is based on adaptive large neighborhood search that adaptively chooses among a number of removal and insertion heuristics to intensify and diversify the search. For CVRP the authors report results within 0.11–2.4% from the best-known with competitive computing times. Kytöjoki et al. [59] present a very efficient variable neighborhood search heuristic that is specifically aimed at solving very large-scale problems. The search is based on seven standard improvement heuristics. In addition, a simple guided local search strategy and several new implementation strategies are used. The authors report competitive results for problem instances up to 20 000 customers within reasonable CPU times.

Jozefowiez et al. [60, 61] study a bi-objective CVRP. In addition to distance, also balance of the tour lengths is considered simultaneously. The authors suggest a parallel evolutionary algorithm that utilizes island model and elitism for diversification and intensification. The results are post-optimized with a tabu search and Or-opt neighborhood. Thammapimookkul and Charnsethikul [62] deal with real-word ATM routing problem with two objectives: total travel time and time of the longest tour. A modification of Clarke and Wright savings and exchange heuristics are applied to the problem.

Sariklis and Powell [63] consider a variant of the CVRP, called Open VRP (OVRP). The major difference to CVRP is that in OVRP each route is a Hamiltonian path instead of Hamiltonian cycle. This difference is due to the fact that the vehicles do not return to the starting depot or, if they do so, they must travel their trip until now backwards. In several problems there is a limit set for the total travel time of vehicles, and the primary objective is to minimize the number of vehicles, followed by distance minimization. The problem is of particular importance for planning fleets of hired vehicles. The authors suggest a two-phase method based on clustering and reassignment of customers between clusters. Brandão [64] study the same variant and suggests a tabu search metaheuristic. The initial solution is generated with nearest neighbor and insertion heuristics and the same heuristics are used also within the tabu search together with the US procedure of Gendreau et al. [37]. Another tabu search approach with different neighborhood structure is presented in Fu et al. [65]. Tarantilis et al. [66] suggest a threshold accepting metaheuristic [67] that makes use of standard 2-opt, relocate and exchange neighborhoods. The search is started from a solution where each customer is served by a separate vehicle. The reported results for benchmark problems outperform previous methods. In addition, a case study from Athens is reported. Pisinger and Röpke [58] report also very competitive results for OVRP with their large neighborhood search method described above.

2.2 Arc routing

In Arc Routing Problems (ARPs), the aim is to determine a least cost traversal of all edges or arcs of a graph, subject to some side constraints. Compared to more common node routing problems, customers are here modeled as arcs or edges. Such problems arise naturally in several applications related to garbage collection, mail delivery, snow clearing, meter reading, school bus routing, police patrols etc. In addition a number of industrial applications such as laser beam plotting

(Ghiani and Improta [68]) and task sequencing (Anily et al. [69]) have recently been described. ARPs have received far less attention than so-called node or vertex routing problems common in collection and delivery operations. A good treaty of the relevant theory for the ARP with variants and solution methods can be found in Dror [70]. For recent surveys on ARPs, see Eiselt et al [71, 72], Assad and Golden [73] and Dror [70].

One of the most typical side constraints is the capacity constraint for the vehicles. The corresponding arc routing problem is called Capacitated ARP (CARP). It can be characterized as follows: A set of customers has to be served by a fleet of vehicles operating from one or more depots. Each vehicle starts and ends its route at the depot it is assigned to. Furthermore, it has given capacities with respect to time and quantity. Additional side constraints may also exist. Important variants of the ARP are the Chinese Postman Problem (CPP), where arc traversal may be duplicated, and the Rural Postman Problem (RPP), where only some edges are required. Ghiani et al. [74] present a survey of some recent algorithmic developments for the RPP and CARP and describe some heuristics for the problems.

Recently Hertz et al. [75] proposed an efficient tabu search algorithm for the CARP. The approach is called CARPET and it is based on four simple procedures that are used to transform between RPP and CARP and reverse the order of subtours. The RPP solutions are generated with Frederickson's [76] algorithm. Another tabu search application can be found in Greistorfer [77]. Mittaz [78] developed a Variable Neighborhood Search (VNS) that is based on CARPET for the directed RPP. The author concludes that for larger instances VNS is better than CARPET in terms of solution quality, and also faster. Genetic Algorithm (GA) was first applied to the CARP in Lacomme et al. [79] with good results.) Beullens et al. [80] report very good results with a Guided Local Search (GLS).

Amberg et al. [81] consider CARP with multiple centers. The objective is to find routes starting from the given depots such that each required arc is served, capacity constraints are satisfied and total travel cost minimized. The authors propose a heuristic transformation of the problem into a multiple center capacitated minimum spanning tree problem with arc constraints, and a route-first-cluster-second algorithm for determining initial feasible solution as well as tabu search and simulated annealing-based improvement procedures for the transformed problem. The CARP with vehicle and site dependencies (CARP-VSD) is described by Sniezek et al. [82].

The CARP-IF variant arises when there are Intermediate Facilities for unloading the vehicles that are disjoint with the depot. A typical example is waste disposal or road gritting. Ghiani et al. [83] describe this problem, and give procedures for two sets of lower and upper bounds. The refuse collection problem for a part of Lisbon is described with a rich model in Mourão and Almeida [84] and heuristically solved to produce near-optimal solutions.

Clossey et al. [85] consider a variant with turn penalties, and Gendreau et al. [86] show how arc routing can be applied to a real-world problem. The clearing of snow on roadways can be modeled as a type of postman problem, with the addition of many real-world based side constraints. A survey of this problem is in Campbell and Langevin [87].

3 Supply side extensions

Less attention has been directed at extensions dealing with the supply side of a VRP. Nevertheless, a heterogeneous fleet of vehicles, most often in accordance with site-dependencies, are minimal requirements for practical applications. An important side-constraint is the maximal route time for the truck driver for instance due to specific industry regulations. The complexity of the VRP is increased when multi-compartment vehicles are considered. Recently, some attention is given to time-dependent travel times. The importance of this topic increases with the growing traffic saturation of highways and metropolitan areas. Finally, the combined problem of location-allocation of facilities and routing is briefly discussed.

3.1 Heterogeneous fleet of vehicles and usage of trailers

Although often assumed in theory, a trucking firm's vehicle fleet is rarely homogenous. Vehicles differ in their equipment, carrying capacity, age and cost structure. The need to be active in different market (e.g. container and bulk transport) causes firms to buy vehicles with a container chassis, dump installation etc. Vehicles of different carrying capacity allow a dispatcher to maximize capacity utilization by deploying smaller vehicles in areas with a lower concentration of customers. Moreover, it is also possible to service customers requiring small vehicles because of accessibility restrictions. The differences in equipment, carrying capacity and the fact that vehicles might differ in age, causes them to have a different cost structure. In contrast to traditional VRP, in Fleet Size and Mix Vehicle Routing Problem (FSMVRP) the vehicles are assumed to have heterogeneous capacities and acquisition costs. The objective is to minimize both routing costs and vehicle costs. Practical applications of FSMVRP include deliveries to grocery stores, pet food and flour delivery problem and the mail collection problem. In the literature one can separate several variants of the problem according to how the variable costs and fleet size are issued. For good reviews on the classical approaches for FSMVRP, we refer to Salhi and Rand [88] and Osman and Salhi [89].

The early solution approaches for FSMVRP include adaptations of the Clarke and Wright [4] savings heuristic, giant tour algorithms and two-stage general assignment based heuristic. For more details, see for example Dullaert et al. [90]. Salhi and Rand [88] developed a seven-phase heuristic based on matching demand and vehicle, eliminating, combining and splitting routes, and moving customers between routes. Osman and Salhi [89] improved the approach by Salhi and Rand [88] by considering also moves that were seen as infeasible in the previous approach using tabu search. Gendreau et al. [90] used also a tabu search heuristic that is based on the well-known GENIUS generalized insertion heuristics (Gendreau et al. [37]) and adaptive memory (Rochat and Taillard [27]) that works as a pool of partial solutions. Ochi et al. [92] present a hybrid metaheuristic that uses parallel genetic algorithms and scatter search coupled with a decomposition-into-petals procedure. Han and Cho [93] introduce a generic intensification and diversification search metaheuristic that incorporates concepts from the threshold accepting, the great deluge and the intensification and diversification strategies. Wassan and Osman [94] present a reactive tabu search metaheuristic with several neighborhood generation mechanisms and special data structures for efficiency, and Renaud and Boctor [95] describe an extension of the basic VRP algorithm of Renaud et al. [50] to heterogeneous fleet case. Lima et al. [96] hybridize a genetic algorithm with GENIUS [37] and λ-interchange [14] heuristics. The authors report 8 new best-known solutions for a set of 20 benchmarks.

Tarantilis et al. [97,98] suggest threshold accepting [67] algorithms to fixed fleet FSMVRP where the number of vehicles of each type is fixed and equal to a constant. The local search is performed with 2-opt and vertex relocate and exchange moves. Li et al. [99] adapt the record-to-record travel algorithm of Li et al [34] to the same fixed fleet problem. The differences between the methods appear small but on the average Li et al. report the best performance.

Liu and Shen [100] designed the first initial heuristics for the FSMVRP with time windows (FSMVRPTW), where the customers have a certain time window in which the service must begin (for more details on routing problems with time windows, see the second part of this paper). Their parallel savings heuristics are inspired by Solomon's [5] sequential insertion heuristics. Instead of linking routes, one route is inserted into another. Dullaert et al. [90] extend Solomon's [5] sequential insertion heuristic I1 with vehicle insertion savings, based on Golden et al. [101] and report significantly better results.

Mechti et al. [102] consider a real-life mail collecting optimization in an urban area. The problem is modeled as FSMVRP with Time Windows FSMVRPTW. The authors introduce a new type of tabu search approach, where on each iteration the best move is selected among a large variety of possible moves. The method varies between local moves involving only one route and global moves that change the solution structure more dramatically. The authors also present quite comprehensive literature review. Similar approach and also an exact algorithm are presented in Mechti et al. [103].

Gerdessen [104] study the VRP with trailers (VRPT), where one has to determine optimal deployment of a vehicle fleet of truck-trailer combinations and present construction and improvement heuristics for the problem. Truck-trailer combinations may encounter maneuvering problems at certain customer sites. Therefore the opportunity is introduced to leave the trailer at a parking-place and visit some customers with the truck only. Interesting application areas for the VRPT are the distribution of dairy products and compound animal feed among farmers. Chao [105] describes a tabu search heuristic coupled with the deviation concept found in deterministic annealing and well-known node and arc exchange neighborhoods. The initial solution is solved by combining relaxed generalized assignment for assigning customers to different types of routes (depending on whether or not trailer is with truck), and a cheapest insertion heuristics for constructing the tours. Scheurer [106] presents two new clustering-type construction heuristics and a tabu search heuristic with swift and swap neighborhoods and candidate list strategies. The search is intensified through restarts from the current best solution. The best-known solutions are reported to all VRPT benchmarks. Tan et al. [107] study a VRPT variant that considers the availability of trailers, time window constraints and multiple objectives. An effective evolutionary algorithm with specialized genetic operators, fitness sharing, variable-length representation and a route merging heuristic is suggested for the problem.

Bodin et al. [108] consider a similar problem in sanitation routing context, where the tractors move large trailers between locations and a disposal facility. The trailers are so large that the tractor can only transport one trailer at a time. The objective is to service all trips such that the number of tractors and nonproductive time are minimized. The authors call the problem Rollon-Rolloff VRP (RRVRP) and present mathematical programming formulation and four heuristics. The heuristics are based on solving set covering and bin packing problems over specific trip types, dynamic programming, simple cheapest insertion combined with route improvement procedures and adaptation of Clarke and Wright [4] algorithm. The same problem is discussed earlier in De Meulemeester et al. [109], who propose two simple heuristics. The first heuristic is the parallel version of the Clarke and Wright [18] savings algorithm for the VRP. The second heuristic is based on the solution of a transportation problem that is used to provide a lower bound.

3.2 Time-dependent travel times

Most available routing models assume that the travel times are constant throughout the day. In real-world conditions, however, the travel times are subject to variations over time. These variations may result from predictable events (e.g., congestion) or from unpredictable events like accidents or vehicle breakdowns. Therefore a solution to a problem assuming constant travel times may be suboptimal. And if customers requested service in tight time windows and/or if the scheduled route contains little slack, the stochastic nature of travel times can even make the schedule infeasible. Therefore, when designing routes, a dispatcher has to consider the stochastic nature of travel times. The interest in time dependent travel times has grown proportionally with the increasing traffic congestion problems. In addition, the importance of time-dependent travel times is largely dependent on the scale of the VRP. The smaller the scale on which to perform the routing, the more important it is to obtain an accurate planning. Discarding time dependent travel speeds in metropolitan areas can result in an underestimation of the total routing time.

The Time Dependent VRP (TDVRP) is a VRP for which the travel time between two nodes depends on the distance between the points and the time of the day. The time-dependency accounts for variations in travel speed caused by congestion etc. All other data is static and known. It is to be noticed that triangle inequality does not hold anymore for the TDVRP. The objective is often to minimize the total time of the routes.

Ratliff and Zhang [110] present methods to approximate driving speed as a piecewise linear function of the distance traveled, and provide also comprehensive literature review on the related approaches. Wunderlich et al. [111] propose a heuristic prediction technique for decentralized route guidance architectures to identify time-dependent link travel times that are then communicated to drivers to create faster paths, consistent with the forecast. Fleischmann et al. [112] present a general framework for the implementation of time-varying travel times in various vehicle routing algorithms. In addition, computational results obtained with savings and insertion heuristics with 2-opt are reported using real traffic data from Berlin. Taniguchi et al. [113,114] consider variable travel times in their time-constrained probabilistic model with multiple trips per day for each vehicle. Dynamic traffic simulation coupled with a genetic algorithm is used to test the model and quantify the benefits of considering the uncertainty of travel times.

Ichoua et al. [115] adapt the tabu search heuristic of Taillard et al. [116] and test it both in static and dynamic context. The authors conclude that fixed approximation of the real travel times is not competitive with usage of time-dependent heuristic. The authors consider the travel time equivalent to estimating the travel speed, and create a model that divides the horizon into several time periods, and satisfies the "first-in-first-out"-property. Donati et al. [117] adapt the multiple ant colony system, originally developed for VRP with time windows by Gambardella et al. [118] for the time dependent variant of the problem. The authors also introduce new segment exchange and relocation local search procedures that allow searching for the feasible moves in constant time. Results are reported with both benchmark and real data.

3.3 Multiple use of vehicles

One drawback of the standard VRP definition is that it implicitly assumes that each vehicle is used only once over a planning period, such as working day. In several contexts, it may be possible to assign several routes to the same vehicle and thus use fewer vehicles. Especially in situations, where the vehicle capacity is relatively small and the number of vehicles is given, multiple use of vehicles is the only option.

The vehicle routing problem with multiple use of vehicles is a variant of the standard vehicle routing problem in which the same vehicle may be assigned to several routes during a given planning period. Taillard et al. [119] describe a tabu search heuristic for this problem. The proposed method is made up of three parts. It first generates a large set of good vehicle routes satisfying the VRP constraints. It then makes a selection of a subset of these routes using an enumerative algorithm. Finally, it assembles the selected routes into feasible working days using several applications of a bin packing heuristic. Brandão and Mercer [120] propose also a tabu search for multi-trip vehicle routing problem. The method combines nearest neighbor and insertion concepts with two-phase tabu search and standard neighborhoods of reinserting a customer of exchanging a pair of customers. Brandão and Mercer [121] used similar heuristic to tackle real-life distribution problem faced by British biscuit manufacturer with multiple trips per vehicle. Petch and Salhi [122] suggest a multi-phase constructive heuristic that first constructs many feasible VRP solutions with a savings heuristic and then assigns routes to vehicles using a bin packing heuristic. The obtained solutions are improved with a set of standard improvement heuristics such as 2-opt, 3-opt and relocate. Olivera and Viera [123] propose an adaptive memory algorithm where new solutions are constructed periodically using a data in the memory and improved by a tabu search metaheuristic that allows infeasible intermediate solutions. The initial solution is constructed with a sweep algorithm and the tabu search works with the US procedure [37] and a new assignment heuristic. The authors report more feasible solutions to the benchmarks than previous papers.

3.4 Location Routing and Location-Arc Routing

While distribution management is traditionally divided into the long-term subproblem of location and the short-term subproblem of routing, it has been shown in the last decades that an integrated approach is more efficient. This is due to the fact that the two subproblems are inter-related in practice. This integrated approach is named location-routing. The Location Routing Problem (LRP) can be defined as follows: A feasible set of potential facility sites and locations and expected demands of each customer are given. Each customer is to be assigned to a facility which will supply its demand. The shipments of customer demand are carried out by vehicles which are dispatched from the facilities, and operate on routes that include multiple customers. There is a fixed cost associated with opening a facility at each potential site, and a distribution cost associated with any routing of vehicles that includes the cost of acquiring the vehicles used in the routing, and the cost of delivery operations. The cost of delivery operations is linear in the total distance traveled by the vehicles. The LRP is to determine the location of the facilities and the vehicle routes from the facilities to the customers to minimize the sum of the location and distribution costs such that the vehicle capacities are not exceeded. A number of issues related to the problem perspective have to be considered in practice. Examples of such issues are: stochasticity, number of facilities, planning horizon, time windows, vehicle and facility capacities, multiple objectives etc.

Location routing models are especially useful for systems where the time horizon for the facility location decisions is not very long, and location costs are comparable to the routing costs. Some application areas include distribution in food and drink industries, delivery to retail shops, delivery of newspapers, distribution of various consumer goods and services, engineering and waste collection.

The heuristic solution methods applied to the LRP can be divided in location-allocation first, route second (locate facilities first, then allocate users to facilities, and define the routes in the end), route first, location-allocation second (set of customers belonging to a vehicle route are first determined, routes are then constructed and facilities are located), savings and insertion heuristics, improvement/exchange procedures, use of route length approximation and tree-tour heuristic. Min et al. [124] present an excellent survey and explore promising research opportunities. Other reviews can be found in Laporte [125], Salhi and Fraser [126] and Salhi and Sari [127]. Berman et al. [128] provides a broad overview of the developments for location-routing problems for which the number and possibly the location of customers are described by a priori probability distributions.

Min [129] proposed both exact integer programming approach and location-allocation-first, route second heuristic with clustering for static real-world application. Bruns and Klose [130] propose a location-allocation-first, route second heuristic for LRP. a hierarchic agglomerative clustering method based approach is used to get an initial estimate of delivery costs, and the location subproblem is solved using a Lagrangean heuristic based on the relaxation of supply and capacity constraints. The routing subproblem is then solved with conventional tour construction heuristics combined with some improvement procedures. Similar study is reported in Klose [131].

Renaud et al. [132] describe a competitive tabu search heuristic. It constructs an initial solution by assigning each customer to its nearest depot and solving the resulting VRPs by mean of the improved petal heuristic (Renaud et al. [50]). Cordeau et al. [133] propose a tabu search heuristic for multi-depot VRP. The used insertion scheme borrows from Gendreau et al. [37]. The approach allows intermediate infeasible solutions, and it employs a diversification scheme based on a penalized function. Cordeau et al. [36] applied a similar tabu search approach for the Multi-Depot Vehicle Routing Problem with Time Windows. Polacek et al. [134] tackle the same time-constrained variant and report very good solutions to all instances with a variable neighborhood search algorithm. The initial solution is constructed with a greedy allocation heuristic and improved with variants of CROSS- and 3-opt heuristics using a threshold accepting [67] type acceptance criterion. Nagy and Salhi [135] develop a new estimation formula for route length approximation and propose a nested heuristic approach, where location is the master problem and routing is a subproblem. A tabu search approach with three simple neighborhoods for changing depot composition is proposed for master problem and subproblems is solved with heuristic of Salhi and Sari [127]. Tuzun and Burke [136] develop a two-phase tabu search based on simple local search moves and savings algorithm by Clarke and Wright [4] for the LRP and Crevier et al. [137] present an effective tabu search metaheuristic with adaptive memory framework. Their model considers also inter-depot routes. Pisinger and Röpke [58] report very competitive results for standard benchmarks with their adaptive large neighborhood search heuristic, described in Section 1.

Su [138] proposes a basic genetic algorithm with random crossover and mutation operators, determining simultaneously the location of distribution centers and fleet size and routing policy. Filipec et al. [139] propose a genetic algorithm coupled with well-known GA operators for TSP to solve non-fixed destination multi-depot VRP, where vehicles originate and terminate at different depots. The authors also set a maximum limit for customers served by one route to increase the reliability of supply. The genetic algorithm consists of three phases: clustering, radial routing and building the complete link structure. Thangiah and Salhi [140] suggest using a genetic algorithm to adjust special geometric shapes that are then used to cluster customers into separate sets. The routes for each set are created with an insertion heuristic, and the obtained solution is post-optimized by swapping and reallocating customers between routes and depots, and combining and splitting routes.

Sumichrast and Markham [141] consider a problem of delivering raw materials to plants, where the objective is to minimize both transportation and material costs, assuming the different sources have different prices. Clarke-Wright savings heuristic combined with procedures to exchange sources and depots between routes, as well as a lower bound obtained from a relaxed binary formulation are presented. Chen et al. [142] introduce a model that combines routing, scheduling and dispatching for daily deliveries in a real-life oil delivery problem. The solution approach consists of three layers (atomic, molecular and individual) and each layer consists of several local search operations with different objectives. Also metaheuristics, such as tabu search and simulated annealing as well as global search, which is realized by a recursive procedure are used to guide the move/exchange operations. Chan et al. [143] study multi-depot, multiple vehicle LRP with stochastically processed demands, generated by a queuing network at each service region. The study was motivated from real-life medical-evacuation case study from U.S Air Force. A three-dimensional space-filling curve-heuristic is proposed for the stochastic model, and its performance is evaluated using deterministic model solved by extended savings heuristic by Clarke and Wright [4].

Salhi and Fraser [77] present an integrated system that tackles simultaneously the location and routing problem for a variant with multiple depots and heterogeneous fleet. Modifications of two previously proposed heuristics by Salhi and Atkinson [144] for the location problem and by Salhi and Sari [127] for multi-depot routing problem are used. The latter is based on determination of borderline customers and others that are easy to allocate to right depot. Wu et al. [145] consider also the LRP with heterogeneous fleet but with limited number of vehicles. They propose an iterative procedure that solves sequentially the subproblems of location-allocation and vehicle routing. For the location-allocation problem, a space-filling curves heuristic is proposed. Correspondingly, the VRP is solved with a cluster-first route-second approach, and solutions to both problems are improved with iterative improvement heuristics, guided by a simulated annealing algorithm. Lim and Fan [146] introduce a model where the number of homogeneous vehicles assigned to each depot is fixed. A new one-stage approach that integrates the assignment of customers with routing is proposed. The routing is done first with a draft-routing method and then with a more detailed routing method.

Wasner and Zäpfel [147] develop an integrated hub location routing model with backhauls and report a case study from Austria. The model considers also the hub-hub transports in addition to number and location of hubs. The suggested hierarchical heuristic solution method is based on simple add and drop procedures and a series of feedback loops. Lin and Kwok [148] explore a multi-objective model with multiple routes per vehicle. Both tabu search and simulated annealing metaheuristics with standard simple neighborhoods are applied and tested with both real and simulated data. Cappanera et al. [149] study the problem of locating obnoxious facilities (dump sites, nuclear reactors, chemical industrial plants etc.) and routing obnoxious materials. The problem is decomposed to location and routing subproblems that are solved with two Lagrangean heuristics. The solutions of the heuristics are then improved with an effective Branch and Bound algorithm. Liu and Lee [150] propose a mathematical model for a variant that takes inventory control decisions into consideration. The suggested heuristic solution method is based on route-first location-allocation second approach and random exchanges of depots. Liu and Lin [151] consider a similar model and propose a hybrid of tabu search and simulated annealing for its solution. The local search moves are based on add and drop procedures.

Location-Arc Routing Problems (LARPs) are encountered in contexts where it is necessary to simultaneously determine a traversal of a subset of edges and arcs of a graph and to also locate facilities on the graph. The main LARP applications arise in the areas of postal delivery, garbage collection and road maintenance. Ghiani and Laporte [152] present a survey on the main LARP applications and algorithms.

As in the case of LRP, most LARP heuristics use a decomposition of the problem into its main subcomponents: facility location, allocation of customers to vehicle routes and routing. Ghiani et al. [74] propose an route first, location-allocation second heuristic in which RPP solution is first determined by means of Frederickson's [76] heuristic. They then simultaneously locate facilities and construct feasible routes by determining an optimal partitioning of the RPP solution into a set of feasible routes. Ghiani et al. [83] propose a slightly different approach for the solution of the CARP with intermediate facilities. In the first step they obtain a feasible CARP solution by means of the CARPET heuristic of Hertz et al [75]. Then they introduce facilities in the CARP solution to make it feasible for the LARP.

4 Acknowledgements

This work was partially supported by the Liikesivistysrahasto and Volvo Foundations, and the TOP project funded by the Research Council of Norway. This support is gratefully acknowledged.

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