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TITLE

Industrial Aspects and Literature Survey: Combined Inventory Management and Routing

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ABSTRACT

This paper describes industrial aspects of combined inventory management and routing, and gives a comprehensive literature review of the current state of the research. First there is a classification of types of supply chains, followed by conditions for when inventory management and routing can be combined, and the current industrial practice. The literature related to the basic problem, as well as extended and related problems is described and classified. The literature is finally contrasted with aspects of industrial applications from a constructive, but critical, viewpoint. Suggestions for where the research in this area should go, is also suggested.

KEYWORDS	ENGLISH	NORWEGIAN
GROUP 1	Logistics, Information Technology	Logistikk, Informasjonsteknologi
GROUP 2	Optimization	Optimering
SELECTED BY AUTHOR	Transportation, Inventory Management	Transport, Lagerstyring
	Operations Research, Inventory Routing	Operasjonsanalyse, Ruteplanlegging
	Literature Survey	Litteraturoversikt

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1 Introduction

The entire manufacturing business, from the extraction of raw materials through production at various stages in the supply chain to the end customers, has witnessed a rising need for efficient behavior due to increased competition and reduced profit margins. The whole chain is affected, and both production actors and service providers in the transport industry are thus facing a more challenging situation than just a few decades ago. In this situation, many companies have been forced to change their focus from optimizing its own business to plan for the benefit of the whole chain. Thus we experience that the competition today is rather between supply chains than between autonomous actors in the supply chain. This has resulted in an increased collaboration between actors within a chain. Some of the companies, who earlier negotiated about prices and tariffs, are today sharing information in order to enhance overall performance.

Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and distributed at the right quantities, to the right location, and at the right time, in order to minimize system-wide costs while satisfying service level requirements (Simchi-Levi et al., 2003). This involves a set of management activities like purchasing, inventory control, production, sales and distribution (Christopher, 2005). Normally the production facilities are found in the beginning of the supply chain, while the consumption facilities exist in the last part of the chain. This paper focuses on the parts of the supply chain consisting of the transportation between production and consumption facilities and the inventory management at some or all of these facilities. Often, inventory management is necessary to consider in just one type of facilities; either production or consumption.

In industry, we observe many cases where one actor is responsible for both transportation and inventory management at one or both ends of a transportation leg. Several transportation companies wish to extend their decision-making area to include inventory management of their customers. By doing this, they can achieve better utilization of their vehicle fleet and offer better prize and service quality to their customers. In addition, this policy gives the production companies the possibility to concentrate on their core business and outsource procurement and transportation. On the other hand, several production companies want to manage the transportation function themselves to remain in power and control. In practice, the actor responsible for the coordinated planning can either be the producer, consumer or the transportation company depending on the type of business. For instance, vertically integrated companies are often responsible for the internal transportation themselves.

Independent of the responsible actor of the integrated planning task, inventory management and routing have traditionally been managed separately in industry. However, more and more supply chains now take advantage of the possibility to synchronize production and inventories at two consecutive facilities. A well established policy in industry and the literature (see for instance Chopra and Meindl (2004)), is to introduce *Vendor managed inventory* (VMI). Here, the vendor is responsible for all decisions regarding product inventories at its customers. As a result, the control of replenishment is placed at the vendor instead of its customers. These policies allow a vendor to choose the timing and size of deliveries. In exchange for this freedom, the vendor agrees to ensure that its customers are never out of stock. In the literature, the VMI concept is often explained and discussed with vendors as the manufacturers or suppliers and the customers as retailers. Ideally, the actors that are practicing VMI should integrate inventory management and transportation planning. However, this is not always the case in reality. One reason may be the lack of support from many of the Enterprise Resource Planning (ERP) systems and other advanced planning systems currently on the market. Very few, if any, of these systems provide decision support for combined inventory management and routing.

There is a large variety of planning problems for combined inventory management and routing. This paper focuses on problems with the following common characteristics:

- The planning is at a tactical and/or operational level.
- One actor in the supply chain is responsible for both the transportation and the inventory management at one or both ends of the transportation leg.
- At the sites where inventory management is necessary, the transported products are produced and/or consumed.
- Inventory capacities are given, together with production/consumption patterns.
- The problem consists of designing a routing and scheduling plan with regard to inventory management at the relevant facilities, while minimizing the relevant costs from this part of the supply chain.

This survey paper will focus on Operations Research (OR) that combines inventory management and vehicle routing. Altogether, we have found more than 90 scientific papers that address this combination, consisting of both case studies based on real applications and theoretical contributions on idealized models. Further, several pure surveys on the topic exist; see for instance Federgruen and Simchi-Levi (1995), Baita et al. (1998), Sarmiento and Nagi (1999) and Moin and Salhi (2006). In addition, there are short literature reviews in several of the technical papers describing research related to combined inventory management and routing. Even though relatively recent surveys exist on the topic, no survey paper focuses on relating the existing literature to industrial aspects in a general way.

The objective of this paper is to give an updated survey on OR literature for combined inventory management and routing problems with a focus on road-based and maritime goods transportation. Compared to airborne and rail transportation, both road-based and maritime transportation are more often concerned with the combined aspects in practice. To our knowledge, no literature on combined inventory management and routing exists within airborne or rail transportation. The case studies reported in previous surveys are concentrated on road-based transportation, while maritime transportation is barely mentioned. This review is exhaustive also on the maritime side. Differences and similarities between the two selected modes will be presented. This survey will be given with a critical eye on the content of the research. The literature is compared with the needs from industry. Further, the paper discusses trends in research and industry within the area and points to needs for future research.
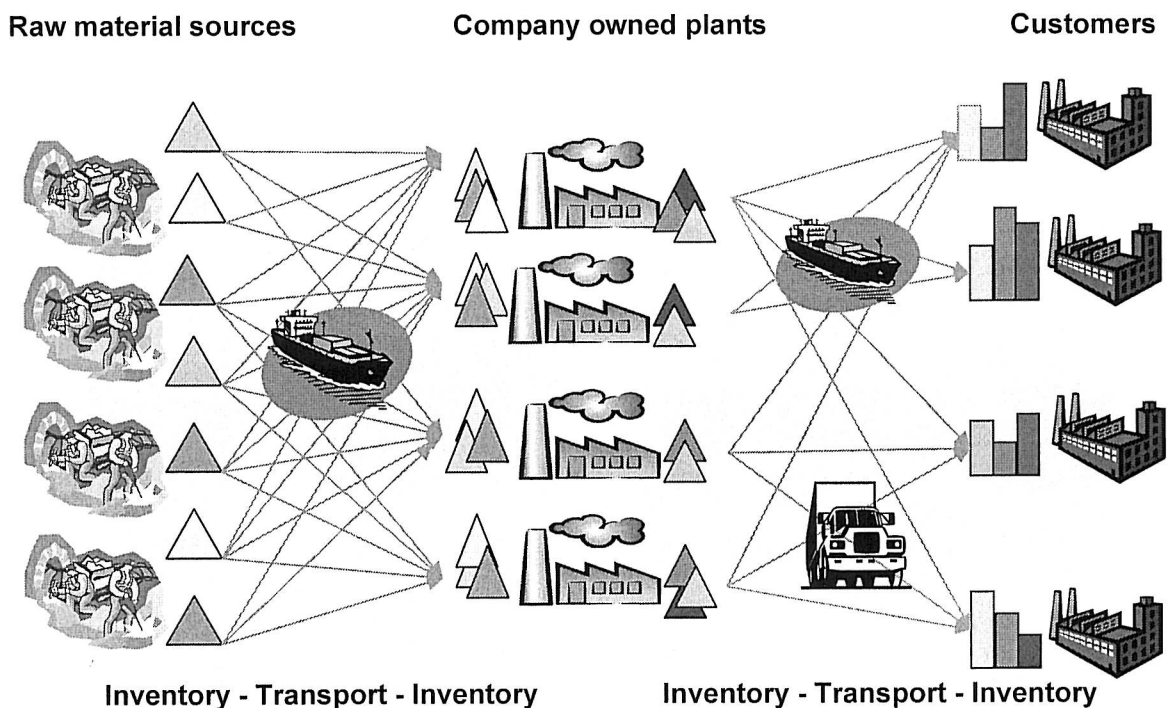
The rest of the paper is organized as follows. Section 2 addresses some important industrial aspects related to combined inventory management and routing problems. In Section 3, we discuss the combined inventory management and routing problems in depth and come up with a classification of the literature that will be used when the literature is reviewed in Section 4. Both problem types and solution methods are discussed. Important trends and perspectives on future developments of the research in this area and the need for optimization-based decision-support systems in the industry with their changing environment are discussed in Section 5. In addition, this section also criticizes the existing research described in the literature and suggests future research with regard to both further development of the research area and industrial needs. Finally, a summary and some concluding remarks follow in Section 6.

2   Industrial aspects

This section gives an overview of combined inventory management and routing in industry. First, different types of supply chains are addressed. Then, it is appropriate to present necessary conditions for achieving integration and positive results from the integration. Further, we give an overview of current industrial practice, before ending up with some differences between the types of transportation modes.

2.1 Types of supply chains

Different types of businesses have dissimilar supply chain structures. Christopher (1998) defines the supply chain as the network of organizations that are involved, through upstream and downstream linkages in the different processes and activities that produce value in the form of products and services in the hands of the customers. This paper focuses on supply chains containing physical products. The chains typically start at the sources producing raw materials, continue with production of main products at production plants and end at the customers. Transportation is normally needed between the main stages in the chain.

Raw material sources　　　**Company owned plants**　　　**Customers**

Inventory - Transport - Inventory　　　**Inventory - Transport - Inventory**

Figure 1: Product flow for a typical supply chain in the metal business

Figure 1 shows an example of a chain for a metal producer with several silicon plants located in different countries and silicon customers all over the world. Combined inventory management and routing might be considered both upstream and downstream from the company. The first combined problem starts at the upstream suppliers where the storage capacity of raw materials might be limited, continues with transportation at sea of heavy raw materials, and then ends at the production plants. At these production plants there should always be sufficient raw materials to allow for a continuous production of silicon products. Then the second combined problem starts at the finished goods inventories at the production plants. It continues with transportation by different transportation modes to various downstream silicon customers. We refer to Ulstein et al. (2006) for the description of a real silicon production chain. To keep the figure simple only direct shipments are indicated. For the real problem it is possible to visit consecutive company owned production plants before visiting a number of customers.

Following the product flow, we have two extreme types of supply chains. The car making industry is part of a typical *convergent* supply chain where many input factors and parts are necessary to make a car. On the other hand, the oil industry faces the opposite type, a *divergent* chain, where the raw oil is processed into a number of oil qualities and chemicals.

The chain structure has impact on the type of combined inventory management and routing problem the involved industry faces. Table 1 demonstrates an overview of the typical responsible actor for the combined inventory management and routing problem for different structures.

Responsible actor → ↓ Type of supply chain	Producer	Transport provider	Customer
One producer – one customer	X	X	X
One producer – many customers	X	X	
Many producers – One customer		X	X
Many producers – many customers		X	

Table 1: Possible responsible actors for various types of supply chains.

In a multi-actor supply chain, the size and power of the actors have vital importance for which actor is taking the overall responsibility. Alternatively, the actors are autonomous and have to collaborate on these management issues. In a vertically integrated company, the company either outsources transportation to specialized providers, or it chooses to operate a fleet of vehicles with the necessary capacity itself. Of course, some vertically integrated companies combine these strategies and are responsible for part of the transportation. Typically, a company provides for the transportation itself in low seasons, but charter in extra transport capacity in peak seasons.

Table 1 lists four main types of supply chains. The simplest problem contains one production facility and one consumption facility. For such a problem the routing is given, but a schedule has to be determined together with pickup and delivery quantities. Further, many real combined inventory management and routing problems have a classical vehicle routing problem (VRP) structure consisting of a central depot and a set of geographically dispersed customers with local inventories and production or consumption rates. One example from road-based transportation is the delivery of gasoline to gas stations from a refinery or central storage. Here the refinery is the production facility, while all the gas stations are consumers. However, milk collection at farms to a dairy has the opposite structure. The customers are producers, and the depot produces dairy products from milk. More seldom this classical VRP structure for combined inventory management and routing can be seen in the maritime sector. However, one example of such a problem with a central depot is the distribution of calcium carbonate slurry from a production facility in Norway to European paper manufacturers (Dauzère-Pérés et al. 2007).

Combined inventory management and routing problems in the maritime sector can most often be found in industrial shipping. Here, the cargo owner or shipper also controls the ships. Industrial operators strive to minimize the cost of shipping all their cargoes.

A typical combined inventory management and routing problem in industrial shipping concerns the transportation of products between a number of production and consumption facilities. Often, it is possible to visit several facilities of the same type in sequence. The products are produced and stored in inventories at given loading ports and transported at sea to inventories at unloading or consumption ports. Large quantities are picked up, transported and delivered. Industrial shipping companies are becoming more aware of the economical benefits of synchronizing inventory management and route planning. We see this trend particularly in the wet bulk segment.

2.2 Conditions for combining inventory management and routing

Inventory management is a key planning issue in all production and manufacturing industries. Controlling inventories is necessary for any company dealing with physical products, including manufacturers, wholesalers and retailers. The use of OR models and decision support systems (DSSs) within inventory management is widespread in industry. The cost structure might be complex including setup, production, holding, ordering, and shortage costs.

Manufacturing and production industries normally have another key planning issue; namely the transportation of raw materials, intermediary products, and final goods. As discussed

above, a company producing or consuming a product can outsource transportation or choose to take this responsibility itself. The transportation is planned using a DSS taking. There exist many DSSs for a variety of routing problems, see for example (Hall and Partyka, 2008). These often plan already given orders and are mainly used for operational planning.

When there are dependencies between inventory management and transportation, it should at least in theory be beneficial to coordinate these functions. We see economical benefits, flexibility in services, and improved robustness as results of such coordination. However, if the actors in the chain are autonomous, the following conditions need to be fulfilled to achieve integration:

- It has to be economically beneficial at the system level.
- It must be possible to split the benefits between the actors in a way that they can agree upon.
- There should be a long term relationship between the producing and consuming actors through long term contracts or common ownership.
- The actors have to be willing to share information and data.

In order to achieve positive results from integration, certain characteristics are preferable. A high volume produced and consumed together with the costs associated with inventory and transportation motivate integration. Another possibility is that the product is critical from a supply chain perspective and large costs are induced if the unavailability of the product causes interruptions to the supply chain flow. A DSS for combined inventory management and routing can be beneficial, given that:

- both the inventory management problem and the routing problem is sufficiently complex such that a DSS will be helpful to solve each of them,
- there should be competence to take a DSS into operation,
- there should be necessary data available,
- the company should be willing to re-organize internally such that the DSS can be fully utilized, and
- the costs of developing, implementing and operating a DSS should be outweighed by the benefits.

Even if a DSS is not available to the responsible actor, it might be appropriate for the actor to consider the inventory management and routing simultaneously.

2.3   Current industrial practice

Researchers have been collaborating with industry on combined inventory management and routing for several decades both within road-based and maritime transportation.

For road-based transportation the application areas described in the literature are transportation of products such as heating oil, beer, soft drinks, industrial gases, and groceries. However, most of this research on road-based transportation has not resulted in implemented DSSs in industry. To the best of our knowledge, there are very few, if any, commercial optimization-based systems in current use that integrate inventory management and routing for road-based transportation. See for example (Aksoy and Derbez, 2003), where supply chain management software are surveyed. Some systems have modules for both vehicle routing and inventory management, but it is unclear whether or not these modules are really integrated or if they work separately.

Still, the industry treats these two planning problems separately. In many cases, there is thus a considerable potential for improvement. Planning systems for inventory management produce order quantities and schedules. This output is used as input to routing systems. However, several integrated systems are under development in the road-based industry. These are among

others projects regarding delivery of gasoline to gas stations from refineries, replenishment of animal fodder from a central depot to farms, and milk collection from farms to dairies.

The maritime sector has lagged behind the other modes of transportation when it comes to implementing optimization-based DSSs for routing and scheduling, see the discussion in Christiansen et al. (2004). Often, manual planning is used for routing, scheduling, and inventory management. However, we now observe a change in the need, awareness, and use of such systems in shipping companies all over the world. According to our experience, many shipping companies that consider implementing routing and scheduling tools want a tailor-made DSS. Inventory management issues are often brought into the discussion. Several real combined ship routing and scheduling problems with inventory management considerations have been studied by researchers; see for instance Christiansen (1999) and Flatberg et al. (2000). These papers focus on industrial shipping of ammonia, where the problem owner is responsible for production, transportation, and consumption. However, this research has not resulted in an operating DSS so far. As far as we know, all companies with combined inventory management and ship routing issues that use ship routing and scheduling tools, currently separate the problems. The inventory management system produces a cargo plan that includes information about the specified quantities, pickup and delivery ports, and the time window for each cargo. Ship routing and scheduling tools, see for instance (Fagerholt and Lindstad, 2007), subsequently take the specified cargoes as planning entities and design routing and scheduling plans. Several integrated systems are now under development for companies with combined inventory management and routing problems in the maritime sector. This is particularly true for the liquefied natural gas industry mainly due to an extraordinary strong growth in high value business.

2.4 Modal differences

Road-based and maritime transportation differ in several ways that are important for integrated planning and control. Here we point to the most important differences in relation to the combined planning challenges.

The industry concerned with combined inventory management and routing normally faces these challenges continuously. For planning purposes these problem are often handled at an operational or tactical planning level. Normally, the planning horizon needs to be set longer in the maritime sector due to time consuming port operations and sailing times. Normally, ships are operated around the clock and no central depot exists. This is also true for some trucks and trailers operating in the full-truckload segment. However, trucks and trailers operating in the less-than-truckload segment are often returning to their home locations at specified times.

Both modes are faced with uncertainties, where time and demand are normally the main uncertain parameters. In general, maritime transportation has more uncertainty in service and traveling times than road-based transportation, due to technical problems and strikes in ports and varying weather conditions at sea. However, road-based transportation in urban areas is often heavily concerned with rush-hours. The degree of uncertainty related to demand is closer connected to type of business than transportation modality. In general, it is a large variation within each modality.

Depending on the type of business, there most often exist just a few classes of vehicles within road-based transportation. Also in the shipping industry, we often find classes of ships according to size and technology for particular segments and fleets. However, ships are long-term investments and the useful life of a ship spans 20-30 years. This means that the diversity in a ship fleet is normally larger than for a fleet of trucks and trailers. Moreover, a particular fleet of ships may consist of ships that are all different in size, cost structure, speed or other characteristics.

As mentioned, the planning problem for road-based transportation often has a classical vehicle routing problem structure, where a central supplier (or depot) serves a set of customers with a local inventory and a consumption rate. Normally, relatively small quantities are delivered to each of the customers. In the maritime sector, the production and consumption at the various

ports are more symmetrical. Large quantities, also relative to the ship capacity, are normally loaded and unloaded. The resulting structure normally becomes a pickup and delivery routing problem, but without a central depot.

Due to the large quantities loaded and unloaded in maritime applications, we believe that inventory aspects are more predominant in the maritime sector than in the road-based. Based on this observation, one can argue that the potential gains of combining inventory management with routing and scheduling is higher in the maritime applications than in road-based applications. However, we can all agree on the potential gains of combining these planning aspects in both transportation modes. Important gains are profitable plans that ensure uninterrupted production and consumption at the facilities.

3 Integrated view and classification

Inventory management and routing are two important planning issues within a logistic system. This makes them interesting to study from a research perspective, where the effectiveness of the planning as well as the similarities and dissimilarities between different industrial sectors are among the studied topics. In this section we will briefly describe the research within inventory management and routing, first separately and then combined and provide a description and a classification of this combined problem.

3.1 Inventory and inventory management

Today inventories are used by most companies as a buffer between processes to even out variations and handle uncertainty. With a volatile market creating stochastic demands and a production process working best under economies of scale, the inventory can be seen as a way to balance these conflicting goals. A low response time to the customers and an effective usage of the production apparatus is often very hard to combine without an inventory. With this in mind, inventory management can be seen as managing the conflicting goal between supply, i.e. procurement and production, and demand.

Even though there usually are inventories both before and after the production, it is often the procurement on the component side that is under consideration in inventory management theory. One reason for this is that the production planning philosophy often overrides the inventory decision made in the finished goods inventories. In a push environment, the inventory levels are direct consequences of the order releases and quantities, while they are ideally fixed and pre-calculated in a pull environment.

The literature on inventory management is vast, reaching from introductory text books in operations research and management science to purpose-made journals, and the use of operations research models and decision support systems (DSS) within inventory management is widespread in industry. The number of software providers that offer DSS for inventory management is large and growing, and the large enterprise resource planning system providers all have inventory management as an integrated part of their systems.

A general inventory management framework is presented in (Hillier et al. 2005) and comprises the following four steps:
1. Formulate a mathematical model describing the behavior of the inventory system.
2. Seek an optimal inventory policy with respect to this model.
3. Use a computerized information processing system to maintain a record of the current inventory levels.
4. Using this record of current inventory levels, apply the optimal inventory policy to signal when and how much to replenish inventory.

Even though all four steps are important and for the whole system to work, it is mainly in step 1 and 2 that operations research expertise is needed. Ranging from tailor-made and very sophisticated systems to off-the-shelf products, some of the systems are based on relatively basic operations research theory, while others include advanced methods and algorithms. Our experience is that very few users in industry are familiar with the underlying assumptions and methods in the inventory management DSS implemented in their organization.

The road to these systems and models started in the early twentieth century with the derivation of the economic order quantity, (Harris, 1913), the production lot size model, (Taft, 1918), and the newsboy problem, (Morse and Kimball, 1951). These models have since then been extended with among other things; multiple products, multiple tiers, defective items and discrete times between shipments. For a clear and precise description of inventory management, the reader is referred to (Silver, Pyke and Peterson, 1998).

The cost structures and supervision of the inventory are very well developed in most systems, and functions for handling for example contracts, discounts, alternative parts, and so on are easily managed. The same can be said about the integration between inventory management and production, especially in make-to-stock environments.

One thing that most models and systems do not address properly is the routing aspect of transportation. Often it is assumed that the transportation is either handled by the supplier, and the cost is thereby included in the unit price, or the transportation cost per shipment is fixed, and therefore a part of the order preparation cost. Studies by for example Carter and Ferrin (1996) and Swenseth and Godfrey (2002) clearly show the differences between a flat transportation cost and one that depends on the size of the shipments when calculating the optimal order quantities. Both papers express the variable transportation cost as a function of freight rates, but none of them make the actual routing an active decision in their models. Later we will see that some inventory management papers where routing is an active decision have been classified as combined inventory management and routing problems.

3.2 Vehicle routing

Where most models in inventory management treat routing aspects in a passive way, by either assuming a fixed cost per shipment or as handled by the supplier, they are the active decisions in vehicle routing problems. A typical routing problem consists of deciding which customers should be visited by each vehicle and in what order so as to minimize total cost subject to a variety of constraints such as vehicle capacity and delivery time restrictions.

In routing problems the decision to order and the routing decision are not made at the same time, and usually not by the same actor. Often the decision to order is made by the consumer, and an order is released to the supplier. The supplier collects all orders and makes a routing plan to fulfill all orders. The quantities to deliver to the consumers are fixed and known to the supplier and must be delivered within the planning period. Depending on the topology of the application, it is common to distinguish between problems with a central warehouse, where all goods are picked up or delivered, and problems without such a warehouse. In many cases all vehicles start and end their route at a depot at the central warehouse, but there are also cases when the depot is located away from the warehouse or where there is no depot and the vehicles do not have a fixed starting position.

The classical vehicle routing problem is to determine a set of vehicle routes, where a route is a tour that begins at the depot, traverses a subset of the customers in a specified sequence, and returns to the depot. Each customer must be assigned to exactly one of the vehicle routes and the total size of deliveries for customers assigned to each vehicle must not exceed the vehicle capacity. The routes should be chosen to minimize total travel cost. For a presentation of the vehicle routing problem and many of its variants, the reader is referred to (Toth and Vigo, 2002).

If the transportation request consists of picking-up a certain quantity at a predetermined pick-up location and transporting this quantity to a predetermined delivery location without

visiting the depot in between, the problem turns into a pick-up and delivery problem (PDP). Time constraints on the pick-up, delivery or the pick-up and delivery pair are common for this type of problem. A general discussion about pick-up and delivery problems can be found in (Savelsbergh and Sol, 1995) and (Berbeglia et al., 2007). A rightful critique to the latter paper is the lack of references to maritime routing and scheduling problems, of which many are PDPs.

One interesting difference between the inventory management literature and the routing literature is the lack of papers combining new theoretical insights with practical problem solving on the inventory management side. In the routing literature, many of the new models and extensions of existing models come from a practical application where these models are not detailed enough to capture all aspects of the problem. Tiwari and Gavirneni (2007) pinpoint this when they write *"Practicing managers increasingly rely on initiatives based on information systems to improve the sizes of their inventories; researchers try to obtain optimal solutions to restrictive analytical models"* as a critique of the current inventory-control research. On the other hand, Hartl et al. (2006) write in their editorial to a special issue on rich vehicle routing problems, *"...the research community has turned to variants of the VRP which before were considered too difficult to handle. The variants include aspects of the VRP that are essential to the routing of vehicles in real-life, ... often called rich vehicle routing problems. ... We are confident that the recent trend in the VRP research community of more efforts devoted to studies of rich problems will increase for many years. Results will certainly make a difference to industry and society"*.

3.3 Combined inventory management and routing

Having inventory management on one side, where the routing aspects of the transportation is not properly treated, and routing on the other, with a number of predefined orders to serve, a natural extension to both problems is to combine them. In the combined problem, key components from both inventory management and routing are modeled appropriately. These problems will henceforth be denoted combined inventory management and routing problem.

The large variety of aspects and assumptions that can arise in combined inventory management and routing is obvious. This shows in the problem formulation and modeling, making the problem very hard to classify. Unlike many other routing problems, for which there are clear definitions with well-defined assumptions, there is almost one new version of the problem for every paper published.

As well as many versions of the problem, there are also a number of different names used in the literature. One of the first papers addressing the combined inventory management and routing problem, (Golden et al., 1984), uses the term inventory/routing problem, and define this problem as a routing problem with explicit inventory features. The name "the Inventory Routing Problem" (IRP) has since then been one of the most commonly used for problems combining inventory management and routing.

A more precise definition is given by Dror et al. (1985), who state that *"The IRP involves a set of customers, where each customer has a different demand each day. ... The objective is to minimize the annual delivery costs, while attempting to ensure that no customer runs out of the commodity at any time."*

Being a nice definition in itself, it does not capture the whole essence of the problem, nor does it give any guidelines about how to characterize the different versions of the problem.

Campbell et al. (1998) give a description of the inventory routing problem that can be the basis for most road-based applications presented by the research community. They state that the IRP is concerned with the repeated distribution of a single product, from a single facility, to a set of customers over a given planning period. The customers consume the product at a given rate and have the capability to maintain a local inventory of the product. A fleet of homogeneous vehicles is available for the distribution of the product. The objective is to minimize the distribution costs during the planning period without causing stock-outs at any of the customers.

Looking at the coordination of inventory management and routing from a practical point of view, it is clear that the combined problem is a long-term, possibly infinite, dynamic problem that is inherently stochastic. Since long-term dynamic and stochastic problems are extremely difficult to solve, the approaches found in the literature have simplified the problem in one way or another.

We classify the literature according to the dimensions introduced below in order to easier discuss similarities and differences between different applications and solution approaches. Many of the dimensions have been used in earlier surveys and reviews, but we have also added extra dimensions to reflect the focus on industrial aspects. In Table 1, the different characteristics and the alternatives we see are presented. The rest of this section is devoted to a detailed description and clarification of the table.

Table 2: The classification criteria used for the inventory routing problem.

Characteristic	Alternatives			
Time	Instant	Finite	Infinite	
Demand	Stochastic	Deterministic		
Topology	One-to-one	One-to-many	Many-to-many	
Routing	Direct	Multiple	Continuous	
Inventory	Fixed	Stock-out	Lost sale	Back-order
Fleet composition	Homogeneous	Heterogeneous		
Fleet size	Single	Multiple	Unconstrained	

Time

Since the problem has a long-term nature, but is used in an operational or tactical planning environment in most applications, the reduction from a long-term horizon to a short-term is crucial. We will use three different modes when classifying the literature in the time dimension, reflecting the planning periods used.

Instant: In these models, the planning horizon is so short that at most one visit per customer is needed. The main decisions are to balance the inventory and routing costs with the costs associated with stock-outs at the customers. If the planning horizon consists of more than one period, visit a customer earlier or postponing a visit may also be considered.

Finite: When more than one visit at a customer may be needed, we talk about a finite problem. This mode can be further divided into sub-modes. If there is a natural and finite end to the horizon, a fixed horizon can be used. Since it is assumed that there is no interaction between the time before and after the horizon, there is no need to handle the long-term effects. If the state after the horizon depends on the decisions made, for example through the inventory levels, the long-term effects must be handled one way or another. The most common way is to use a rolling horizon and solve the problem for a longer period than is actually needed for the immediate decisions.

Infinite: When analyzing a problem with infinite planning horizon, distribution strategies rather than schedules are the decisions.

Demand

Since combined inventory management and routing problems, seen as practical problems rather than theoretical constructs, are stochastic by nature, a classification trying to capture stochasticity must focus on if and in that case when a deterministic realization is used in the algorithm. On one side, there are cases where the demand is treated as deterministic from the very start. On the other side are cases where stochasticity is an important component and integrated in the solution framework. Between these two extremes there are of course many ways to treat the stochastic demand. We have chosen to use a classification based on uncertainty. The cases when

the method proposed incorporates uncertainty with respect to the demand will be called *stochastic*, while the cases with no incorporation are called *deterministic*.

Topology
We follow the classification by Baita et al. (1998) to classify the topology of the problem. This means that we use three modes; *one-to-one*, *one-to-many* and *many-to-many*, to describe the problem. The many-to-one case is included in the one-to-many case; it is not well studied and can easily be transformed to a one-to-many topology. The one-to-many mode is the dominant one for road-based inventory routing, where a single facility serves a set of customers using a fleet of vehicles. The central facility is a depot where the vehicles start and end their routes and where the goods are stored before delivered to the customers.

In a maritime setting, usually there is no central facility, but the vehicles can load and unload at any port. Neither is there any fixed position where the routes start and end. This means that the many-to-many mode is the dominant one in maritime applications.

Routing
The routing component of a combined inventory management and routing problem can be seen as either a VRP or a PDP. In the VRP setting, the depot serves both as a depot where all routes start and end and as a central warehouse from where the goods are distributed. Most road-based applications utilize this kind of routing. In a PDP setting, there is no central warehouse, but all distribution is from a pick-up customer to a delivery customer, possibly via other customers. In most road-based cases, there is a depot where the vehicles start and end their routes, while a PDP setting without a central depot is more common in maritime applications.

It is natural to distinguish three cases when classifying the routing component in an inventory routing problem. In the first case, a vehicle picks up goods at the central warehouse and then distributes all goods to a single customer before returning to the warehouse. We will use the term *direct* for this case, since it is often referred to as direct shipping in the literature. We denote the case when a vehicle can visit more than one customer on a trip *multiple visits*. In both cases, the trip starts and ends at the central warehouse and the underlying problem is a vehicle routing problem. In the pick-up and delivery setting, the trip can be seen as continuous, with no start or end. We will denote this case *continuous*.

Inventory
There are many different inventory management decisions that have to be made in a combined inventory and routing problem. In this classification, we will focus on the decisions concerning the customers. The main decision to make is whether to allow the inventory level to become negative. In many applications, this is not allowed and we will denote this case *fixed*, since the lowest inventory level is fixed either to zero or a level based on the safety stock. Sometimes failure to satisfy the demand occurs, which can be seen as either a stock-out, followed by an emergency delivery to the customer where the stock-out occurs or simply as lost sales. We will denote these cases *stock-out* and *lost sales*, respectively. The last possibility is to simply postpone the demand and satisfy it later, we will denote these cases *back-order*.

Vehicle fleet
The fleet used to distribute or collect the goods can be classified according to composition and size. In a homogeneous fleet, all vehicles have the same characteristics such as speed, fixed cost, variable cost, equipment, and size. If the fleet is heterogeneous, some, or all, of the characteristics of the vehicles differ. This means that a solution may become infeasible if two vehicles are interchanged. We will use the terms *homogeneous* and *heterogeneous* to describe the two possibilities.

The size of the fleet is an important aspect of the problem. If the fleet consists of one vehicle we will use the term *single*. If the fleet consists of a number of vehicles and this might be

a constraining factor, we will use *multiple* to describe this situation. This is the case for example if the distributor owns the fleet and cannot purchase extra distribution capacity. In both road-based and maritime applications, extra capacity may often be hired on a short-term basis. Exceptions are applications where highly specialized vehicles are needed. If it is possible to purchase extra distribution capacity, the planner will always have enough vehicles and we will use the term *unconstrained* for these situations.

Solution approach
We will have a pragmatic attitude when classifying the literature according to solution approach. Since a number of different approaches have been suggested, we will not give predefined alternatives, but describe each approach as well as possible.

4 Literature survey

In this section the literature on combined inventory management and routing problem will be presented. We have mainly focused on papers published in scientific journals and book chapters, working papers and technical reports are only occasionally included. Still, more than 60 papers are reviewed. Each part in this section ends with a table where the central papers are presented and classified according to the classification scheme from Section 3.

The section starts with a review of earlier surveys, followed by the regular papers classified according to the length of the planning horizon. There is a table summarizing the most important papers after each section. If one paper is followed by other papers, outlining parts of the proposed method in greater detail, these accompanying papers are not included in the tables.

4.1 Other surveys and reviews

A number of more or less comprehensive surveys and reviews about inventory routing problems have been published. In the first published review, (Baita et al., 1998), the author's focus is on the simultaneous presence of three aspects, routing, inventory, and dynamics when they define dynamic routing-and-inventory (DRAI) problems. In short, they write, "*DRAI problems deal with how to manage the activity of supplying goods from origins to destinations during some time horizon, considering both routing and inventory issues*". The main categorization is based on whether the decisions are in the frequency domain, where the main decisions are delivery frequencies, or in the time domain, where the main decisions are schedules, routes, and quantities. Compared to the time dimensions used in this survey, the frequency domain corresponds to an infinite planning horizon and the time domain to instant or finite planning horizons.

In a whole chapter about the inventory routing problem, Campbell et al. (1998) present the complexity of a stochastic version of the problem for the cases with one and two customers. The literature is not classified, but the focus of the presentation is on different solution approaches that have been used. The chapter ends with a useful presentation of different practical issues related to among other things the measurement of inventory levels and usage rates, multi-product environments, and heterogeneous vehicle fleets.

A logistical overview of inventory routing problems is claimed in (Moin and Sahli, 2006), but very little is said about the application oriented works within the field. The paper classifies the literature according to the number of time periods; single-period, multi-period and infinite horizon models, and treats problems with stochastic demands separately. The authors also point out a number of research avenues worthwhile exploring. Only one paper related to maritime inventory routing is reviewed, a clear deficiency bearing in mind that maritime logistics is critical for most global supply chains.

The first review of combined inventory management and routing problems is part of a longer chapter that analyzes vehicle routing problems and inventory routing problems,

(Federgruen and Simchi-Levi, 1995). The review is mainly focused on problems with an infinite planning horizon where it is distinguished between settings with or without inventory management at the central depot.

Besides these three surveys entirely devoted to combined inventory management, a number of other surveys and reviews partly treat the subject.

Sarmiento and Nagi (1999) reviews integrated production-distribution systems, but the main part of the paper deals with settings without production decisions. The classification used has two levels. On the first level, the authors classify the problems according to which logistics functions, i.e. inventory, distribution, and production, are present. On the second level, topology and length of the planning horizon are used as classifiers. A distinction is made between the single depot/multiple retailers problem and the inventory routing problem based on a system versus depot perspective. In light of this survey, the distinction seems to be between problems with finite time horizon, seen from a depot perspective, and problems with an instant or infinite time horizon, seen from a system perspective.

An introduction to combined inventory management and routing problems is presented by Bertazzi et al. (2007). Instead of surveying the literature, they develop a small deterministic example and investigate several different settings related to the inventory capacities, holding costs and continuous production. A reference list covering many of the important papers is also included.

Cordeau et al. (2007) focus on broad modeling and solution approach areas when discussing the combined inventory management and routing literature. The areas are: integer programming models for problems with a finite time horizon, single customer analysis of instant time horizon problems, asymptotic analysis of problems with infinite time horizon, and the formulation of problems with an infinite planning horizon as Markov decision processes. Comparing this classification with the one proposed in this survey we notice that many papers do not fit in any of the areas, but that the areas are well chosen and representative for the combined inventory management and routing problem. This is also the only one of the above reviews with references to maritime applications.

Two surveys addressing ship routing and scheduling discuss combined inventory management and routing problems. In (Christiansen and Fagerholt, 2007), which is fully devoted to maritime inventory routing problems, a basic version of the problem is modeled and used as a starting point for an extensive discussion about different extensions. Many of the papers discussed are application oriented and classified according to the type of extensions they involve.

In a survey of ship routing and scheduling, Christiansen et al. (2004) review the literature addressing maritime combined inventory management and routing problems. The paper surveys ship routing and scheduling and discusses maritime inventory routing problems mainly under tactical and operational problems in industrial shipping. This classification of maritime problems is still valid today.

4.2   Instant time horizon

Since the combined inventory management and routing problem can be seen either as an extension of an inventory management problem with the inclusion of a routing component, or as a routing problem extended with an inventory management component, the research on inventory routing problems is highly colored by the background of the researchers. This is evident, especially when discussing combined inventory management and routing problems with an instant time horizon. Still there are common elements; the balancing between the inventory and transportation cost and the risk of stock-out is one of the most prominent features of these problems. In most analyses, the customer's demands are considered stochastic and not known at the time of planning. Basically two approaches have been used to deal with this. The stochasticity is either included as a part of the main algorithm or it is used in a pre-analysis to decide the

optimal replenishment days for the customers and the cost of changing these days. In the latter approach, the main problem becomes deterministic.

Federgruen and Zipkin (1984) adopt the first approach and have random consumption as a part of the objective function. They analyze a problem where the inventory levels are known to the planner, who has to decide how much of a scarce resource to deliver to each customer and how to route the fleet of vehicles. After the deliveries are made, the demand is realized and the inventory, transportation and stock-out costs are calculated. The proposed nonlinear mixed integer programming model has three different sets of decision variables. The first determines the movements of the vehicles, the second the assignment of customers to routes, and the third the amount delivered to each customer. The solution approach is based on the observation that if the second set of variables is fixed, the problem decomposes into an inventory allocation problem and one traveling salesman problem for each vehicle. The algorithm starts with an initial set of routes, i.e. a feasible assignment of customers to vehicles, and then evaluates changes, using r-opt methods, in the assignment. As a second part, an exact algorithm based on generalized Benders decomposition is presented.

The general problem presented in (Federgruen and Zipkin, 1984) was later extended with multiple products in (Federgruen et al., 1986) as a way to handle perishable products. The different products reflect the age of the original product. If a product is considered old, an out-of-date cost is paid for each item not sold, while no such cost exists for fresh products.

Instead of including the decision about whether to visit a customer or not, these decisions can be made beforehand. Golden et al. (1984) use a threshold to decide; all customers with a relative inventory level less than the threshold are considered potential customers. In the next step, which customers to actually visit is decided by solving a time constrained traveling salesman problem with a modified objective function reflecting the urgency of resupplying the customers. Then a vehicle routing problem over the selected customers is solved using a Clarke and Wright-based method (Clarke and Wright, 1964) and the routes generated are combined to form day-long work schedules for the vehicles. If it is impossible to form day-long routes, the time constrained traveling salesman problem is solved once more with a tighter time constraint. When a feasible set of day-long routes has been generated, the amount distributed to each customer is calculated. The heuristic is embedded in a multiple period simulation, where the single day problem is solved repeatedly. Instead of presenting the objective values, the amount of goods deliver per time unit and number of stock-outs are the main performance measures. Using other measures related to utilization or effectiveness instead of the objective value is one way of dealing with the long-term effects, for a thorough discussion about performance measure for combined inventory management and routing problems, the reader is referred to (Song and Savelsbergh, 2007).

Considering the customer selection, the resource allocation and the routing of vehicles in the same model is done by Chien et al. (1989) who formulate the problem as a linear mixed integer programming model. Using Lagrangean relaxation, the problem decomposes into one inventory allocation problem and one customer assignment/vehicle utilization problem. Both subproblems are solved to optimality and then a subgradient method is used to update the multipliers. Using the solutions from the relaxed problem, a heuristic in two phases is used to find feasible solutions. In the first phase, an initial set of vehicle routes is constructed based on the inventory allocation and the customer assignments. In the second phase the routes are improved in a greedy fashion. As in (Federgruen and Zipkin, 1984), the inventory at the depot is not enough to cover all demands.

The way the long-term effects are modeled differs between the papers. Federgruen and Zipkin (1984) and Golden et al. (1984) model the demand using a distribution function and realize the demand at the end of the day. After the demand is realized, a stock-out cost is incurred if the demand is higher than the current inventory level. Chien et al. (1989) use a revenue-penalty cost function where each delivered unit earns revenue and each unit of unsatisfied demand incurs a penalty. By transferring the penalty cost in one period to revenue the next, the periods are linked. Only the maximum demand for each customer is given, and hence no stock-outs can occur.

When the planning period is longer than one period, the question is not only if a customer should be visited or not, but also when it should be visited. Both Dror et al. (1985) and Bard et al. (1998b) use a single customer cost-function as the basis for their analysis. Dror et al. (1985) derive the expected future cost as a function of when the next visit takes place, while Bard et al. (1998b) derive an expression for the optimal constant replenishment interval. The solution approach taken by Dror et al. (1985) is based on first dividing the customers into one set that must be visited and one set that can be visited based on their mean consumption. The single customer cost-function is then used when constructing the objective function. For a customer that must be visited, the cost reflects the difference in future cost between visiting the customer earlier than the latest possible day and visiting the customer on the latest possible day. For the other customers, the cost reflects the difference in future cost between visiting the customer and not visiting the customer. Given these costs, the customers are assigned to different days, and then a vehicle routing problem is solved for each day. The algorithm ends with a node exchange improvement phase.

Instead of having the possibility to visit a customer whose optimal replenishment time falls outside the planning period, Bard et al. (1998b) extend the planning period. Only customers with the optimal replenishment time within the planning period are assigned a delivery day. The vehicle routing problem solved for each day in the first part of the planning period is extended with satellite facilities, where a vehicle can be refilled. The solution approach is embedded in a rolling horizon framework, where a two-week planning period is used, but only the first week is implemented. A bi-criteria objective is used where the trade-off between distance and annual cost is analyzed.

The reader is referred to (Dror and Ball, 1987) and (Dror and Levy, 1986) for details of the single customer cost function as well as the node exchange improvement phase, and to (Jaillet et al., 2002) and (Bard et al., 1998a) concerning their single customer cost function and the vehicle routing problem with satellite facilities.

The setting in (Dror et al., 1985) was later extended by Trudeau and Dror (1992) who modify the single customer cost-function and introduce stochastic demands and route failures. If a stock-out occurs prior to the day of replenishment, an emergency route is made and the customer is removed from the planned route. The algorithms from (Dror et al., 1985), (Dror and Levy, 1986), (Dror and Ball, 1987), and (Trudeau and Dror, 1992) are combined and tested on a real-life case in (Dror and Trudeau, 1988). Later the single customer cost function analyses in (Dror and Ball, 1987) are discussed from the perspective of the present value of the cash flow by Dror and Trudeau (1996).

When the setting analyzed by Dror et al. (1985) is extended with fixed ordering, holding and shortage costs, Herer and Levy (1997) denote the problem the metered inventory routing problem. They state that in the standard IRP, the customers pay for the delivery when it is made, while in the metered version, the customers pay for the inventory they use, as they use it. Thus under the metered IRP formulation, the supplier, not the customer, pays for inventory held at the customer. Herer and Levy (1997) use a single customer cost function to derive functions for the different cost components and then define a temporal distance, besides the spatial, between the customers that is used in the modified Clarke and Wright algorithm proposed.

Given a fixed sequence of customers with unknown demands, another problem with an instant time horizon is how to allocate the products given a truck with limited capacity. The difference between this problem and that of Federgruen and Zipkin (1984) is that the amount to unload is decided on at the customer's premises in this problem, while in the latter problem, the allocation is done before the vehicles leave the depot. Bassok and Ernst (1995) formulate a version of this problem with multiple products as a linear program with stochastic demands. A similar problem was later analyzed by Berman and Larson (2001), the major differences being the introduction of an ideal refill percentage and the knowledge about the inventory level at the current customer. Berman and Larson (2001) discuss cases with knowledge about the inventory levels, but also when the levels are unknown.

Table 3: Characteristics of central papers with instant time horizon.

Author (Year)	Demand	Topology	Routing	Inventory	Fleet composition	Fleet size	Approach
Federgruen and Zipkin (1984)	Stochastic	One-to-many	Multiple	Stock-out	Heterogeneous	Multiple	Routing/allocation decomposition, interchange heuristic, Generalized Benders decomposition
Golden et al. (1984)	Stochastic	One-to-many	Multiple	Stock-out	Homogeneous	Multiple	Customer selection/routing heuristic, Clarke and Wright heuristic
Dror et al. (1985)	Stochastic	One-to-many	Multiple	Fixed	Homogeneous	Multiple	Customer selection/routing heuristic, single customer analysis
Federgruen et al. (1986)	Stochastic	One-to-many	Multiple	Stock-out	Heterogeneous	Multiple	Routing/allocation decomposition, interchange heuristic
Chien et al. (1989)	Deterministic	One-to-many	Multiple	Back-order	Heterogeneous	Multiple	Mixed integer program, Lagrangean heuristic
Trudeau and Dror (1992)	Deterministic	One-to-many	Multiple	Stock-out	Homogeneous	Multiple	Customer selection/routing heuristic, single customer analysis
Bassok and Ernst (1995)	Stochastic	One-to-many	Multiple	Stock-out	Homogeneous	Single	Linear program, dynamic programming, Monte-Carlo simulation
Herer and Levy (1997)	Stochastic	One-to-many	Multiple	Stock-out	Heterogeneous	Unconstrained	Single customer analysis, Clarke and Wright heuristic
Bard et al. (1998)	Stochastic	Many-to-many	Multiple	Fixed	Homogeneous	Multiple	Single customer analysis, assignment/routing heuristic, local search
Berman and Larson (2001)	Stochastic	One-to-many	Multiple	Stock-out	Homogeneous	Single	Stochastic dynamic programming

4.3 Finite time horizon

When more than one visit per customer is possible, we talk about planning over a finite time horizon. This is the case in most maritime applications, see for example (Christiansen, 1999), and in road-based applications when routes cover multiple time periods, see (Savelsbergh and Song, 2005).

Besides the general characteristics described in Section 3, we add transportation mode as an aspect in this section. There are so many differences between maritime and road-based applications that we have chosen to separate the description of the different modes. For further details, the interested reader is referred to (Christiansen et al., 2004) for a discussion on the difference between modes of transportation. In the classification, *maritime* will be used when the application considered is a maritime application, and *road-based* will be used in road-based applications. In many papers not describing an industrial application but focusing on more generic models, the mode of transportation could be either maritime or road-based. We classify these papers in a third category, denoted *generic*.

In maritime inventory routing problems, there is often a number of products involved. Due to their characteristics, sometimes they must be transported in different compartments in the vessels. The products are supplied at different ports and consumed at others, i.e. a many-to-many topology. The fleet is often highly heterogeneous since each vessel is divided in a number of compartments of different sizes. The objective is to minimize the distribution costs during the planning period without causing stock-outs at the consumption ports or interrupted production at the production ports.

A general setting for a road-based inventory routing problem with a finite time horizon is a one-to-many topology dealing with a single product. The depot has enough goods to supply the customers whose demands are known to the planner at the beginning of the planning period. A homogeneous fleet of vehicles is available for the distribution of the problem and neither the depot nor the customer faces any ordering or inventory costs. The objective is to minimize the distribution costs during the planning period without causing stock-outs at any of the customers.

Because of the complexity of the problem, only small instances can be solved to optimality. Therefore, almost all solution approaches proposed in the literature are heuristics, either pure heuristics or optimization methods ended before proven optimality.

Distinguishing between optimization-based methods and heuristics gives two classes, where the first use techniques and algorithms for solving solutions to mathematical programs, while the other typically explore the solution space using neighborhood structures. The few cases where a combination of neighborhood exploration and mathematical programs has been used will be classified as heuristics.

When an optimization-based approach is used, two different modeling approaches can be distinguished; arc flow and path flow formulations. In an arc flow formulation, what constitutes a feasible schedule for a vehicle is explicitly given through constraints on the arcs, while in a path flow formulation; only the feasible schedules are included in the model. This means that some kind of generation of feasible schedules is needed for solving path flow formulated models.

Depending on how the problem is modeled, certain solution approaches are preferable and different decisions are tied closer together. An advantage with a path flow formulation is that all constraints concerning an individual schedule can be handled outside the mathematical program; the drawback being the huge number of variables representing the feasible schedules. On the other hand, arc flow formulations are considered to be better suited for polyhedral studies and need fewer variables, but all constraints are explicit in the mathematical program.

Path flow formulations have been used extensively in the maritime sector, starting with the pioneering work of Appelgren (1969) and Appelgren (1971), but most of these applications have only considered routing and scheduling issues and not inventory management. Christiansen and Nygreen present a combined ship scheduling and inventory management problem faced by a company in the fertilizer industry (Christiansen, 1999), (Christiansen and Nygreen, 1998a), and (Christiansen and Nygreen, 1998b). The problem is formulated using an arc flow formulation, but is reformulated using Dantzig-Wolfe decomposition. This gives a problem where both ship schedules and the inventory management decisions can be expressed as columns. In the master problem, columns from schedule and inventory subproblems are combined into feasible solutions. Recently, Christiansen and Nygreen (2005) extend the problem by introducing soft inventory and time window constraints as a way of dealing with uncertainties in sailing time and time consumption at ports.

Persson and Göthe-Lundgren (2005) have used a path-based approach when modeling a maritime inventory routing problem. They consider a multi-product application, and also include variable production at the loading ports. The structure of the problem does not allow for an implicit treatment of the inventory management, but explicit inventory constraints are used in the master problem.

The only paper with a road-based application that uses a path flow formulation is (Bell et al., 1983), in which the set of vehicle routes is generated beforehand using a heuristic. Embedded in a Lagrangean relaxation framework, routes with up to four customers are generated.

Considering the success with path flow formulations in other routing applications, it is strange that so few solution approaches are based on path flow formulations. One reason for this could be the problem with path feasibility; due to the inventory flow, the feasibility of the paths cannot be ascertained in the generation phase, it depends on the status of the master problem solution. This means that extra constraints in the master problem are needed to cover up. Hence, many of the advantages with path flow formulations are lost.

Bell et al. (1983) circumvent the problem by pre-generating the set of feasible routes and introducing continuous variables representing the delivered amount. A more attractive way to solve the problem is proposed by Christiansen (1999), who includes information about quantities and visit start times in the vessel paths and also generates corresponding harbor visit sequences to assure inventory feasibility at the harbors. Persson and Göthe-Lundgren (2005) do not address this problem, but use a model with explicit constraints for path feasibility.

The arc flow formulation used by Miller (1987) is not used to directly solve the maritime inventory routing problem under investigation, but instead to evaluate changes made to an already existing solution. Using a constructive heuristic to generate a feasible schedule for the fleet of vessels, this solution is then fed into a decision support system where it can be manipulated both

interactively and automatically. After each manipulation, the arc flow model is used to evaluate the consequences. A different approach have been proposed by Al-Khayyal and Hwang (2007), who start with the arc flow formulation proposed by Christiansen (1999) but extends it to include multiple products and ships with multiple dedicated compartments.

On the road-based side, until recently there were no papers that discussed arc flow formulations for the inventory routing problem. Savelsbergh and Song (2005) motivate their extension to the general setting with practical considerations from the industrial gas sector. Three areas are under consideration; limited product availability, customers cannot be served using out-and-back tours and tours that last for several days. The name "inventory routing problem with continuous moves" is used to describe this setting. Starting from a multi-commodity flow model, a number of reductions are applied to make it more tractable. A branch and cut framework is developed around delivery cover inequalities and super-arc branching. Another extension to the general setting is discussed in (Yugang et al., 2007), where the possibility to split deliveries is included. The proposed model is relaxed using Lagrangean multipliers. The relaxed problem does not decompose, but can be approximately solved by fixing the routing solution while solving the inventory part and vice versa. The solutions from the relaxed problem are then used to construct feasible solutions to the original problem.

Many of the heuristics proposed for the multi-period inventory routing problem are based on decomposing the problem into hierarchical subproblems, where the solution to one subproblem is used in the next and so forth. Chandra (1993) extends the general road-based setting to a problem where the depot faces both ordering and inventory costs. The algorithm proposed starts by solving a depot ordering problem, which gives the preferred replenishment times and quantities for the depot. Then a distribution planning problem, given these quantities, is solved to distribute the goods to the customers. In a consolidation phase, the distribution plan is heuristically improved, and the perturbed quantities are evaluated in the depot ordering problem. If an improvement is found, the process is repeated.

Another way to decompose the inventory routing problem is to see it as one allocation problem, where the decisions are how much, and when to deliver to the customers, and one routing problem where the routes are determined. This approach has been used by Carter et al. (1996) and Campbell and Savelsbergh (2004a). The idea proposed by Carter et al. (1996) is to solve an allocation problem with a capacity constraint reflecting the total vehicle capacity, followed by a VRPTW to decide the routing. If the VRPTW is infeasible, the total capacity in the allocation problem is changed and the process is repeated. Periodic delivery patterns are used and a rolling horizon framework is used to handle the long-term effects. The difference between the approaches presented by Carter et al. (1996) and Campbell and Savelsbergh (2004a) is in the allocation phase. Carter and co-workers deal with individual customers, while Campbell and Savelsbergh define clusters of customers that can be served efficiently by a single vehicle for a long time. Based on these clusters, an allocation problem is solved to decide the amount delivered to each customer. These amounts are guidelines for the second phase where a VRPTW is solved for the first days of the planning period. See also (Campbell and Savelsbergh, 2004b) and (Campbell and Savelsbergh, 2004c) for more details on the insertion heuristic used to solve the VRPTW and the volume optimization performed once the routing is decided.

Gaur and Fisher (2004) analyze a periodic version of the problem where they assume a time-varying demand repeated over a week-long period. Fixed partitioning policies are considered and the routing part of the problem is solved by a randomized sequential matching algorithm, while the inventory management part is handled by stating a maximum time between deliveries. The model is part of a larger system, and the system and its implementation at a supermarket chain is also discussed. Rusdiansyah and Tsao (2005) also work with a periodic schedule, but assume a stable and deterministic demand and time windows at the customers. Another assumption not in (Gaur and Fisher, 2004) is that the replenishments are performed at equal intervals and with equal delivery sizes. The number of visit-day combinations for each customer

is limited, and they are used to define neighborhood structures in the proposed tabu search heuristic, (Gendreau, 2003).

Imposing different inventory policies is another way to make the general problem more tractable. Bertazzi et al. (2002) investigate how a policy where the inventory is always completely full after a delivery, i.e. an order-up-to level policy, performs and propose a two-step heuristic to solve the problem. The heuristic starts with a construction phase where a feasible solution is constructed by inserting the retailers one by one into the schedule using a shortest path algorithm to find the best delivery periods. In the second phase, the solution is improved by removing and then reinserting pairs of retailers. Later, Archetti et al. (2007) continue the research on order-up-to level policies. They propose an arc flow formulation for the problem and develop a branch and cut algorithm to solve it.

Savelsbergh and Song (2007) also use a greedy constructive heuristic to solve the inventory routing problem with continuous moves, but instead of working with delivery schedules for each retailer, only the next visit to each customer is under consideration. An urgency measure is used to decide which customer to visit next, and a cost minimizing procedure is used to choose the visiting vehicle. The heuristic is enhanced with a GRASP (Resende and Ribeiro, 2003), and a linear program is developed to optimize the delivery volumes once a schedule has been constructed.

Even though the inventory routing problem with continuous moves has many similarities to the maritime inventory routing problem, the algorithms to solve these problems are different. Flatberg et al. (2000) analyze the problem discussed in (Christiansen, 1999) and propose an algorithm combining a combinatorial local search heuristic with a linear program. The problem is decomposed into one ship routing problem solved by the heuristic and one scheduling problem where the routes of the vessels are given and the arrival times and quantities are optimized. The algorithm iterates between the two problems until no more improvements are found.

Dauzère-Pérès et al. (2007) use a great deal of practical knowledge about the problem in order to formulate a model that is less complex than many other models for the inventory routing problem but still captures the essence of the problem. The model excludes multiple visits and only works with departure and arrival times on the routing side. Due to a nonlinear objective function, only an approximation of the model can be solved. The model is merely for describing the problem and no solution approach is developed for it. Instead a memetic algorithm (Moscato and Cotta, 2003) is developed. The core component of the algorithm is an ordered list of visits to the customers and a local search that determines the best order. The local search is also combined with genetic algorithm operators to better search the solution space, (Reeves, 1993).

Another way to use genetic algorithms to solve an inventory routing problem is presented in Abdelmaguid and Dessouky (2006), where the problem is decomposed into one routing problem for each time period and one inventory problem. The link between the routing and inventory problems is the delivery quantities. These quantities are also the information in the chromosome. The crossover operator exchanges the delivery schedules for a set of customers between two chromosomes. This will preserve the customer inventory feasibility, but may introduce infeasibility in the vehicle capacity constraints. Once the infeasibilities are taken care of, a mutation operator that transfers part of the delivery quantities to different periods is applied. Kim and Kim (2000) only consider direct deliveries, but allow for more than one trip per vehicle and time period. They formulate the problem as a mixed integer linear program and propose a Lagrangean heuristic to solve it.

Table 4: Characteristics of central papers with finite time horizon

Author (Year)	Demand	Topology	Routing	Inventory	Fleet composition	Fleet size	Mode	Approach
Bell et al. (1983)	Stochastic	One-to-many	Multiple	Fixed	Heterogeneous	Multiple	Road-based	Lagrangean heuristic
Miller (1987)	Deterministic	One-to-many	Continuous	Fixed	Heterogeneous	Multiple	Maritime	Constructive heuristic, network flow
Chandra (1993)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Unconstrained	Generic	Ordering/distribution decomposition, sequential heuristic, local search
Carter et al. (1996)	Deterministic	One-to-many	Multiple	Back-order	Homogeneous	Multiple	Road-based	Allocation/routing decomposition, iterative heuristic
Christiansen (1999)	Deterministic	Many-to-many	Continuous	Fixed	Heterogeneous	Multiple	Maritime	Dantzig-Wolfe decomposition, column generation, branch and price
Flatberg et al. (2000)	Deterministic	Many-to-many	Continuous	Fixed	Heterogeneous	Multiple	Maritime	Routing/inventory decomposition, iterative heuristic
Kim and Kim (2000)	Deterministic	One-to-many	Direct	Fixed	Heterogeneous	Multiple	Road-based	Lagrangean heuristic
Bertazzi et al. (2002)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Single	Generic	Constructive heuristic, local search
Campbell and Savelsbergh (2004)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Multiple	Road-based	Allocation/routing decomposition, sequential heuristic
Gaur and Fisher (2004)	Stochastic	One-to-many	Multiple	Fixed	Homogeneous	Multiple	Road-based	Fixed partitioning policy, randomized sequential matching algorithm
Persson and Göthe-Lundgren (2005)	Deterministic	Many-to-many	Multiple	Fixed	Heterogeneous	Multiple	Maritime	Column generation, tree search
Rusdiansyah and Tsao (2005)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Multiple	Road-based	Tabu search
Savelsbergh and Song (2005)	Deterministic	Many-to-many	Continuous	Fixed	Homogeneous	Multiple	Road-based	Valid inequalities, branch and cut
Abdelmaguid and Dessouky (2006)	Deterministic	One-to-many	Multiple	Back-order	Heterogeneous	Multiple	Generic	Constructive heuristic, genetic algorithm
Al-Khayyal and Hwang (2007)	Deterministic	Many-to-many	Multiple	Fixed	Heterogeneous	Multiple	Maritime	Branch and bound
Archetti et al. (2007)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Single	Generic	Branch and cut
Dauzère-Pérès et al. (2007)	Stochastic	One-to-many	Continuous	Fixed	Heterogeneous	Multiple	Maritime	Greedy heuristic, genetic local search
Savelsbergh and Song (2007)	Deterministic	Many-to-many	Continuous	Fixed	Homogeneous	Multiple	Road-based	Greedy randomized adaptive search procedure
Yugang et al. (2007)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Multiple	Generic	Lagrangean heuristic

4.4 Infinite planning horizon

When the inventory routing problem with an infinite planning horizon is studied the cost minimization is no longer over the planning period, but instead the long-run average costs are minimized. Chan and Simchi-Levi (1998) state the problem as "*a single warehouse serves many retailers which are geographically dispersed in a given area. Stock for a single item is delivered to the retailers by a fleet of vehicles of limited capacity. Each retailer faces a deterministic, retailer specific, demand rate. Inventory holding costs are accrued at a constant rate, which is assumed to be identical for all retailers. ... The objective is to determine an inventory policy and a routing strategy such that each retailer can meet its demands and the long-run average transportation and inventory costs are minimized*".

Anily and Federgruen (1990) analyze this setting for a certain class of replenishment strategies. A strategy in the class divides the customers into regions, and each region is served independently. If a customer is visited, all customers in the region are visited. A customer can belong to more than one region, but the partitioning of its demand is done beforehand. Lower bounds on the optimal cost within the class are derived and a heuristic based on a geographical division of the customers is proposed. The proposed heuristic is shown to be asymptotically optimal for the given class of strategies. For further information, see also the comment by Hall (1991) and the rejoinder by Anily and Federgruen (1991a). In a later paper by Anily and Federgruen (1991b), details about calculating the lower bounds are described. Recently, Anily and Bramel (2004) derived lower bounds for the special case when a customer can only belong to one region.

Another replenishment strategy is direct shipping, which under the general setting is analyzed by Gallego and Simchi-Levi (1990), and later commented by Hall (1992) and Gallego and Simchi-Levi (1994). The main conclusion is that direct shipping is an effective alternative to more complex strategies when the economic lot sizes for all customers are close to the capacities of the vehicles. Direct shipping, with stochastic demands, was later analyzed by Barnes-Schuster and Bassok (1997) who derived conditions for when the strategy is powerful.

Extending the model analyzed by Gallego and Simchi-Levi (1990) with ordering costs depending on the number of truck loads, Jones and Qian (1997) show that direct shipping sometimes is optimal and also strengthen the result by Gallego and Simchi-Levi (1990).

The analysis in (Anily and Federgruen, 1990) was extended in (Anily and Federgruen, 1993), where the central depot also keeps an inventory. Here, the depot faces fixed ordering costs as well as inventory costs, and its replenishment strategy must be coordinated with the strategies of the customers. For the same class of replenishment strategies as in (Anily and Federgruen, 1990) restricted to power-of-two structures, the proposed heuristic is almost asymptotically optimal. In a power-of-two-structure, the replenishment periods for all customers and the warehouse are power-of-two multiples of a base planning period, see (Roundy, 1985) for details on the power-of-two structure.

With the addition of a customer ordering cost, Bramel and Simchi-Levi (1995) propose a location based heuristic for the general case. The heuristic is based on reformulating the problem into a location problem with a cost structure approximating the routing cost. The solution to the location problem partitions the customers into disjoint subsets, which can be served separately and independent of all other subsets. This class of strategies is denoted fixed partition strategies.

Chan and Simchi-Levi (1998) analyze the general setting under the class of fixed partition strategies. Lower bounds for any feasible policy are derived and compared to the asymptotic effectiveness of the fixed partitioning policies. The results are used to motivate the heuristic presented in (Bramel and Simchi-Levi, 1995) as well as the modified version presented in the current paper.

A power-of-two policy for the multiple product version of the problem investigated by Anily and Federgruen (1990) is proposed by Viswanathan and Mathur (1997), who analyze both uncapacitated and capacitated cases. Unlike Anily and Federgruen (1990), a fixed ordering cost is assumed. The heuristic by Viswanathan and Mathur (1997) is used by Custódio and Oliveira (2006) in an application concerning the delivery of frozen products in Portugal. A deterministic version of the problem is solved using the heuristic, and stochastic parts as well as sporadic demands are treated in a post-process phase.

Jung and Mathur (2007) extend the problem discussed in (Anily and Federgruen, 1993) by allowing different reorder intervals for each customer in a cluster. Their three-step heuristic is based on first clustering the customers into clusters and then sequencing the customers in a cluster according to the reorder intervals. In the last step, a relaxed integer linear program is solved and the solution is rounded to fit the power-of-two policy constraints. Another extension to the (Anily and Federgruen, 1993) problem is discussed by Zhao et al (2007). Here, the customers are not split between different partitions, but the clustering problem is embedded in a tabu search framework to find the optimal partitions.

Allowing the vehicles to drive more than one route, and thus decreasing the number of vehicles used, is analyzed by Aghezzaf et al. (2006). The general assumptions are as in (Anily and Federgruen, 1990) and a heuristic column generation approach is used to solve the problem.

All approaches described above are based on dividing the customers, or the demand, into sets that can be served separately and independent of each other. This means that there is no coordination between visits to different sets, and hence that a limited number of vehicles cannot be considered.

One way to avoid the coordination problem is to assume only one vehicle. Herer and Roundy (1997) assume an uncapacitated vehicle but follow besides this the assumptions in (Anily

and Federgruen, 1993). By approximating the routing cost with a submodular function, the optimal power-of-two reordering intervals are calculated.

Another way to handle the coordination between different replenishment decisions is to formulate the problem as a Markov decision process. Here, the state is the inventory levels of all the customers and the actions are the different routing decisions. The goal is to minimize the short-term cost, i.e. the cost of the action taken, and the expected long-term cost as a function of the action.

Minkoff (1993) was the first to use this approach on a setting with stochastic demands. He bases the analysis on approximating the value function by a sum of value functions, one for each customer and proposes a three-step heuristic. In the first step, a linear program is solved to allocate transportation costs from a set of complex routes to direct shipping routes. The second step consists of solving a delivery problem for each customer, and in the last step, dispatches are calculated based on the solutions in step two.

Even though Minkoff (1993) assumes an unlimited number of vehicles, the approach points toward a way to handle a limited fleet. Kleywegt et al. (2002) analyze the same problem as Minkoff (1993) but change the assumptions to include a limited fleet and only allow for direct deliveries. In order to solve the problem, three tasks are identified; computing the optimal value function, estimating the long-term costs, and maximizing the expected total discounted value. A dynamic programming-based approximation method is developed for the first task, where the problem almost decomposes into customer specific subproblems. The solutions to these problems are then used to estimate the expected value using well-known methods for high-dimensional integration. The third task is solved using a greedy heuristic. Stock-outs are allowed, but no backordering is possible.

Later, Kleywegt et al. (2004) extend the problem analyzed in (Kleywegt et al., 2002) to allow for multiple deliveries per trip. The same three tasks as in (Kleywegt et al., 2002) are identified, but the first and the third are handled differently. The first task is approached by solving subproblems with fewer customers. The subproblems are also Markov decision processes, which hopefully make a good representation of the overall process. When these subproblems are solved, they can be combined into an approximation of the value function by solving a partitioning problem including the number of vehicles constraint. The third task is solved by a local search procedure starting from a solution with one customer per trip and then adding customers to the trips.

Adelman (2003) and Adelman (2004) use the same approximation of the value function as Minkoff (1993) but interpret the value functions as marginal prices for the transportation. The optimality equations for the value function are reformulated into a linear program and the value function approximation is used to reformulate the linear program to a more tractable problem. The dual prices to this linear program are the optimal prices used to calculate the optimal policy. In (Adelman, 2003), a deterministic setting is analyzed and the proposed way of calculating the value function is compared with existing methods from the literature. Adelman (2004) derive the linear program for the stochastic case and outline an algorithm for the solution.

Instead of solving the problem by approximating the value function, as in the above papers, a heuristic finite horizon approach is proposed by Hvattum et al. (2007) and Hvattum and Løkketangen (2007). The main ideas are that even though the problem has an infinite planning horizon, most of the stochasticity can be captured in a finite scenario tree, and that solving a scenario tree problem can give good routing and inventory decisions. In Hvattum et al. (2007) a GRASP is developed for the scenario tree problem, where the construction is based on successively increasing the delivery volumes. In the second paper, (Hvattum and Løkketangen, 2007), the assumption that all identical sub-paths in the scenario tree should be assigned the same decisions is used in a progressive hedging algorithm. This decomposes the problem into one-scenario problems. A penalty scheme is then used to steer the one-scenario solutions to an implementable solution for the original problem.

A simpler version of the stochastic inventory routing problem with only a single vehicle is studied by Reiman et al. (1999). An interesting extension is that both demands and traveling times are stochastic. By only allowing traveling salesman tours visiting all customers and direct deliveries, the three versions studied are modeled as queuing control problems and solved using heavy traffic analysis. The versions considered are fixed routing using either only traveling salesman tours or direct deliveries and dynamic routing where both types can be used.

A problem similar to the one discussed by Reiman et al. (1999), but with deterministic traveling times, is addressed in (Schwartz et al., 2006). The proposed heuristic starts with calculating the best route given that only one visiting sequence is used, this is the preferred route. Each time a scheduling decision is made, it is based on the assumption that the preferred route will be the subsequent route.

Another research area with an infinite planning horizon is the investigation of strategic issues related to the coordination of inventory management and routing. The main focus here is to derive results concerning the general structure of the logistics systems under consideration, often by the use of analytical models. Blumenfeld et al. (1985) use models based on the economic order quantity to analyze different topologies, including consolidation terminals, where all distribution is direct deliveries. The effects of coordinated and uncoordinated production scheduling is also discussed, something which is further developed in Blumenfeld et al. (1991). In a companion paper, (Burns et al., 1985), a simpler topology, one-to-many without consolidation terminals, is analyzed and the focus is on more advanced routing. Using approximations of the round-trip distance for customers randomly located in an area, analytical expressions for the total cost of routes with multiple visits are derived and compared with similar expressions for the direct delivery case. The results come from the development of a decision support system together with General Motors, which is further described in (Blumenfeld et al., 1987).

As noted in (Speranza and Ukovich, 1994), the analytical methods presented by Blumenfeld et al. (1985) often give dispatch frequencies that can be hard to implement in operations. Instead, they analyze the situation when a number of products are shipped from a depot to a customer with vehicles that may only leave at some given frequencies. A number of cases related to the splitting of products to different frequencies and combining products on the same vehicle are analyzed. Mixed integer linear programs are formulated for the cases, and upper and lower bounds are derived. Later, the case with several products and frequencies is further analyzed by Speranza and Ukovich (1996), who also propose a branch and bound-based algorithm for the problem.

The results in (Speranza and Ukovich, 1994) and (Speranza and Ukovich, 1996) are used by Bertazzi et al. (1997) where the multiple customer case is analyzed. The proposed heuristic is based on solving the one customer case for each customer and then identify customer which are served by shipments having the same frequency. These customers are aggregated into routes and new frequencies are calculated.

When analyzing periodic replenishment policies, the infinite problem can be solved as a finite problem with a planning horizon equal to the least common multiplier of the replenishment periods of all products. Qu et al. (1999) formulate the problem with variable periods, i.e. a variable planning horizon, and solve it by iterating between an inventory problem and a routing problem. The solution to the inventory problem gives which supplier to visit each period and is used in the routing problem to find the optimal tours. The transportation cost is then fed back to the inventory problem and the process is repeated. A similar problem extended with capacitated vehicles and storage constraints is analyzed by Stacey et al. (2007), who develop two heuristics based on the heuristic by Bramel and Simchi-Levi (1995). The heuristics differ in how the storage constraints are handled.

Table 5: Characteristics of central papers with infinite planning horizon

Author (Year)	Demand	Topology	Routing	Inventory	Fleet composition	Fleet size	Approach
Blumenfeld et al. (1985)	Deterministic	One-to-many Many-to-many	Direct	Fixed	Homogeneous	Unconstrained	Analytical method
Anily and Federgruen (1990)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Unconstrained	Lower bound, fixed partitions
Gallego and Simchi-Levi (1990)	Deterministic	One-to-many	Direct	Fixed	Homogeneous	Unconstrained	Lower bound, analytical
Anily and Federgruen (1993)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Unconstrained	Lower bound, fixed partitions
Minkoff (1993)	Stochastic	One-to-many	Multiple	Lost sale	Homogeneous	Unconstrained	Markov decision process, customer decomposition
Speranza and Ukovich (1994)	Deterministic	One-to-one	Direct	Fixed	Homogeneous	Unconstrained	Lower and upper bounds, Mixed integer programs
Bramel and Simchi-Levi (1995)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Unconstrained	Lower bound, fixed partition, cluster first/route second
Barnes-Schuster and Bassok (1997)	Stochastic	One-to-many	Direct	Back-order	Homogeneous	Unconstrained	Lower bound, analytical
Bertazzi et al. (1997)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Unconstrained	Sequential heuristic, single customer analysis, local search
Herer and Round (1997)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Single	Submodular approximation, local search
Jones and Qian (1997)	Deterministic	One-to-many	Direct	Fixed	Homogeneous	Unconstrained	Lower bound, analytical
Viswanathan and Mathur (1997)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Multiple	Constructive insertion heuristic
Chan and Simchi-Levi (1998)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Unconstrained	Lower bound, fixed partitions
Qu et al. (1999)	Stochastic	One-to-many	Multiple	Stock-out	Homogeneous	Single, uncapacitated	Lower bound, modified periodic policy, routing/inventory decomposition
Reiman et al. (1999)	Stochastic	One-to-many	Direct, multiple	Back-order	Homogeneous	Single	Heavy traffic analysis, Monte-Carlo simulation
Kleywegt et al. (2002)	Stochastic	One-to-many	Direct	Lost sale	Homogeneous	Multiple	Markov decision process, iterative heuristic
Adelman (2003)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Multiple	Markov decision process, linear reformulation, dual information
Adelman (2004)	Stochastic	One-to-many	Multiple	Lost sale	Homogeneous	Multiple	Markov decision process, linear reformulation, dual information
Kleywegt et al. (2004)	Stochastic	One-to-many	Multiple	Lost sale	Homogeneous	Multiple	Markov decision process, iterative heuristic
Custódio and Oliveira (2006)	Stochastic	One-to-many	Multiple	Fixed	Heterogeneous	Multiple	Constructive insertion heuristic
Schwartz et al. (2006)	Stochastic	One-to-many	Multiple	Back-order	Homogeneous	Single	Analytical model, simulation, change-revert heuristic
Hvattum and Løkketangen (2007)	Stochastic	One-to-many	Multiple	Stock-out	Homogeneous	Multiple	Markov decision process, scenario tree, GRASP
Hvattum et al. (2007)	Stochastic	One-to-many	Multiple	Stock-out	Homogeneous	Multiple	Markov decision process, scenario tree, progressive hedging
Jung and Mathur (2007)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Unconstrained	Lower bound, fixed partition integer linear program
Stacey et al. (2007)	Deterministic	One-to-many	Multiple	Fixed	Homogeneous	Multiple	Cluster first/route second location based heuristic

4.5   Industrial cases

Solving real practical problems is one of the corner stones in operations research. In this chapter we have so far classified the literature on a scientific basis while here we focus on the industrial applications presented. First of all we have to define what we mean with an industrial application. We have chosen a restrictive view and only consider those papers where the focus is on the

industrial application and where the structure of the paper emphasizes the application. Papers where a generic model is exemplified through industrial cases are therefore not included.

A table of the papers discussed ends this chapter. The products as well as some special features concerning the routing and inventory management of each application are shown. The table is in no way comprehensive, but shows only some of the more prominent features.

Naturally, the industrial applications presented all deal with products having a relatively high consumption rate. In the cases with typical bulk products, like industrial gases and ammonia, it is natural to treat the delivered volumes as continuous, but also general cargo, like groceries and automobile components, is treated as flow due to the high consumption rate.

The early applications deal with gas, (Bell et al. 1983), (Golden et al. 1984), chemicals (Miller, 1987) and automobile components (Blumenfeld et al. 1987). All but (Golden et al. 1984) present already developed decision support systems. Later applications deal with ammonia, (Christiansen, 1999), groceries, (Gaur and Fisher, 2004), industrial gases, (Campbell and Savelsberg, 2004), bitumen (Persson and Göthe-Lundgren, 2005), calcium carbonate slurry, (Custódio and Oliveira, 2006), frozen products, (Dauzère-Pérès et al., 2007), and petrochemical products (Al-Khayyal and Hwang, 2007). Of these, only (Gaur and Fisher, 2004) and (Dauzère-Pérès et al., 2007) present implemented decision support systems.

There are probably many reasons for the trend towards less focus on the system. One reason could be that the implementation phase of a project does not actively involve the researcher any longer as the system must interact with so many other systems that special competence is needed in the implementation phase. Another reason is the journal ratings in various countries, often favoring journals focusing on methods and algorithms above more practice oriented journals. A third reason is the status within the scientific community, where theoretical studies often are valued higher than more practical work.

The uniqueness of the industrial cases is important. There are often some factors that are not included in already existing models that can either complicate or sometimes also simplify the problem. Bell et al. (1983), Gaur and Fisher (2004) and Dauzère-Pérès et al. (2007) all use knowledge about the number of customers that are normally visited on a route to simplify the generation of routes, while Christiansen (1999) use the consumption/production rates and inventory bounds to explicitly model each possible port call and to define time windows for each visit. In all these cases, information from the industrial cases plays a crucial role in the development of the model and solution approach. Even though Al-Khayyal and Hwang (2007) have used the same modeling idea as Christiansen (1999), the lack of time windows in their model does not make it possible to a priori restrict the number of port calls and hence make the solution approach cumbersome.

Among the maritime applications, Miller (1987) and Dauzère-Pérès et al. (2007) discuss the problem with sloshing, i.e. the back-and-forth splashing of a liquid in a tank, which is critical during transportation of liquid bulk and chemicals. In applications with multiple products, the stowage planning is another important issue. Miller (1987) uses a product mix dependent vessel capacity to handle the stowage problems, while Persson and Göthe-Lundgren (2005) and Dauzère-Pérès et al. (2007) approximate the compartments onboard the vessels with continuous sizes. The most rigorous treatment of the stowage planning is found in (Al-Khayyal and Hwang, 2006) where each compartment is dedicated to a product. The application in (Christiansen, 1999) only concerns one product, and hence the stowage is no problem.

Stowage planning can be a problem in road-based applications as well, especially when bulk products are transported. In the road-based applications in this study, either a single bulk product is transported or the products are packed and can easily be separated after mixing.

All decision levels are present in the industrial applications. From the strategic model presented in (Blumenfeld et al., 1987) and (Custódio and Oliveira, 2006), via the tactical model of Fisher and Gaur (2004) to the operational problems discussed in the other papers. This is also evident in the methodology use. Both Blumenfeld et al. (1987) and Custódio and Oliveira (2006) use continuous approximation models without and with a routing component, respectively. In

Gaur and Fisher (2004) the fact that a weekly schedule is repeated is used to formulate a periodic model. On the operational level, the difference in method is bigger, ranging from pure integer programming-based methods in for example (Christiansen, 1999) and (Persson and Göthe-Lundgren, 2004) to heuristics and meta-heuristics in (Golden et al., 1984) and (Dauzère-Pérès et al., 2007), respectively.

Table 6: Routing and inventory management aspects of industrial cases

Paper	Products	Routing aspects	Inventory management aspects
Bell et al., (1983)	Industrial gases	Restricted routing Few visits on each route	Bounds on delivered quantities Bounds on number of visits
Golden et al., (1984)	Industrial gases	Heuristic routing Savings-based heuristic	Fixed quantity to deliver
Miller, (1987)	Chemicals	Restricted routing Perturbations of existing solution	Inventory balance and capacities
Blumenfeld et al., (1987)	Automobile components	Flow-based routing	Economic order quantity assumption
Christiansen, (1999)	Ammonia	Exact routing	Inventory balance and capacities
Gaur and Fisher, (2004)	Groceries	Restricted routing Few visits on each route	Fixed quantities to deliver
Campbell and Savelsbergh, (2004)	Industrial gases	Restricted routing Cluster-based approach	Bounds on delivered quantities
Persson and Göthe-Lundgren, (2005)	Bitumen	Exact routing	Inventory balance and capacities
Custódio and Oliveira (2006)	Frozen products	Restricted routing Greedy heuristics	Economic order quantity assumption
Al-Khayyal and Hwang, (2007)	Oil	Exact routing	Inventory balance and capacities
Dauzère-Pérès et al., (2007)	Calcium carbonate slurry	Restricted routing Direct shipping	Inventory balance and capacities

5   Future trends and directions

In this section we tie together the most important findings from the literature review and compare them with the current industry status. Moreover, we describe trends and future directions within combined inventory management and routing. The next section contrasts the research literature and industry status. It pinpoints both similarities, where a close collaboration between the research community and the industry has a positive effect on both, and differences where the gap between industry needs and research results is too large.

In the following two subsections we outline the trends in both industry and research. Here, the ongoing developments in industry as well as the new areas within research are presented. The last subsection is a look in the crystal ball, where we describe in which direction we think the industry will head and where the most promising, and important, research paths are.

5.1   Status summary

Gain in interest due to increased focus on supply chain management
We have seen an increased interest in combined inventory management and routing problems in both industry and academia in the last years. With the development of supply chain management and the realization that a company's performance depends to a high extent on the performance of its customers and suppliers, the demand for decision support systems working with more than one logistic process has increased. This demand from industry has to some degree been met by academia, see section 4.5 about industrial cases, but there are still too few published papers dealing with real industrial cases.

There is also a gap between what industry needs and what the decision support system providers offer.

Many different problems, no clear definitions

Combined inventory management and routing is not one single well-defined problem, but a whole class of problems trying to describe certain links and relations between actors in a supply chain. Even the inventory routing problem is not as well defined as the capacitated vehicle routing problem and many other versions of routing problems.

This is not a bad thing in itself; it is very hard to capture all aspects of an industrial application in a stylized model, but it may harm the algorithmic development. Without a well-defined problem and relevant and rich benchmarks, it is hard to do good computational studies in order to test an algorithm's performance and generality.

A gap between industry and academia

Depending on transportation mode, there are large differences in industrial relevance of the scientific publications. When looking at the published literature, two things are evident. All papers considering maritime applications are based on industrial cases, while the overall fraction of papers with a maritime focus is small. On the other hand, many of the papers discussing a generic setting tend to have assumptions, such as a classical vehicle routing structure, relatively short travel distances, and many customer visits between each visit to a depot. These assumptions give a clear flavor of road-based applications. We can clearly criticize both industry and academia for this mismatch, but it is more important to try to increase collaboration and decrease the gap between research and industrial needs.

Different planning horizons

The lack of published papers on problems with a very short planning horizon in recent years is clear. There are many potential reasons. One is that problems with short horizon fall between what is industrially relevant and what is scientifically interesting.

From an industrial perspective, it is more common to plan on a tactical level and decide on when to visit the customers and then have routing software working with fixed orders for the execution of the day-to-day operations. This favors models with a longer time horizon and other richer routing problems without an inventory management aspect. The same is true for the majority of companies that do not have routing software.

Some versions of the instant period problem resemble stochastic and dynamic versions of other routing problems. These are better defined and may therefore be more interesting from a mathematical and computational perspective.

Most industrial cases presented are modeled with a finite time horizon. This is natural since combined inventory management and routing problem are often identified on a tactical or operational level. The models are usually detailed enough to give solutions that can be implemented in day-to-day planning, and also give an idea about the long term effects of the operational decisions. Still there are aspects missing, especially in the treatment of uncertainty and risk. These aspects are partially handled by introducing rolling time horizon frameworks, but introducing elements from stochastic programming into the industrial cases has not been done yet. In spite of their shortcomings, we see these lines of research as the most promising from an industrial perspective.

Stochastic demand has been better treated in the literature when an infinite planning horizon is assumed, especially by using Markov decision processes. In spite of this there is still a long way to go before stochastic models are ready to be implemented in decision support systems.

Literature promises more than it actually delivers

There are cases from the literature where more than delivered is promised in the title, abstract and introduction. What often happens is that assumptions and simplifications are imposed that changes the problem from the rich problem described in the abstract and introduction to a more idealized one. These flattering descriptions of the then studied problem is unfortunately a general

trend, not only seen in this literature review. Presumably, it reflects the growing need to "sell" the paper to journals and other publication channels. This may be good marketing in the short run and a necessity given the publication-based rules for funding in many countries, but we believe that it is harmful for the scientific community in the long run. The referees also have a responsibility here, and should be observant of this problem and clearly point out if these tendencies are found.

Little work on exact methods
There is relatively little work on exact methods in the current literature. One reason for this might be that the models describing the problems the research community wants to solve become too complex to solve exactly, except for small instances. On the other hand, exact methods have been used heuristically, for example early termination in branch and bound-based methods or as a part of a heuristic, with good results, especially for problems with a finite time horizon. A consequence of this is that few dual bounds and bounding procedures exists, which makes it hard to evaluate the proposed heuristics.

As mentioned in Section 4.3, another interesting observation is that so few solution approaches are based on path flow formulations and column generation frameworks, which very common approaches for other similar routing problems. A reason for this could be that the inventory considerations introduce problems with the path feasibility. Since the feasibility of the paths cannot be ascertained in the generation phase, extra constraints are needed in the master problem. Hence, many of the advantages with path flow formulations are lost.

5.2   Trends in industry

Section 2 described some industrial aspects related to combined inventory management and routing problems, and a glimpse at the current industrial practice was given. Current trends in industry were mentioned when appropriate. The focus here is on the major trends we see in industry that probably will influence the need for DSSs for combined inventory management and routing, as well as the industry's acceptance of and benefits from such systems.

Data availability, visibility and information sharing
One major trend in the transportation industry is the increased availability of data and information, both in quantity and quality. New technologies, such as GIS, GPS, automatic metering of inventory levels and RFID (Radio Frequency Identification) enable this trend. This helps in tracking goods and vehicles, as well as in handling orders and communicating with customers.

Company data, which used to be spread across numerous legacy systems, is now collected centrally and made available in larger parts of the supply chain, continuously and in real-time.

Development in software and hardware
A general and well known trend is the developments in information systems and DSSs within supply chain management and the increased number of companies using such systems in their regular planning. Along with this, there has been significant algorithmic progress. The fast technological growth in computers and communications has contributed heavily to the introduction of successful DSSs in industry and has made it possible to include important problem characteristics and provide a good user interface.

Increased cooperation along the supply chain
As argued in the introduction there is a clear trend of increased collaboration between actors within a chain in order to stay competitive and survive. The supply chain view is therefore dominating when managing and planning activities within the chain. As an example, there exists an increasing number of VMI contracts between actors in supply chains.

Increased globalization

Increasingly, companies procure raw materials and components worldwide, produce in low-cost countries, and sell their products in many different markets (Christopher 2005). This leads to more complex and costly transportation and material flows, larger amounts of inventory in the supply chain, higher uncertainty and variability, and higher risk. Manufacturing companies must therefore put more emphasis on efficient design and management of their global supply chains (Ghiani et al. 2004).

Acquisitions and mergers

There is also a trend in industry, through acquisitions and mergers, to form larger companies. To reduce the imbalance between the size of the production companies and the transport providers, we have witnessed consolidation also among transport providers. This has increased their market power and enabled a larger flexibility in the services offered. Most often competing companies or companies operating in the same market are potential mergers in order to gain a stronger position on specific links in the supply chain. However, we also see fusions along a supply chain giving the merged company control of a larger part of the supply chain.

Extended transport contracts

Many traditional transport providers extend their services and become third-party logistics providers. These companies provide third party services to companies for part or all of their supply chain management function.

Increased number of DSSs under development

The number of inventory management DSSs and routing DSSs, as well as supply chain DSSs, is vast. Often the supply chain DSSs contain modules for both inventory management and routing, but it is unclear if these modules are integrated or used sequentially. However, there is a pronounced and increasing need for tools that combine inventory management and routing and produce optimal or near-optimal plans in both the road-based and maritime industry. Several systems are under development in both modes of transportation, and we will see several in use in the years to come.

Modern planners

Finally, we see a trend of more planners with a college or university education. In the old days, a planner typically had started working with transportation at a practical level, and then finally ending up doing the company's planning. The new trend is a planner with a college or university education, who knows a lot about economics and optimization in general. He or she is more able to use DSSs, better knowing their virtues and limitations.

Environmental focus

There is an increased focus around environmental aspects in general. Also, environmentall y related requirements for transportation systems have increased considerably over the last years. There are strong rules for handling hazardous materials along the chain. There will be more restrictions on emission of dangerous gases, as well as noise and dust, from transportation. In addition to costs, the stronger environmental requirements demand efficient supply chains.

5.3 Trends in the research literature

Both the current status and some trends have briefly been discussed in section 4. In this section, the most important and prominent trends will highlighted and commented.

Richer models
With the increased number of papers describing industrial cases as well as more complexity in the scientific-oriented papers, we see a trend towards using richer models, being more capable of describing problems that are closer to the real world.

Uncertainty and risk
Including stochasticity and elements of uncertainty into the models will be more and more important, especially in the theoretical papers working analyzing generic models. This is very good for the general understanding of combined inventory management and routing problem, but we do not think this will lead to any imminent changes in the way industrial cases are modeled and solved. Both the problem sizes that can be solved and the general lack of knowledge about stochastic models and solution approaches are obstacles that need to be overcome.

Industrial cases
Of the papers we have identified as industrial cases, more than half are published the last four years. Together with an overall increase in the number of papers addressing combined inventory management and routing issues, this is a very positive trend that we expect to continue.

Advanced heuristics
With the complexity seen in combined inventory management and routing problems, heuristics may be the most reasonable solution methods to industrial relevant problems. A number of solution approaches based on meta-heuristics have been published recently, and there is a clear trend in the routing literature to develop more sophisticated heuristics combining traditional heuristics with optimization-based approaches.

5.4   Future research directions

Based on the status and trends described so far in this section, we suggest a few research directions based on needs both from the research community and from industry.

Richer models and integrated systems
It is clear that the industry needs models that reflect their real planning challenges. This means that researchers should include more and more elements of the real planning challenges in their research. In this development, it is necessary with a close cooperation between industry and researchers.

In addition, the models should more often contain combined planning aspects. For the combined inventory management and routing problem, which is a combined problem in itself, this includes among other aspects fleet composition, location issues, and contract evaluation depending on the particular real problem.

There is a clear trend of increased collaboration between actors within a chain resulting in new and extended planning issues. This means that new and richer models should be developed to cope with the new challenges. The increased data availability is important in this development. As mentioned, there is an increasing number of VMI contracts between actors in supply chains. However, few of them combine the vendor inventory management aspects with routing. ERP systems with VMI modules should integrate routing into the planning.

These future research directions need, in addition to development of relevant and efficient models, further development of solution methods, DSSs and hardware.

Focus on uncertainty, robustness and flexibility
Within combined inventory management and routing there are technical uncertainties due to transportation conditions and equipment, as well as economical or market uncertainties. In many businesses the market conditions have changed dramatically over the last years with new market

opportunities arising continuously. As a result the demand for products becomes highly uncertain in some business areas. Researchers should focus on development of models and methods that match the industries need of robust and flexible plans to handle the uncertainties. However, most companies are not aware of the possibilities of introducing stochastic elements in the planning. Neither are they familiar nor confident with stochastic planning systems. Here, researchers have a huge responsibility in convincing industry about the potential of stochastic planning.

Better benchmarks

To enhance the development of methods for solving real-world combined inventory management and routing problems, benchmarks suited for this task should be developed and made available. In order to develop such benchmarks, researchers need real data from industry and a full and rich problem description. At present, most benchmarks are too simplistic to address the industrial aspects in a proper way. Another problem is that many case descriptions in the literature are not specific enough for reconstruction of the underlying problem model. Finally, companies have traditionally been less than willing to make their operational data available to the competition. If the data is masked or transformed, the resulting test-cases are often less rich, or detailed, than what is required for describing real-world problems.

In order to overcome the problem of diversity and number of variants of real-world combined inventory management and routing problems, it is important to choose an appropriate structure for the data.

6 Concluding remarks

Cooperation and integration along the supply chains is becoming increasingly more common, as most managers can clearly see the benefits. There is also an increasing number of mergers leading to larger players controlling more of the supply chains. This leads to the need for methods and tools to facilitate and control this integration, especially in industrial settings.

This paper has first given an overview of combined inventory management and routing problems, describing the various types of supply chains encountered. Conditions for when inventory management and routing can be combined are described, together with current industrial practice.

To bring order into the research literature on combined inventory management and routing, a classification scheme has been developed. The structure of the literature review is based on this classification. In the review, around 90 papers are discussed. In section 5, the current status is summed up and trends within both industry and research are presented. The section ends with a look into the future and identifies future research directions.

Both the industry and the research community are now well aware of the importance of combined inventory management and routing, and the synergies that can be achieved by integrating these two logistical processes. With the increased collaboration between actors in a supply chain that is seen today, information sharing and integration will be key aspects of the planning of tomorrow.

There is still a long way to go on the path to fully integrated decision support systems where the decisions made are not only based on local information, but where the whole supply chain is considered. We believe that combining inventory management and routing decisions is a small, but important, step in the right direction. With the combined efforts of the industry and the research community, where information is shared and ideas exchanged, we see large potential improvements in logistics performance through better supply chain coordination.

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