Arne Johannesen

SINTEF RESEARCH 1

An Intelligible and Practicable Methodology for Power System Dynamic Analysis

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Part 1: System Modeling
Part 2: Component Modeling

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Preface

Electrical power systems all over the world are steadily being tied more closely together by strengthening of local national connections, as well as more ties across borders to neigh-boring countries.

It is a challenge both in design and operation of an expanding interconnected power system, to ensure that geographically distributed power supply and demand becomes matched in an optimal way. – I.e. a way that provides for proper buy/sell situations for all participants of the power market, and where agreed-upon qualities of delivery conditions are met.

To succeed in the stated optimal large-scale matching of electrical power production and demand, mathematical models have to be applied on two main and interrelated levels:

On economy level *market driven optimal power flow analyses* have to be applied to match distributed and partly price sensitive demand, to distributed and «competing» production facilities. Such facilities may e.g. include large scale thermal plants cost-evaluated via *defined fuel costs*, reservoir hydro plants that are cost-evaluated via computed *time-variable water values*, and *forced power input* from distributed facilities based om power from firstly sun/wind/small waterfalls. See SINTEF Energy Research Report TR A4651, [11].

World-wide considered, there is strong motivation for more sustainable behaviour within the energy sector. Such behaviour is first of all achieved via properly specifying the terms *defined fuel cost* above, and by prioritising increasing the world-wide capacity of what above is termed *forced power input*.

On technical level *power system dynamic analyses* have to be conducted as part of the processes of initially defining proper power quality constraints, and next following up by checking quality conditions during operation.

This report deals with power system dynamic analysis. Central to the presented methodology is the development of a stock of compact sub models for modeling of power system components.

Formulation and solution-wise, problem complexity becomes thereby largely confined to local *component* level rather than overall *system* level.

Contents

Part 1 System Modeling

Part 2 Component Modeling

Abstract

This paper summarizes a compact process of power system dynamic analysis that – indifferent of the level of detail observed in the electrical modeling of any given component – allows for appropriate power network modeling and analysis in a way that appears conceptually straight forward as well as practicable.

Based on describing all power system components in terms of discrete elements, the power network related modeling is conducted within the d–q axis frame of reference. Central to the methodology is the definition of a standard electrical circuit model to act as common network building block for all power network components.

Part 1 gives an overview of component- and system modeling as well as model application. A small example illustrates the methodology.

Part 2 treats component modeling in depth. Some overview observations on the detailed analysis of unbalanced conditions are also made.

Part 1
System Modeling

1. Introduction

The paper summarizes a methodology that – independent of the complexity level observed for describing any given component – allows for befitting power system dynamic modeling and analysis in a way that appears conceptually straight forward as well as practicable.

Central to the methodology is the development of a stock of compact *sub models* for modeling of power system components. See Table I for illustrations.

Formulation- and solution-wise, problem complexity becomes thereby largely confined to *component* level rather than overall *system* level.

Transparency is retained throughout studies, via running access to prevailing algorithmic as well as numeric content of what is termed *the primitive system*. See coming Section 2.

Publications [1]–[5] serve very well among the up-to-date references that both in depth and width, deal with *Power System Dynamic Analysis*. It emanates that component- and system modeling normally is being dealt with in such a way that the complete system model consists of a large set of *ordinary differential* equations plus a large set of *sparse algebraic* equations. In the integrated solution process which may be based on different schemes, the processing part relating to solving the algebraic equations, is similar to the iterative process met with in power flow analysis.

In the present paper a loop current approach is applied to describe how power system components interact in operation. The complete system model may then getting close to being a large set of *ordinary differential* equations, the solution of which may take place without, or with only marginal inclusion of, the element of iterative processing. The solution process implies frequent generation and inversion of system loop matrices, the computational burden of which may increase rapidly with increasing size of the power system. It is envisaged that the use of parallel processing together with tailored mathematical processes exploiting e.g. matrix sparsity and diagonality, would contribute to retaining practicability of the proposed scheme of analysis.

2. Conceptual overview

A. Approach to network analysis

In basic circuit analysis electrical circuit models accounting for components like resistors, inductors, capacitors and sources, are interconnected into a model network to afford study of the performance of some given physical process.

In such analyses a well known and intelligible three-stage task sequence affords building the desired network model [6]:

- Arrange the set of electrical circuit models associated with the network into (what Gabriel Kron denoted) «the primitive network». The latter network comprises 3 main parts; 1) a set of oriented graph elements defining the graph structure of the primitive network, 2) a set of square and for the most part diagonal matrices containing component parameter figures, and 3) a vector matrix comprising figures that describe the sources associated with resp. electrical circuit models.
- Describe how the electrical circuit models of the network are to be tied together, e.g. by a loop incidence matrix, or a node-related incidence matrix.
- Produce the desired network model via standard matrix operations related to the primitive network and the incidence matrix.

With the electrical circuit model formally defined as common building block to all power system components, the above three-stage process is being retained in the outlined methodology.

B. The electrical circuit model

The power network related modeling is conducted within the d-q axis frame of reference.

Network-wise, any power system component is then represented in terms of one or more *electrical circuit model(s)*, each comprising a 2x2 resistance matrix $\mathbf{R}$, a 2x2 inductive reactance matrix $\mathbf{X}_L$, and an 2x1 *electromotive force* (emf.) matrix $\Delta\mathbf{E}$. All component-specific complexity is «hidden» within the confines of the circuit terms ($\mathbf{R}$, $\mathbf{X}_L$, $\Delta\mathbf{E}$). Depending on which system component a given circuit term contributes to describing, it may be a zero matrix, a constant matrix, or a matrix containing elements that are functions of one or more of the variables that relate to the system component at hand.

To illustrate: As a *network component*, a lossy capacitor bank will appear as an emf. represented by an *electrical circuit model* of generic terms ($\mathbf{R}_C$, $\mathbf{X}_L$=0, $\Delta\mathbf{E}_C$). $\Delta\mathbf{E}_C$ being in this case a set of 2 (d-q axis) state variables governed by a separate *capacitor voltage model*. The main content of the latter model being a 2x2 capacitive reactance matrix $\mathbf{X}_C$ defining the size of the capacitor bank. See Section 1 of Part 2 for further details.

From the preceding illustration it is incidentally observed that a capacitor bank is to be modeled by a set of *two sub models*, - namely the stated *electrical circuit model* which accounts for the lossy capacitor emf. in the network equations, and the *capacitor voltage model* describing the «inner life» of the ideal capacitor emf. Collate Table I.

C. State variables

They comprise the power network state variables and the remaining or «local» state variables:

- *The power network state variables* are the defined network loop currents together with the capacitor voltages of the network. All state variables being implied by the usual single line diagram of the power network. Given a) the arrangement of involved electrical circuit models into what previously was termed «the primitive network», b) topological information describing how the primitive network's graph elements are to be tied together, and c) 2x2 capacitive reactance matrices $\mathbf{X}_C$ characterizing respective ideal capacitors of the network diagram, power network modeling is readily afforded by generating an appropriate set of network equations. In the present scheme of analysis a system loop matrix $\mathbf{B}$ is defined and applied to the «machinery» of generating the differential equations that describe the performance of the power network state variables.
- *The remaining or «local» state variables* may illustration-wise be fluxes and angular speed of rotating machines, electrical angle of synchronous machines, and variables associated with involved control systems. A state variable is here termed «local» when no foreign, but only one or more of the power system component's own variables appear explicit in the differential equation that describes the state variable.

  Differential equations that model «local» state variables can thus be formulated independent of the *network* related task at hand. See Table I plus footnote.

D. System model and model application

The system model is here the aggregate of simultaneous first order, ordinary differential equations describing the behaviour of the set of all system state variables.

Model application normally implies initial condition analysis followed by eigenvalue- and/or time response analysis:

- *Initial condition analysis* means setting d/dt = 0 in all of the differential equations of the system model, and solving for the particular steady state solution that fulfils the initial power flow requirements. An efficient gradient technique is used iteratively to converge sufficiently close to the desired initial solution. See Section 5 below.
- *Eigenvalue analyses* are conducted to learn about the power system's inherent dynamic characteristics when incrementally disturbed from its initial state. A linearized formulation $d\Delta\mathbf{z}/dt=\mathbf{A}\cdot\Delta\mathbf{z}$ is established. Self and mutual elements of matrix **A** are developed on general algorithmic form for main types of power system components. For further on such analyses, see [9].
- *Time response analyses* implies solving the model numerically over some given time horizon. To account for the fact that electrical circuit models themselves may be functions of state variables, the stated 3-stage task of power network modeling must be repeated sufficiently often during processes of numerical integration.

To illustrate, a tiny power system is modeled and exposed to a temporary three phase short circuit. See Section 6.

Unbalanced conditions can be studied as well. Section 2 of Part 2 exemplifies dealing with 2 cases; forced opening of one of three phases of a power transmission, and line-to-line short circuit.

For further on e.g. start/loading up/ disconnection of rotating machines, and islanding, see [9].

3. Component modeling

A stock of *component sub models* have been established for modeling of the common power network components like overhead lines, cables, «the (remote) infinite bus», capacitor banks, transformers, synchronous machines and asynchronous machines. Table I illustrates how *component sub models* may add up to model main power system components.

The component sub model that is a common network building block, is *the electrical circuit model,* the formal description of which is given in Fig. 1. *The electrical circuit model* comprises *three* main parts:

TABLE I
Overview Of How *Component Sub models* May Add Up To Model Main Power System Components

Main power system components	**Component sub models**
Inductive series impedance	Electrical circuit model
Inductive impedance load	Electrical circuit model
The infinite bus	Electrical circuit model
Capacitor bank	Electrical circuit model Capacitor voltage model
Overhead line / Cable	Electrical circuit model Capacitor voltage models
Transformer	Electrical circuit model(s) (Capacitor voltage models)
Synchronous machine – «Ordinary» version – Adjustable speed version	Electrical circuit model Machine flux model*) Electromechanical model*) Control system models *)
Asynchronous machine – Singly-fed (ie. «ordinary») version – Doubly-fed version	Electrical circuit model Machine flux model *) Electromechanical model*)

*) Component submodels associated with «local» state variables

- *An oriented terminal graph,* showing circuit model structure and positive direction of the circuit model variables (**i**,**e**) that connect electrically with the external network [7]. For a 2-terminal circuit model the oriented terminal graph becomes an oriented line segment. See Fig. 1a.
- *Impedance terms $\boldsymbol{R}$ and $\boldsymbol{X}_L$*, describing the power network related «passive» electrical properties of the circuit model. Subscript 'L' denotes inductive character of the reactance. **v** is the voltage across the serial interconnection of **R** and $\mathbf{X}_L$. Fig. 1b.
- *A voltage source* ***e***, giving the power network related source impact of the *electrical circuit model*. In the context of Table I, a few introductory comments on the interpretations of **e** are given next: In applying the electrical circuit model for network-wise representing;
  - *an inductive series impedance* or *an inductive impedance load,* **e** is zero.
  - *an infinite voltage «behind» some series impedance*, **e** is a fixed phasor.

- *a lossy capacitor bank*, **e** is the voltage across the ideal capacitor. The active losses are accounted for by **R**, while the model term $\mathbf{X}_L$ per definition is zero.
- *a synchronous machine*, **e** is a formal electromotive force (emf.) contributing to modeling of the machine.
- *an asynchronous machine*, **e** is a formal emf. contributing to modeling of the machine.
- **u** is the voltage across the terminals of *the electrical circuit model*, see Fig. 1b.

Example *electrical circuit models* for network-wise description of various power system components, are given in Section 1 of Part 2. The main steps of model development are also covered.

In addition other component sub models may be required for the full description of a given power system component. See Table I. Section 1 of Part 2 also deals with such sub models to the extent they are implicated.

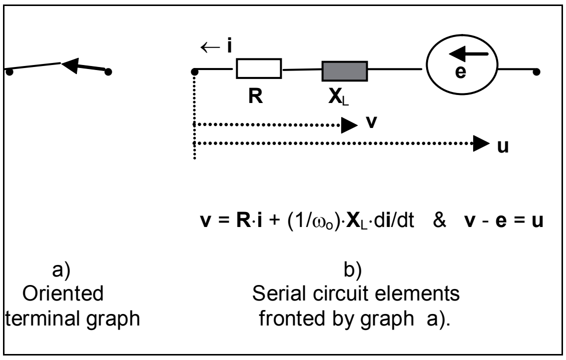


Fig. 1 The electrical circuit model; formal structure of sub model that is made common to all power network components.

4. Network Modeling

The system model can be viewed as the aggregate of *two* system sub models, namely 1) the set of differential equations describing the power network state variables, and 2) the set of such equations describing the remaining state variables.

The ensuing outline focuses on the development of the first system sub model. The second such model comprises differential equations that algorithmically are based on component-specific information only. Comments on the second system sub model is left to Section 1 of Part 2, where component modeling is further dealt with.

The algorithmic basis for modeling of the behaviour of the power network state variables – which are the chosen loop currents ($\mathbf{i}_{loop}$) and the capacitor voltages ($\mathbf{e}_{tc}$), – is next summarized via entries under *four* headings:

A. «Line-up» and «fill-in» of the «primitive network»

From the stock of component sub models – see Section 1 of Part 2 for illustrations – the proper *electrical circuit models* each comprising terms ($\mathbf{R}$, $\mathbf{X}_L$, $\Delta\mathbf{E}$) plus an oriented graph element, are fetched and «lined up» to form the *primitive network:*

- The collection of graph elements describes the topology of the primitive network.
- For description of terms ($\mathbf{R}$, $\mathbf{X}_L$) the aggregate of terms **R** are organized into a diagonal resistance matrix termed $\mathbf{R}_{primitive}$. Similarly, the aggregate of inductive reactances $\mathbf{X}_L$ are arranged into a reactance matrix $\mathbf{X}_{Lprimitive}$ that often also is diagonal. Off-diagonal elements may occur here when there is significant electromagnetic coupling between power system components of adjacent circuits.
- For description of the effect of sources, the aggregate of voltage source terms $\Delta\mathbf{E}$ are arranged into a voltage source vector $\mathbf{e}_{primitive}$.

Based on estimated/current value of all state variables, the content of all the model terms are computed and «filled in» to produce current description of *the primitive network*.

Thus *the primitive network* is the place for updating of model terms due to e.g. saturation effects, or model terms' derived functional dependencies of component variables.

B. Description of model network loop currents

The oriented graph of the model network is established by connecting together the oriented graph elements of the primitive system, as implied by the single line diagram of the power network.

The formal description of the interconnection of oriented graph elements is now afforded by the *network loop matrix **B***, which here is defined on the basis of a chosen *tree* and *co-tree* of the network graph. Thus **B** describes the incidence of independent graph loops – as defined by the set of *co-tree elements* (or *chords*) – and the set of all graph elements of the connected graph. The labels attached consecutively to the co-tree elements can conveniently identify also the set of independent

network loop currents $\mathbf{i}_{loop}$. Furthermore, the orientation of the co-tree elements can suitably define positive direction of the loop currents.
$\mathbf{B}$ can be partitioned into a sub matrix $\mathbf{B}_{cotree}$ that describes the incidence of *loops* and *co-tree elements*, and submatrix $\mathbf{B}_{tree}$ that gives the incidence of *loops* and *tree elements*. Given the conventions above, $\mathbf{B}_{cotree}$ will always be a unit matrix. For illustration, see Fig. 4 and associated text.

In present compact notation where unit entities are the terms ($\mathbf{R}$, $\mathbf{X}_L$, $\Delta\mathbf{E}$) of the *electrical circuit model*, entries in $\mathbf{B}$ are ($\mathbf{1}$,$\mathbf{-1}$,$\mathbf{0}$). $\mathbf{1}$ is a 2x2 unit matrix and $\mathbf{0}$ is a 2x2 zero matrix.

The network loop currents must fulfil the following set of equations [7], [9]:

$$\mathbf{E}_{loop} = \mathbf{R}_{loop}\cdot\mathbf{i}_{loop} + (1/\omega_o)\cdot\mathbf{X}_{Lloop}\cdot d\mathbf{i}_{loop}/dt \qquad (1)$$

where;

$\mathbf{E}_{loop} = -\mathbf{B}\cdot\mathbf{e}_{primitive}$ = driving voltage of resp. loops

$\mathbf{R}_{loop} = \mathbf{B}\cdot\mathbf{R}_{primitive}\cdot\mathbf{B}^t$ = loop resistance matrix.

$\mathbf{B}^t$ is the transpose of $\mathbf{B} = [\mathbf{B}_{cotree}, \mathbf{B}_{tree}]$ (2)

$\mathbf{X}_{Lloop} = \mathbf{B}\cdot\mathbf{X}_{Lprimitive}\cdot\mathbf{B}^t$ = loop inductance matrix

C. Description of model network capacitor voltages

As summarized in Table I, a capacitor bank of the power network is to be modeled by a set of *two* component sub models:

– Circuit-wise, the lossy capacitor is accounted for in equations (1) by its *electrical circuit model* with generic terms ($\mathbf{R}_C$, $\mathbf{X}_L$=0, $\Delta\mathbf{E}_C$). See Section 1 of Part 2, where sub model development is dealt with. The state variables $\Delta\mathbf{E}_C = [\Delta E_{Cd}, \Delta E_{Cq}]^t$ account for the voltage across the *ideal* capacitor involved. The set of all such capacitor voltages is denoted $\mathbf{e}_{tc}$. To bring forth $\mathbf{e}_{tc}$ in (1), the driving voltage vector $\mathbf{E}_{loop}$ should be further developed. To this end it is here presumed that all graph elements that represent capacitors, are contained in the chosen *tree* of the network graph. Sub matrix $\mathbf{B}_{tree}$ is then expressed in terms of 2 sub matrices: $\mathbf{B}_{tree} = [\mathbf{B}_{tc}, \mathbf{B}_{t\text{-}rest}]$, where $\mathbf{B}_{tc}$ describe the incidence of loops and tree elements that symbolize capacitors, and $\mathbf{B}_{t\text{-}rest}$ the incidence of loops and the «rest» of the tree elements. The voltage source vector $\mathbf{e}_{primitive}$ is correspondingly partitioned as follows: $\mathbf{e}_{primitive} = [\mathbf{e}_{cotree}, \mathbf{e}_{tc}, \mathbf{e}_{t\text{-}rest}]$.

Introducing the above definitions into $\mathbf{E}_{loop}$, (1) takes on the form given in (3).

$$d\mathbf{i}_{loop}/dt = \omega_o\cdot\mathbf{X}_{Lloop}^{-1}\cdot[\,-\mathbf{R}_{loop}\cdot\mathbf{i}_{loop} - \mathbf{B}_{tc}\cdot\mathbf{e}_{tc} - \mathbf{e}_{cotree} - \mathbf{B}_{t\text{-}rest}\cdot\mathbf{e}_{t\text{-}rest}\,] \qquad (3)$$

– The second component submodel is the capacitor voltage model. See Figure 3 of Part 2. Equation (16) there, should be extended to deal with all the capacitor voltages $\mathbf{e}_{tc}$. To this end the corresponding set of capacitive reactances $\mathbf{X}_C$ is organized into a diagonal reactance matrix $\mathbf{X}_{Cprimitive}$. In the same way the matrix $\mathbf{1}_C$ of Fig. 3 of Part 2, is repeated into a diagonal matrix $\mathbf{1}_{tc}$ of the same dimension as $\mathbf{X}_{Cprimitive}$. It is also relevant to observe that the subset of (*tree*-element related) capacitor currents $\mathbf{i}_{tc}$ can be expressed by the loop currents: $\mathbf{i}_{tc} = \mathbf{B}_{tc}^{\,t}\cdot\mathbf{i}_{loop}$. Based on the preceding outline the following extension of the just stated eq. (16) provides the basis for modeling of the set of network capacitor voltages:

$$d\mathbf{e}_{tc}/dt = \omega_o\cdot(\,\mathbf{X}_{Cprimitive}\cdot\mathbf{B}^t_{tc}\cdot\mathbf{i}_{loop} + \mathbf{1}_{tc}\cdot\mathbf{e}_{tc}) \qquad (4)$$

D. The system sub model describing the power network state variables

The sought system sub model is found by formulating (3) and (4) as one simultaneous set of equations:

$$\begin{bmatrix} d\mathbf{i}_{loop}/dt \\ d\mathbf{e}_{tc}/dt \end{bmatrix} = \omega_o\cdot\begin{bmatrix} -\mathbf{X}_{Lloop}^{-1}\cdot\mathbf{R}_{loop} & -\mathbf{X}_{Lloop}^{-1}\cdot\mathbf{B}_{tc} \\ \mathbf{X}_{Cprimitive}\cdot\mathbf{B}^t_{tc} & \mathbf{1}_{tc} \end{bmatrix}\cdot\begin{bmatrix} \mathbf{i}_{loop} \\ \mathbf{e}_{tc} \end{bmatrix} - \omega_o\cdot\begin{bmatrix} \mathbf{X}_{Lloop}^{-1} \\ \mathbf{0} \end{bmatrix}\cdot\mathbf{e}_{cotree} - \omega_o\cdot\begin{bmatrix} \mathbf{X}_{Lloop}^{-1}\cdot\mathbf{B}_{t\text{-}rest} \\ \mathbf{0} \end{bmatrix}\cdot\mathbf{e}_{t\text{-}rest} \qquad (5)$$

In re-computing the right hand side of (5) during *initial condition analysis* or *integration*, *two* main numerical processes are involved; fill-in of network model terms into *the primitive network,* based on current value of the state variables, and matrix operations as formally directed by (2) and (5). For efficient computation the prospects of smart matrix generation and parallel processing should be thoroughly exploited.

5. Initial Condition Analysis

Whether eigenvalue- or time dynamical analysis is to be conducted next, an appropriate initial state has to be defined for the system. The initial value of machine variables then has to be set or computed in accordance with the specified situation at hand. Illustrations:

- If *a synchronous machine (SM)* is to be started, its initial per unit (pu.) speed $\Omega_{SM(o)}$, currents and fluxes are set to zero – and conveniently also the electrical machine angle $\beta_{SM(o)}$. The SM's excitation system will have a pré-set «agenda», and initially the excitation voltage $E_{f(o)}$ may also be zero, if the field circuit is kept short-circuited during the first phase of start-up.
- If *a synchronous machine* initially is running at synchronous speed $\Omega_{SM(o)}=1$, it is in present detailed electrical modeling context, natural to specify initial conditions in terms of absorbed power $P_{SM(o)}$ (assuming *motor* operation as the default mode of operation) and voltage $U_{SM(o)}$ at the machine terminals. The initial values ($P_{SM(o)}$, $U_{SM(o)}$) are in principle to be decided on a preceding economy level analysis. Then $\beta_{SM(o)}$ and $E_{f(o)}$ should be determined so as to contribute to fulfilling the specified values $P_{SM(o)}$ and $U_{SM(o)}$. Computationally, this is afforded by an iterative solution process in which $\beta_{SM(o)}$ and $E_{f(o)}$ are simultaneously corrected (together with other such «control variables») until stated initial conditions are reached to required accuracy. Absorbed (or produced) *reactive* power is then in principle a by-product from this solution process.
- If *an asynchronous machine (AM)* is to be started, its pu speed $\Omega_{AM(o)} = 0$, and so also all machine currents and flux variables.
- If *an asynchronous machine* is initially in a steady state mode of operation, it may be appropriate to specify initial conditions in terms of absorbed motor power $P_{AM(o)}$. Thus $\Omega_{AM(o)}$ should be specified so as to fulfil this requirement. Computationally, this is afforded by including $\Omega_{AM(o)}$ as one of the simultaneously corrected «control variables» of the above sketched iterative solution process. Absorbed *reactive* motor power will again flow as a by-product.

With final or tentative setting of resp. «control variables» ($\boldsymbol{\beta}_{SM(o)}$, $\mathbf{E}_{f(o)}$, $\Omega_{SM(o)}$, $\Omega_{AM(o)}$), the premises are given for computing initial value of the remaining pertinent power system variables $\mathbf{z}_{(o)}$. Vector $\mathbf{z}_{(o)}$ comprises in present context the network loop currents $\mathbf{i}_{loop(o)}$, the capacitor voltages $\mathbf{e}_{tc(o)}$, the synchronous machine fluxes $\phi_{SM(o)}$, and the asynchronous machine fluxes $\phi_{AM(o)}$.

$\mathbf{z}_{(o)}$ is found by simultaneously solving the network model (5) and the involved sets of synchronous- and asynchronous machine flux models. The latter models are exemplified in Fig. 6, resp. Fig. 11 of Part 2. After placing the models together and setting the derivative terms to zero, the set of equations to describe initial steady state conditions, may in compact notation appear as follows:

$$\mathbf{H}_{syst(o)} \cdot \mathbf{z}_{(o)} = \mathbf{G}_{syst(o)} \tag{6}$$

In essence, the load flow computation task can be exemplified as follows: With a set of target values

$$(\mathbf{P}_{SMtarget(o)}, \mathbf{U}_{SMtarget(o)}, \mathbf{P}_{AMtarget(o)}) \tag{7}$$

and a corresponding set of «control variables»

$$(\boldsymbol{\beta}_{SM(o)}, \mathbf{E}_{f(o)}, \boldsymbol{\Omega}_{AM(o)}), \tag{8}$$

determine a configuration of the latter variables that – when applied to the above process equations (6) – produces an electrically valid solution that observes the specified load flow premises.

The iterative solution process comprises 3 main steps:

1) Set/stipulate the «control variables» (8).
2) Solve (6) with respect to $\mathbf{z}_{(o)}$, compute the consequences in terms of variables (7), and register current deviations $\mathbf{D}$ from target values. If the deviations are acceptable, the initial balance has been established. If not acceptable, go to step 3).
3) Adjust the «control variables» (8) incrementally, so that an improved initial power flow balance is attained. Then return to step 2).
4) To evaluate proper simultaneous corrections to the «control variables» a sensitivity analysis is conducted to find current value of the elements of the sensitivity matrix $\mathbf{S}$ of the defined relationship (9):

$$\begin{bmatrix} \Delta\mathbf{P}_{SM} \\ \Delta\mathbf{U}_{SM} \\ \Delta\mathbf{P}_{AM} \end{bmatrix} = \begin{bmatrix} \mathbf{S} \end{bmatrix} \cdot \begin{bmatrix} \Delta\boldsymbol{\beta}_{SM} \\ \Delta\mathbf{E}_{f} \\ \Delta\boldsymbol{\Omega}_{AM} \end{bmatrix} \tag{9}$$

If $\mathbf{S}$ is of dimension (mxm), then m intermediate sensitivity analyses are required to define the content of **S**: By increasing the ith «control variable» marginally while the rest are kept unaltered at current «base» value, and solving (6), the numerical value associated with the elements of column 'i' of **S** can readily be determined.

With established sensitivity matrix **S** and prevailing deviations $\Delta\mathbf{D}$ relative to target values, (9) is next applied to estimate the set of increments that will contribute to eliminating the unwanted deviations: Using $-\Delta\mathbf{D}$ as «excitation» on the left side in (9), and solving wrt. the desired simultaneous increments, the corrections for updating the prevailing set of «control variables» are made available. Following the update of these variables, return is made to step 2).

6. Illustration Of Methodology

The methodology applied to system modeling is summarized by way of a small illustration: Given the task of modeling the detailed electrical performance of the tiny power system of Fig. 2.

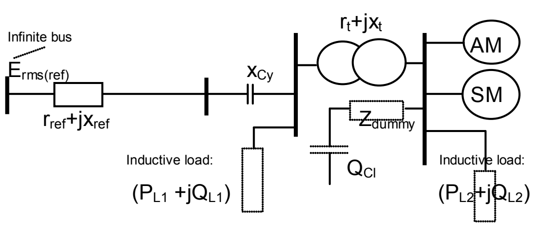


Figure 2 Tiny example power system

A. Component data

The system comprises *nine* power system components, the power network related description of which are exemplified on a common MVA base. Figures and equations referred to next as part of component description, relate to Part 2 of the paper:

– Asynchronous motor ('1'):

Data: $X_{a\sigma} = 0.08$ $X_{r\sigma} = 0.08$ $X_m = 2.5$
$r_a = 0.03$ $r_r = 0.03$ $\kappa = 2.0$ $T_a = 4s$
(For definitions, see Fig.10,12)

Electrical circuit model: ($\mathbf{R}_{AM}$, $\mathbf{X}_{AM}$, $\Delta\mathbf{E}_{AM}$)
See Fig. 10

– Synchronous motor ('2'):

Data: $X_{a\sigma} = 0.12$ $X'_d = 0.34$ $r_a = 0.005$
$X_d = 1.20$ $X''_d = 0.20$ $T'_{do} = 6.0s$
$X_q = 0.75$ $X''_q = 0.30$ $T''_d = 0.04s$
$T''_q = 0.16s$ $T_a = 5.0s$ $\cos\varphi_N = 0.9$
(For definitions, see equations (39))

Electrical circuit model: ($\mathbf{R}_{SM}$, $\mathbf{X}_{SM}$, $\Delta\mathbf{E}_{SM}$) *Fig. 8*

– Impedance type inductive load (P_{L1}+ jQ_{L1}) ('3'):

Data: $P_{L1} = 0.60$ $Q_{L1} = 0.20$ at 1.0 voltage
$\supset$: $r_{L1} = 1.5$ $x_{L1} = 0.5$ (ind.)

Electrical circuit model: ($\mathbf{R}_{L1}$, $\mathbf{X}_{L1}$, **0**) *Fig. 1*

– Impedance type inductive load (P_{L2}+ jQ_{L2}) ('4'):

Data: $P_{L2} = 0.25$ $Q_{L2} = 0.80$ at 1.0 voltage
$\supset$: $r_{L2} = 0.3559$ $x_{L2} = 1.1388$ (ind.)

Electrical circuit model: ($\mathbf{R}_{L2}$, $\mathbf{X}_{L2}$, **0**) *Fig. 1*

– Dummy connection – to enhance definition of loops ('5'):

Data: $r_{dummy} = 0.01$ $x_{dummy} = 0.005$
(See last few sentences of Part 1)

Electrical circuit model: ($\mathbf{R}_{dummy}$, $\mathbf{X}_{dummy}$, **0**) *Fig. 1*

– Shunt capacitor bank (P_{Cl} - jQ_{Cl}) ('6'):

Data: $P_{Cl} = 0.0$ $Q_{Cl} = 0.70$ at 1.0 voltage
$\supset$: $r_{Cl} = 0.0$ $x_{Cl} = 1.4286$ (cap.)

Electrical circuit model: ($\mathbf{R}_{Cl}$, **0**, $\Delta\mathbf{E}_{Cl}$) *Fig. 2*
Capacitor voltage model: ($\mathbf{X}_{Cl}$) *Fig. 3*

– Series capacitor bank (r_{Cy} - jx_{Cy}) ('7'):

Data: $r_{Cy} = 0.0$ $x_{Cy} = 0.025$ (cap.)

Electrical circuit model: ($\mathbf{R}_{Cy}$, **0**, $\Delta\mathbf{E}_{Cy}$) *Fig. 2*
Capacitor voltage model: ($\mathbf{X}_{Cy}$) *Fig. 3*

– Transformer ('8'):

Data: $r_t = 0.01$ $x_t = 0.07$

Electrical circuit model: ($\mathbf{R}_t$, $\mathbf{X}_t$, **0**) *Fig. 1*

– Series impedance (r_{ref} + jx_{ref}) & infinite bus $E_{rms(ref)}$ ('9'):

Data: $r_{ref} = 0.03$ $x_{ref} = 0.125$ $E_{rms(ref)} = 1.05$ $\gamma_{ref} = 0$

El.circuit model:($\mathbf{R}_{ref}$, $\mathbf{X}_{ref}$, $\mathbf{e}_{DQ(ref)}$).
Fig.1, (20), (21)

B. The primitive network

The primitive network for any considered point in time, is the chosen suitable line-up of *the electrical* *circuit models* of the network components, valid at that point in time. With the chosen model sequence above as the key for line-up, the primitive network for the model system of the power network of Fig. 2, can take on the form shown in Fig. 3.

C. Model network topology

The oriented network graph of the example power system is shown in Fig. 4a. It is formed by connecting the primitive network graph elements of Fig. 3 as advised by the single line diagram of Fig. 2. Which direction is chosen as positive for the variables (**i**,**e**) of respective electrical circuit models, is in principle arbitrary.

The topological info of Fig. 4a is formally described by the network loop matrix $\mathbf{B} = [\mathbf{B}_{cotree}, \mathbf{B}_{tc}, \mathbf{B}_{t\text{-}rest}]$, which is suitably partitioned into the sub matrices $\mathbf{B}_{cotree}$, $\mathbf{B}_{tc}$, $\mathbf{B}_{t\text{-}rest}$.

D. The model network sub model

The model network sub model (5) includes two interlinked sets of differential equations describing the behaviour of *the power network state variables*; the defined *loop currents* $\boldsymbol{i}_{loop}$ and the *capacitor voltages* $\boldsymbol{e}_{tc}$.

The equations for $\mathbf{i}_{loop}$ are established by applying the loop matrix **B** to the primitive system, as advised by equations (1) to (3).

The equations for $\mathbf{e}_{tc}$ are produced by applying the capacitive reactance matrix $\mathbf{X}_{Cprimitiv}$ and the sub matrices $\mathbf{B}_{tc}$ and $\mathbf{e}_{tc}$, as specified by equations (4). As outlined in Section 4 under sub heading C, $\mathbf{X}_{Cprimitiv}$ is the line-up on diagonal form of the capacitive 2x2 diagonal reactances $\mathbf{X}_C$. In the present example as described by (10).

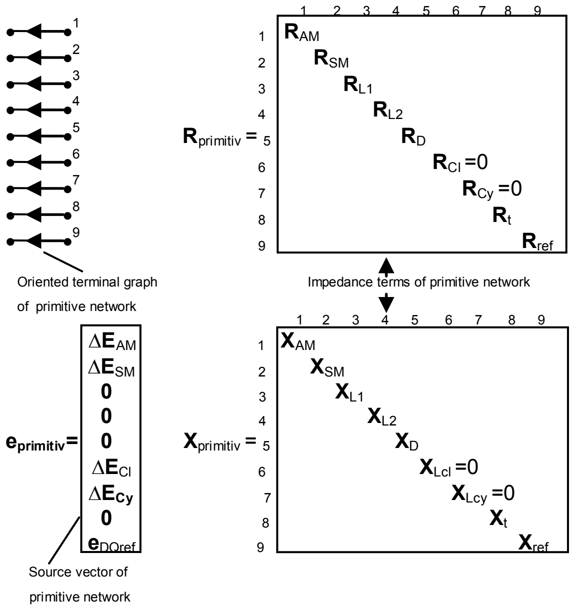
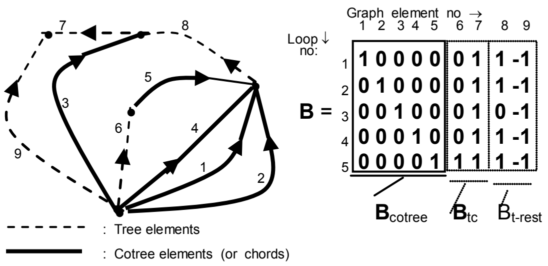


Figure 3 The primitive network of the system of Figure 2



a) Oriented network graph b) Network loop matrix **B**

Figure 4 Network loop matrix **B** for topological description (See Section 4/B for further details on **B**)

$$\mathbf{X}_{Cprimitiv} = \begin{bmatrix} \mathbf{X}_{Cl} & \\ & \mathbf{X}_{Cy} \end{bmatrix} \qquad (10)$$

E. The system model

The size and substance of respective sub models that together constitute the *system model* of the study case, are summarised as follows:

– The (5x2+2x2=14) differential equations (5) describe the performance of the *model network* state variables $\mathbf{i}_{loop}$ and $\boldsymbol{e}_{tc}$. The remaining primitive system element currents $\mathbf{i}_{tree}$ are given as linear combinations of the state variables $\mathbf{i}_{loop}$; $\mathbf{i}_{tree} = \mathbf{B}^t_{tree} \cdot \mathbf{i}_{loop}$. See (2) for fitting partitioning of **B**.

– The (2+1=3) differential equations (11) and (12) describe *«local» asynchronous motor* state variables; namely the motor's flux linkages

$\mathbf{\Phi}_{AM}$, and the pu. speed $\mathbf{\Omega}_{AM}$ of its rotating part.

$$d\mathbf{\Phi}_{AM}/dt = \omega_o \cdot (\mathbf{F}_{AMi} \cdot \mathbf{i}_{AM} + \mathbf{F}_{AM\varphi} \cdot \mathbf{\Phi}_{AM}) \qquad (11)$$

$$d\Omega_{AM}/dt = J_{AM} \cdot (T_{AMel} - T_{AMmec}) \qquad (12)$$

Equations (11) are copied from (68) of Fig.11 of Part 2. Fig.11 summarizes the *flux model* of the AM. (12) is similarly a copy of (71) from the *electromechanical model* of the AM. See Fig. 12 of Part 2.

– The (3+1+1=5) differential equations (13), (14), (15) describe *«local» synchronous motor* state variables; i.e. the motor's flux linkages $\mathbf{\Phi}_{SM}$, the pu speed Ω_{SM} of its rotating part, and the electrical angle β_{SM} associated with the SM. See (33) together with its related text in Part 2, for closer comments on β_{SM}.

$$d\mathbf{\Phi}_{SM}/dt = \omega_o \cdot (\mathbf{e}_{SM} + \mathbf{F}_{SMi} \cdot \mathbf{i}_{SM} + \mathbf{F}_{SM\varphi} \cdot \mathbf{\Phi}_{SM}) \qquad (13)$$

$$d\Omega_{SM}/dt = J_{SM} \cdot (T_{SMel} - T_{SMmec}) \qquad (14)$$

$$d\beta_{SM}/dt = \omega_o \cdot (1 - \Omega_{SM}) \qquad (15)$$

Equations (13) are copied from (35) of Fig. 6 of Part 2, which describes the *flux model* of the SM. (14) and (15) are copies of respectively (56) and (57), from the *electromechanical model* of the SM. See Fig. 9 of Part 2.

– The n_{AVR} differential equations (16) describe the SM voltage control state variables $\Delta\mathbf{E}_{SM(AVR)}$, which belong to the group of *local* state variables. For details, see [9].

$$d\Delta\mathbf{E}_{SM(AVR)}/dt = f(\Delta\mathbf{E}_{SM(AVR)}, \Delta U_{SM(ref)}, \Delta U_{SM}, \Delta\Omega_{SM}) \qquad (16)$$

The incremental field voltage response ΔE_f found in (36) as well as (50), see Part 2,– go into vector $\Delta\mathbf{E}_{SM(AVR)}$. $\Delta U_{SM(ref)}$ is the change (if any) of the voltage reference, and ΔU_{SM} resp. $\Delta\Omega_{SM}$, is deviation from target value of the controlled voltage, resp. angular speed. In the study $n_{AVR} = 4$. Since it is here considered outside the scope of presentation to delve into sub models that yield *control* responses, it is referred to [9] for further details on (16) and (17).

– The n_{LFC} =3 differential equations (17) model the local SM power control state variables $\Delta\mathbf{W}_{SM(LFC)}$, presuming a *hydro generator unit* at hand.. ΔS_{ACE} is the applied *area control error signal*.

$$d\Delta\mathbf{W}_{SM(LFC)}/dt = g(\Delta\mathbf{W}_{SM(LFC)}, \Delta S_{ACE}) \qquad (17)$$

All together, the system model applies (14+3+5+4+3)=29 state variables to describe the dynamical performance of the system of Fig. 2.

F. Initial state analysis

The following operational status is specified for the two rotating machines of Fig. 2:

– Power supplied to the *asynchronous motor*:
  $P_{AMt(o)} = 0.5$
– Power supplied to the *synchronous motor*:
  $P_{SMt(o)} = -0.8$
– Voltage at the *synchronous motor* bus:
  $U_{SM(o)} = 1.0$

With the initial load flow specified in operational terms, the iterative solution process outlined in Section 5 is called upon for targeting the implied electrical state to required accuracy:

The *«load flow control variables»* are ($\beta_{SM(o)}$, $E_{f(o)}$, $\Omega_{AM(o)}$) in the present case. Starting values are arbitrarily set to (0,1.5, 1). End values (-0.38016rad, 1.79243pu, 0.98387pu) that observe the accuracy constraint, are reached after 6 iterations.

Main characteristics of the established initial load flow:

Load bus I:

Voltage	: 1.005pu	
Active load '1'	: 0.606pu	(impedance type)
Reactive load '1'	: 0.202pu	(inductive character)

SM/AM bus:

Voltage	: 1.000pu	(specified)
Active SM power	: -0.800pu	(specified)
Re- " " "	: -0.441pu	(SM acts as capacitor)
Active AM power	: 0.499pu	(specified value: 0.5pu)
Re- " " "	: 0.417pu	(AM acts as inductor)
AM slip	: 1.613%	
Capacitor bank	: -0.700pu	(impedance type)
Active load '2'	: 0.250pu	(impedance type)
Reactive load '2'	: 0.800pu	(inductive character)

G. On presentation of main variables in power system dynamic analyses

During computation processes all currents and voltages are by default instantaneous variables. They comprise d–q axis variables and to the extent involved – zero sequence variables.

For suitable presentation of results relating to 3-phase circuits, currents and voltages are transformed back into their 3-phase (RST) variables via (3) of Part 2:

- In case of analysis of *unbalanced* network conditions, instantaneous traces of current and voltage of individual phases may be of prime interest. Corresponding r.m.s. traces are readily generated from the instantaneous records. See Section 2 of Part 2.
- In case of analysis of *balanced* conditions, it may be considered appropriate to compute current and voltage associated with only *one* of the phases. Moreover, instead of registering instantaneous traces, it may then be more relevant to keep track of the *r.m.s*. records of per phase current and voltage. Such records are readily computed from (18).

$$I_{rms} = [\,½\cdot(\,i_d^{\,2} + i_q^{\,2}\,)]^{0.5} \qquad (18)$$
$$U_{rms} = [\,½\cdot(\,u_d^{\,2} + u_q^{\,2}\,)]^{0.5}$$

Power network currents and voltages presented below in the case of a three phase short circuit, are rms. values from (18).

Other diagram variables such as e.g. absorbed motor power, motor speed, electrical torque, field- and damper currents, and synchronous motor angle, are instantaneous variables that may attain positive as well as negative values.

In the included diagrams a variable is described in terms of its time response curve, plus *three* numbers; its *initial* value, its *maximum* value within the time range analyzed, and its correspondingly defined *minimum* value.

H. Three phase short circuit

Referring to Fig. 2 a three phase short circuit of duration 0.25s, is implemented by temporarily replacing impedance load ’L2’ by a short circuit impedance (0.001+j0). The short circuit is applied at t=0.05s and removed at t = 0.30s. Integration time step: 0.0005s.

The study is repeated for two analysis duration times t_{max} to illustrate how results may appear different due to a given logic of result presentation: Figures 5–13 give sample results for t_{max} = 0.5s, while Figures 14–19 are repeat presentations of Figures 5–10 for t_{max} =3s. (Within the time interval (up to 0.5s) that is common to Figures 5–13 and 14–19, characteristic values (like max. and min.) of any variable may or may not be registered the same for both durations of analysis: Regardless of t_{max}, 1000 discrete values of each variable are retained for drawing etc., causing an increasing no of «intermediate» variables to be omitted with increasing t_{max}.)

As the fault here is implemented via setting of new parameters for load ’L2’, the short circuit current appears as the current supplied to load ’L2’, see Fig. 12.

The asynchronous motor contributes to the fault current with some «peak supply» capability. See Fig. 10. Short-circuiting the shunt battery connected to the motor bus, implies a current pulse that will also contribute to increase the peak of the short circuit current. This is evidenced from Fig. 13. To limit this current pulse, the dummy series impedance has been set to (0.01+j0.005).

7. Conclusions

Based on describing all power system components in terms of discrete elements, the paper outlines a compact methodology for *Power System Dynamic Analysis*. The merit of a suitable scheme for such analyses, depends strongly on two inherent features of the scheme; its *intelligibility,* resp. its *practicability*.

A. Intelligibility

Main focus of the paper has been on the intelligibility aspect. The paper’s chief features in this respect is next summarized under two sub-headings:

1. Power system component modeling

Central to the methodology/intelligibility is the development of a stock of *sub models* for modeling of power system components. Table I gives an overview of how such *sub models* may add up to model main power system components:

The *sub model* termed *electrical circuit model* is formally a *two-terminal serial impedance,* comprising component -specific terms $\boldsymbol{R},\boldsymbol{X}_L,\boldsymbol{E}$. This impedance acts as the common network building block for all power network components. Formulation- and solution-wise, problem complexity becomes thereby largely confined to *component* level rather than overall *system* level.

An illustration on how *sub models* may interplay to model a power system component: A capacitor bank will require a set of two *sub models*; the stated *electrical circuit model* which accounts for the lossy capacitor bank in the power network equations, and the *capacitor voltage model* describing the «inner life» of the ideal capacitor emf. $\boldsymbol{E}=\boldsymbol{E}_c$ of the first stated *sub model*.

In addition to furnishing the above *sub models* for modeling of the *interconnected electrical power network*, another stock of *sub models* are required for modeling of associated «local» variables such as eg. fluxes and angular speed of rotating machines, electrical angle of synchronous machines, and variables associated with involved control systems. Collate Table I.

2. Power system modeling

The system model can suitably be viewed as the aggregate of two *system sub models*, namely a) the set of equations describing *the power network state variables*, and b) the set of equations describing *the remaining or «local» state variables*:

a) Modeling of the power network state variables

Capacitor voltages together with the (here) defined power network loop currents form the *power network state variables*.

The modeling of these variables can fittingly be organized into three main steps;

– Based on estimated/current value of all state variables at considered point in time; fetch from stock the proper set of *electrical circuit models* associated with the power network at hand, update and line up their elements $\boldsymbol{R},\boldsymbol{X}_L,\boldsymbol{E}$ into (what Gabriel Kron denoted) *the primitive network* [6]. Repeated updating of elements of *the primitive system* is required to handle saturation effects, and model element's derived functional dependencies of own component state variables.
– Describe how the electrical circuit models of the power network are to be tied together, eg. by a loop incidence matrix, or a node related incidence matrix. The present report applies a loop incidence approach.
– Produce the current network model (5) via smart matrix operations related to *the primitive network* and the incidence matrix.

b) Modeling of the remaining or «local» state variables

In modeling of a «local» state variable no foreign, but only one or more of the power system component's own variables appear explicit in the equation(s) that describe the considered state variable.

Equations that model «local»state variables can thus be formulated independent of the network related task at hand. See Table I plus touching foot note.

B. Practicability

The practicability aspect is here commented on chiefly from an overview systems analysis point of view, – for the most part due to limited access to proper facilities allowing for large scale parallel processing.

1. On main status of the day

In the up to date literature on practical schemes for power system dynamic analysis, it seems that most often a nodal admittance formulation strategy is applied to describe how power network components interact in operation. Inherently, for any given point in time – this modeling approach seems to imply simultaneous solution of a large set of *ordinary differential* equations plus a large set of *sparse algebraic* («load flow type») equations. The following overview characterization would seem apt to make regarding this power network modeling approach:

It applies a «loose-grip» (ie.nodal admittance based) strategy to iteratively converge upon the proper network flow situation at the given point in time. A such strategy brings inherently in an additional «cost» in terms of eg. increased number of iterations. In this case however, the ease and speed with which the network equations can be generated, modified and applied, seem to far outweigh any detrimental consideration.

2. On the presented methodology

In the present paper a loop current approach is applied to describe how power system components interact in operation. The complete system model may then getting close to being alone a large set of *ordinary differential* equations. A compact overview characterization of this modeling approach, could likely take on this form:

It applies a «firm grip» (ie. loop current based) strategy to directly – or with limited inclusion of the element of iterative processing – evaluate the proper network flow situation. A such strategy incurs inherently an additional «cost» in terms of a more cumbersome network modeling task, – as evidenced by equations (2) and (5): The solution process will imply frequent build-up and inversion of system loop sub matrices, the computational burden of which may increase rapidly with increasing size of the power system.

The just stated equations may point to a prospect of sizable time saving, in accessing computer facilities of capability to simultaneously generate / update the individual contents pertaining to the proper set(s) of sub matrices.

It is envisaged that the use of parallel processing together with tailored mathematical processes exploiting eg. matrix sparsity and diagonality, would contribute to retaining practicability of the proposed scheme of analysis.

To further investigate the latter presumption, the aspect of smart matrix generation together with matching use of parallel processing should be developed and tried out on a large-scale technical level.

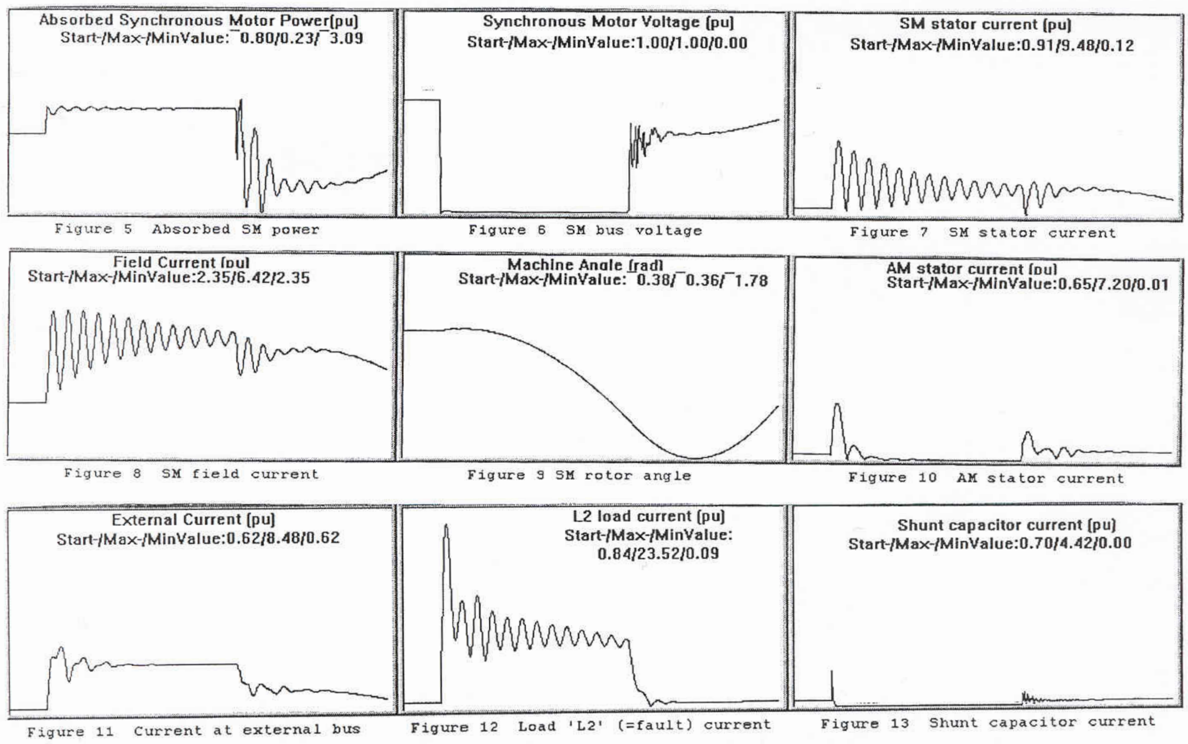


Figure 5-13 **Three phase short circuit** at the machine bus of the system of Figure 2. Sample results for a period of analysis of t_{max} **= 0.5s**. The short circuit is applied at t = 0.05s and removed 0.25s later at t = 0.30s.

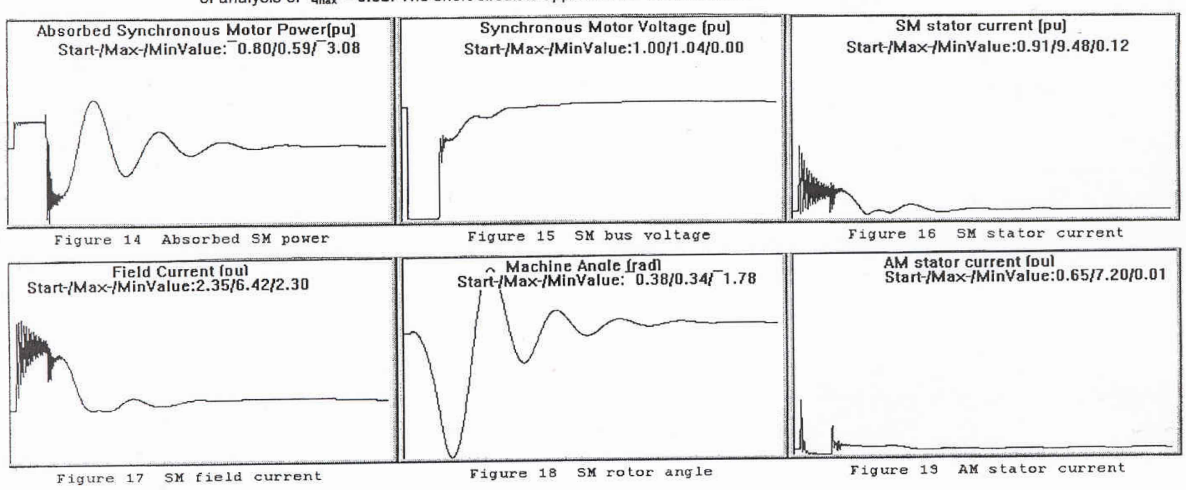


Figure 14-19 **Repeat presentation** of previous 6 Figures 5-10, now with analysis period t_{max} **= 3s**

7

Part 2
Component Modeling

1. Modeling of power system components

This Section focuses on the *component sub models* required for modeling of the main power system components of Table I of Part 1 of the paper.

Network-wise, any such power system component is to be represented by one (or more) sub model(s) termed *the electrical circuit model(s)*, the structure of which is given in Fig. 1 of Part 1. The task then being to develop relevant power component-specific content of terms (**R**, $\mathbf{X}_L$, **e**) of the *electrical circuit model(s) to apply*.

To the extent other component sub models are required for modeling of a given power system component, this Section will deal also with such sub models.

A. The «Symmetrical Lossy Inductor»

For brevity of presentation the «Symmetrical Lossy Inductor» is introduced to cover both *the Inductive series impedance* and *the Inductive impedance load*.

Transformers, overhead lines and cables are modeled by suitably arranging together electrical circuit models of *the symmetrical lossy inductor* and the corresponding *lossy capacitor bank*. See text following heading 'B' next.

Currents ($\mathbf{i}_{dqo}$), voltages ($\mathbf{v}_{dqo}$) and fluxes (ψ_{dqo}) within the d–q axis frame of reference, may definition-wise be related to their corresponding 3-phase (RST) variables in the following way:

$$\mathbf{i}_{dqo} = \mathbf{P}\cdot\mathbf{i}_{RST} \quad \mathbf{v}_{dqo} = \mathbf{P}\cdot\mathbf{v}_{RST} \quad \mathbf{\Psi}_{dqo} = \mathbf{P}\cdot\mathbf{\Psi}_{RST} \quad (1)$$

P is the Park transformation which here is defined as follows [8], [9]:

$$\begin{array}{cccc} \text{R} & \text{S} & \text{T} & \\ & & & \end{array}$$

$$\mathbf{P} = 2/3 \cdot \begin{array}{c} \begin{array}{ccc} \text{R} & \text{S} & \text{T} \end{array} \\ \begin{bmatrix} \cos\theta & \cos(\theta\text{-}2\pi/3) & \cos(\theta\text{-}4\pi/3) \\ \text{-}\sin\theta & \text{-}\sin(\theta\text{-}2\pi/3) & \text{-}\sin(\theta\text{-}4\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array} \begin{array}{c} \\ d \\ q \\ o \end{array} \quad (2)$$

θ is the angular displacement of the axes of the 3-phase reference frame relative to the axes of the (dq) variable's reference frame.

Presuming the existence of the inverse of **P**, it is observed from the foregoing that;

$$\mathbf{i}_{RST} = \mathbf{P}^{-1}\cdot\mathbf{i}_{dqo} \quad \mathbf{v}_{RST} = \mathbf{P}^{-1}\cdot\mathbf{v}_{dqo} \quad \mathbf{\Psi}_{RST} = \mathbf{P}^{-1}\cdot\mathbf{\Psi}_{dqo} \quad (3)$$

where;

$$P^{-1} = \begin{array}{ccc|c} d & q & 0 & \\ \cos\theta & -\sin\theta & 1 & R \\ \cos(\theta-2\pi/3) & -\sin(\theta-2\pi/3) & 1 & S \\ \cos(\theta-4\pi/3) & -\sin(\theta-4\pi/3) & 1 & T \end{array} \quad (4)$$

In the physical three phase (RST) reference frame, one can for (say) phase 'R', express pu voltage v_R across the considered *lossy inductive impedance* $(r+j\cdot x)$ as;

$$v_R = i_R\cdot r + d\psi_R/dt \quad (5)$$

where i_R and ψ_R is – respectively – current and flux linkages of phase 'R'.

By applying (4) into (3), the per phase variables (v_R,i_R,ψ_R) are replaced by their axis variables, zero sequence variables and θ. Using these replacements in (5) one gets the following version of (5) in terms of (d,q,o) variables plus θ:

$$\begin{aligned} 0 = &[-v_d + r\cdot i_d + d\psi_d/dt - \omega\cdot\psi_q]\cdot\cos\theta \\ &+ [v_q - r\cdot i_q - d\psi_q/dt - \omega\cdot\psi_d]\cdot\sin\theta \\ &+ [-v_o + r\cdot i_o + d\psi_o/dt] \end{aligned} \quad (6)$$

For general validity of (6), the following conditions must hold true;

$$\begin{aligned} v_d &= r\cdot i_d + d\psi_d/dt - \omega\cdot\psi_q \\ v_q &= r\cdot i_q + d\psi_q/dt + \omega\cdot\psi_d \\ v_o &= r\cdot i_o + d\psi_o/dt \end{aligned} \quad (7)$$

In the present context it is assumed that zero sequence circuitry is absent. Then the last equation of (7) can be omitted in the current overview presentation. Presumed component symmetry allows furthermore for definition of the following pu relationships: $\psi_d = L_d\cdot i_d = L\cdot i_d$ and $\psi_q = L_q\cdot i_q = L\cdot i_q$. With $x = \omega_o\cdot L$, where $\omega_o = 2\pi f_o$ = nominal angular speed, and with the synchronous phasor as reference, the two equations (7) may take on this form:

$$\begin{aligned} v_d &= r\cdot i_d - x\cdot i_q + (1/\omega_o)\cdot x\cdot di_d/dt \\ v_q &= x\cdot i_d + r\cdot i_q + (1/\omega_o)\cdot x\cdot di_q/dt \end{aligned} \quad (8)$$

(8) implies the following *electrical circuit model* of the defined *lossy inductor*, hereby indexed 'L'. See Figure 1.

From (8) it is definition-wise understood that vectors $\mathbf{v}_L$ and $\mathbf{i}_L$ are as follows, where superscript 't' stands for «transpose»; $\mathbf{v}_L = [v_{Ld}, v_{Lq}]^t$ & $\mathbf{i}_L = [i_{Ld}, i_{Lq}]^t$.

B. The «Symmetrical Lossy Capacitor Bank»

The Symmetrical Lossy Capacitor Bank models directly the three phase, lossy series capacitor and the ditto lossy shunt capacitor. It also contributes to the modeling of other power network components as pointed to above.

A brief 2-step development is next given of the *two* component sub models required for modeling of the capacitor bank*; the electrical circuit model* and *the capacitor voltage model*:

1) The electrical circuit model

Observing the conventions of Fig.1 of Part 1, one can – in the 3-phase (RST) reference frame – for (say) phase 'R', express the voltage u_R across the considered lossy capacitor as;

$$u_R = i_R\cdot r - \Delta E_R \quad (9)$$

where

$$\Delta E_R = (1/C)\cdot\int i_R\cdot dt \quad (10)$$

i_R and ΔE_R is – respectively – current of phase 'R', and voltage across the ideal capacitor element C of phase 'R'. r and C are the per phase parameters that describe the capacitor bank electrically.

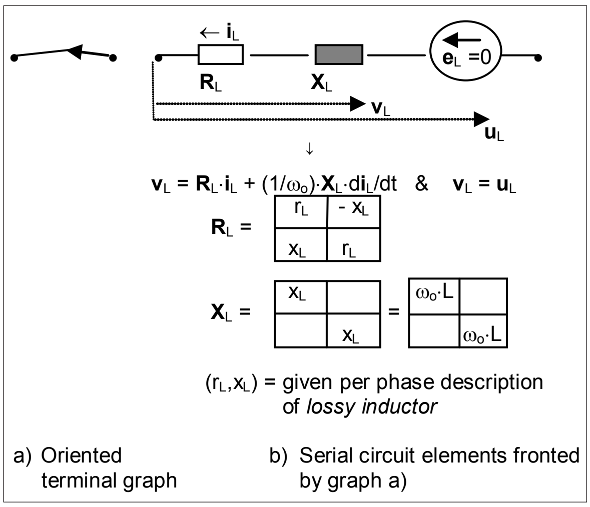


Fig. 1 *The electrical circuit model* of the Symmetrical Lossy Inductor. d–q axis frame of reference.

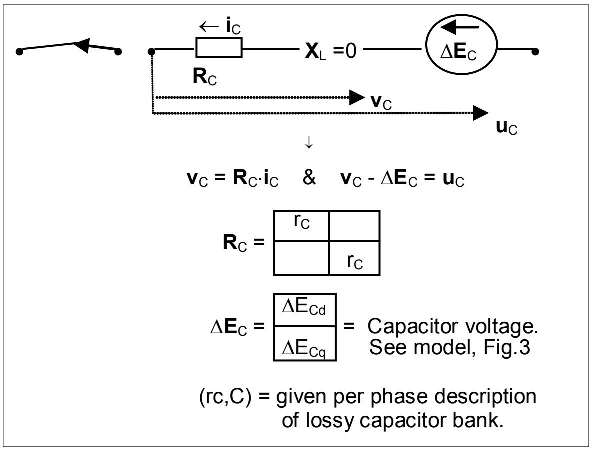


Fig. 2 *The electrical circuit model* of the Symmetrical Lossy Capacitor Bank. d–q axis reference.

The per phase variables u_R, i_R ,and ΔE_R are related to their respective d-q axis components in the following way, see (3) – (4):

$$
\begin{aligned}
u_R &= u_d\cdot\cos\theta - u_q\cdot\sin\theta + u_o \\
i_R &= i_d\cdot\cos\theta - i_q\cdot\sin\theta + i_o \\
\Delta E_R &= \Delta E_d\cdot\cos\theta - \Delta E_q\cdot\sin\theta + \Delta E_o
\end{aligned}
\qquad (11)
$$

Inserting from (11) into (9), one gets the following version of the latter equation;

$$
\begin{aligned}
0 = {} & [\, -u_d + r\cdot i_d - \Delta E_d \,]\cdot\cos\theta \\
& + [\, u_q - r\cdot i_q + \Delta E_q \,]\cdot\sin\theta \\
& + [\, -u_o + r\cdot i_o - \Delta E_o \,]
\end{aligned}
\qquad (12)
$$

For general validity of (12), the following conditions must be observed:

$$
\begin{aligned}
u_d &= r\cdot i_d - \Delta E_d \\
u_q &= r\cdot i_q - \Delta E_q \\
u_o &= r\cdot i_o - \Delta E_o
\end{aligned}
\qquad (13)
$$

Presuming as above that zero sequence circuitry is absent, the remaining two equations of (13) implies the following *electrical circuit model* of the defined *lossy capacitor bank,* hereby indexed $_{'C'}$, see Fig. 2.

From (13) it is definition-wise understood that vectors $\mathbf{v}_C$ and $\mathbf{i}_C$ are as follows ; $\mathbf{v}_C = [v_{Cd}, v_{Cq}]^t$ & $\mathbf{i}_C = [i_{Cd}, i_{Cq}]^t$.

2) The Capacitor Voltage Model

Returning to (10) it is observed that $d\Delta E_R/dt = i_R/C$. Inserting into this equation the expression for i_R from (11), and the expression for $d\Delta E_R/dt$ also derived from (11), one arrives at the following «d–q–o version» of (10):

$$
\begin{aligned}
0 = {} & [\, i_d/C - d\Delta E_d/dt + \omega\cdot\Delta E_q \,]\cdot\cos\theta \\
& + [\, -i_q/C + d\Delta E_q/dt + \omega\cdot\Delta E_d \,]\cdot\sin\theta \\
& + [\, i_o/C - d\Delta E_o/dt \,]
\end{aligned}
\qquad (14)
$$

For general validity of (14), the following conditions must be fulfilled:

$$
\begin{aligned}
d\Delta E_d/dt &= i_d/C + \omega\cdot\Delta E_q \\
d\Delta E_q/dt &= i_q/C - \omega\cdot\Delta E_d \\
d\Delta E_o/dt &= i_o/C
\end{aligned}
\qquad (15)
$$

Again it is presumed that zero sequence currents and voltages are inconsequential. With the synchronous phasor as reference, the two first equations of (15) then implies the *capacitor voltage model* of the symmetrical, lossy capacitor bank, – here indexed $_{'C'}$ –, as shown in Fig. 3.

$$d\Delta\mathbf{E}_C/dt = \omega_o\cdot(\,\mathbf{X}_C\cdot\mathbf{i}_C + \mathbf{1}_C\cdot\Delta\mathbf{E}_C)$$

with initial condition: $\Delta\mathbf{E}_{C(o)} = (\mathbf{1}_C\cdot\mathbf{X}_C)\cdot\mathbf{i}_{C(o)}$

$$
\mathbf{X}_C = \begin{bmatrix} 1/(\omega_o\cdot C) & \\ & 1/(\omega_o\cdot C) \end{bmatrix}
\qquad
\mathbf{1}_C = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix}
\qquad (16)
$$

C = given per phase description of ideal capacitor of *lossy capacitor bank*.

Fig. 3 *Capacitor voltage model* of the Symmetrical Lossy Capacitor Bank. d–q axis reference.

From (15) it is definition-wise understood that vectors $\Delta\mathbf{E}_C$ and $\mathbf{i}_C$ are as follows; $\Delta\mathbf{E}_C = [\Delta E_{Cd}, \Delta E_{Cq}]^t$ & $\mathbf{i}_C = [i_{Cd}, i_{Cq}]^t$.

Initial value of $\Delta\mathbf{E}_C$ in (16) flows from that same equation for t= -0. Then one has $d\Delta\mathbf{E}_C/dt = 0$, and the equation yields $\Delta\mathbf{E}_{C(o)} = -(\mathbf{1}_C)^{-1}\cdot\mathbf{X}_C\cdot\mathbf{i}_{C(o)} = (\mathbf{1}_C\cdot\mathbf{X}_C)\cdot\mathbf{i}_{C(o)}$.

C. The Synchronous Voltage Reference in power network modeling

At some chosen network bus the following symmetrical, synchronous three phase voltage $\mathbf{E}_{RST(ref)}$ may be specified;

$$\mathbf{E}_{RST(ref)} = \sqrt{2}\cdot E_{rms} \cdot \begin{array}{|c|}\hline \cos\alpha \\ \hline \cos(\alpha - 2\pi/3) \\ \hline \cos(\alpha - 4\pi/3) \\ \hline \end{array}\begin{array}{c} R \\ S \\ T \end{array} \qquad (17)$$

E_{rms} is the *root mean square (r.m.s.)* value of the three phase voltage. $\alpha = (\omega_o \cdot t + \gamma)$, where γ accounts for an arbitrary phase shift of the voltages relative to zero time. For convenient final expressions – see (19) – γ is chosen equal to (γ_{ref} $+\pi/2$).

The transformation of $\mathbf{E}_{RST(ref)}$ of (17) into global $\mathbf{e}_{DQ0(ref)}$ of the D-Q axis frame of reference, is afforded by the Park transformation (2). See also (33) and associated text:

$$\mathbf{e}_{DQ0(ref)} = \mathbf{P}\cdot\mathbf{E}_{RST(ref)} \qquad (18)$$

In present synchronous phasor context the angle θ of $\mathbf{P}$ is defined equal to $(\omega_o \cdot t)$. Evaluating the right hand side of (18), the sought infinite bus voltage in the D-Q-0 frame of reference is found;

$$\mathbf{e}_{DQ0(ref)} = \begin{array}{|c|}\hline e_{D(ref)} \\ e_{Q(ref)} \\ e_{0(ref)} \\ \hline \end{array} = \sqrt{2}\cdot E_{rms} \cdot \begin{array}{|l|}\hline -\sin\gamma_{ref} \\ \cos\gamma_{ref} \\ 0 \\ \hline \end{array} \qquad (19)$$

In many practical studies the remote part of the power system is represented in terms of an infinite bus voltage fronted by a given series impedance (r_{ref} ,x_{ref}) – The latter often estimated from the short circuit capacity of the adjoining «foreign» part of the system.

With appropriate interpretation of terms, Fig. 1 may serve as *the electrical circuit model* of the infinite bus located «behind» some specified series impedance: The emf. sub vector $\mathbf{e}_{DQ(ref)}$ of $\mathbf{e}_{DQ0(ref)}$ of (19), is shown in (20) and should be inserted for $\mathbf{e}_L$ in Figure 1. Likewise, the impedance terms ($\mathbf{R}_L$,$\mathbf{X}_L$) there, should be replaced by ($\mathbf{R}_{ref}$,$\mathbf{X}_{ref}$) of (21).

$$\mathbf{e}_{DQ(ref)} = \begin{array}{|c|}\hline e_{D(ref)} \\ \hline e_{Q(ref)} \\ \hline \end{array} = \sqrt{2}\cdot E_{rms} \cdot \begin{array}{|c|}\hline -\sin\gamma_{ref} \\ \hline \cos\gamma_{ref} \\ \hline \end{array} \qquad (20)$$

$$\mathbf{R}_{ref} = \begin{array}{|c|c|}\hline r_{ref} & -x_{ref} \\ \hline x_{ref} & r_{ref} \\ \hline \end{array} \qquad \mathbf{X}_{ref} = \begin{array}{|c|c|}\hline x_{ref} & \\ \hline & x_{ref} \\ \hline \end{array} \qquad (21)$$

D. The Synchronous Motor (SM)

Formal basis for model development is the d–q diagram of a generalised linear machine as e.g. presented by B. Adkins [8]. As a compromise in view of desired precision of analysis, computational burden and availability of data, *a 5-coil, salient pole generalised machine* is applied as basis for the ensuing «default» development.

For special or more detailed analyses modeling based on e.g. the corresponding *6-coil* generalised machine may be appropriate. See e.g. [9], where this extended modeling basis is being used to also describe the performance of *the adjustable speed synchronous machine* as well as *the doubly fed induction machine*.

The well-known diagram of the 5-coil generalised model machine, is shown in Fig. 4. Model development in the following presumes *motor* operation as the default mode of operation.

The three phase main (stator) winding is assumed to be the rotating part, while the d-q axes with associated windings are considered fixed. The «pseudo-stationary» d-and q coils equivalence the electromagnetic effects of the main 3-phase winding. The currents, voltages and fluxes associated with the stated two coils, are definition-wise related to their corresponding physical phase variables via the Park transformation. See (1) – (2). The fixed coil ’f’ of the diagram represents the field circuit of the synchronous motor. The fixed coils denoted ’k_d’ and ’k_q’, aim at equivalencing the effects of all damper circuits in the motor.

The five-coil representation implies 5 state variables to describe the electrical performance of the synchronous motor. As such variables are chosen the coil currents (i_d,i_q) and the flux linkages associated with respectively the ’f’-, ’k_d’- and ’k_q’-coil. The modeling takes place in a suitable per unit (pu) setting.

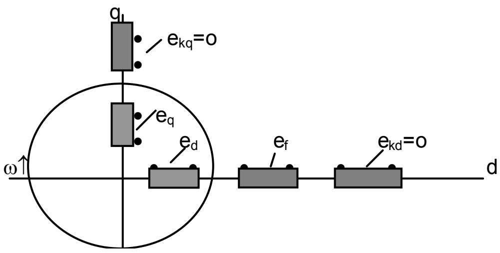


Fig. 4 Diagram of 5-coil salient pole generalised linear machine.

The elaboration of the set of component sub models required for modeling of the synchronous motor, – see Table I of Part 1 – is presented in four steps: Step 1 develops *the basic synchronous motor equations* that establishes the platform for the ensuing (SM) algorithmic development. Step 2 generates the sub model denoted the *flux model*, Step 3 produces the component sub model labelled the e*lectrical circuit model*, and Step 4 the sub model termed the e*lectromechanical model*.

1. The basic synchronous motor equations

A summary outline of these equations for the above 5-coil machine in motor mode of operation, follows. For more on premises and conventions, reference is made to [8].

In the three phase (RST) reference frame one can for (say) phase 'R' of the motor, express the voltage balance as;

$$e_R = i_R \cdot r_a + d\psi_R/dt \tag{22}$$

where e_R, i_R, ψ_R and r_a is – respectively – impressed voltage, current, flux linkages and resistance of motor phase 'R'.

The per phase variables e_R, i_R, and ψ_R are determined from their corresponding (d,q,o) components via transformation $\mathbf{P}^{-1}$, see (4) and (3).

From a formal viewpoint equation (22) is identical to (5). Expressing (22) in terms of (d,q,o) variables, the transformation process becomes identical to that shown from (5) to (7): In the d-q axis frame of reference, (22) then becomes;

$$\begin{aligned} e_d &= r_a \cdot i_d + d\psi_d/dt - \omega \cdot \psi_q \\ e_q &= r_a \cdot i_q + d\psi_q/dt + \omega \cdot \psi_d \\ e_o &= r_a \cdot i_o + d\psi_o/dt \end{aligned} \tag{23}$$

ω is the electrical angular speed of the motor's rotating main winding. For each of the three fixed coils 'f', 'kd' and 'kq' of the model machine, the voltage balance can readily be formulated as follows:

$$\begin{aligned} e_f &= r_f \cdot i_f + d\psi_f/dt \\ e_{kd} &= 0 = r_{kd} \cdot i_{kd} + d\psi_{kd}/dt \\ e_{kq} &= 0 = r_{kq} \cdot i_{kq} + d\psi_{kq}/dt \end{aligned} \tag{24}$$

Equations (23) and (24) form together the voltage equations of the 5-coil model machine in motor mode of operation. As it is presumed that zero sequence circuitry is absent in the present context, the last equation of (23) is being disregarded in the ensuing developments.

On the adopted modeling premises the following defining pu relationships are set up between flux linkages and currents within respective axes:

$$\begin{aligned} \psi_d &= L_d \cdot i_d + L_{ad} \cdot i_f + L_{ad} \cdot i_{kd} \\ \psi_q &= L_q \cdot i_q + L_{aq} \cdot i_{kq} \\ \psi_f &= L_f \cdot i_f + L_{ad} \cdot i_d + L_{ad} \cdot i_{kd} \\ \psi_{kd} &= L_{kd} \cdot i_{kd} + L_{ad} \cdot i_f + L_{ad} \cdot i_d \\ \psi_{kq} &= L_{kq} \cdot i_{kq} + L_{aq} \cdot i_q \end{aligned} \tag{25}$$

Equations (23) – (25) comprise per se the sought basic motor equations. To ease the further processing of expressions, the equations are rewritten in matrix notation, see Fig. 5.

$$\begin{bmatrix} e_d \\ e_q \\ e_f \\ e_{kd} \\ e_{kq} \end{bmatrix} = \begin{array}{c} \begin{matrix} d & q & f & kd & kq \end{matrix} \\ \left[\begin{array}{cc|ccc} r_a & & & & \\ & r_a & & & \\ \hline & & r_f & & \\ & & & r_{kd} & \\ & & & & r_{kd} \end{array}\right] \end{array} \cdot \begin{bmatrix} i_d \\ i_q \\ \hline i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} + \begin{bmatrix} d\psi_d/dt \\ d\psi_q/dt \\ d\psi_f/dt \\ d\psi_{kd}/dt \\ d\psi_{kq}/dt \end{bmatrix} + \omega \cdot \begin{array}{c} \begin{matrix} d & q & f & kd & kq \end{matrix} \\ \left[\begin{array}{cc|ccc} & -1 & & & \\ 1 & & & & \\ \hline & & & & \\ & & & & \\ & & & & \end{array}\right] \end{array} \cdot \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_f \\ \psi_{fd} \\ \psi_{fq} \end{bmatrix} \tag{26}$$

$$\downarrow$$

$$\begin{bmatrix} \mathbf{e}_{dq} \\ \hline \mathbf{e}_{fk} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{r}_a & \\ \hline & \mathbf{r}_{fk} \end{array}\right] \begin{bmatrix} \mathbf{i}_{dq} \\ \hline \mathbf{i}_{fk} \end{bmatrix} + \begin{bmatrix} d\boldsymbol{\psi}_{dq}/dt \\ \hline d\boldsymbol{\psi}_{fk}/dt \end{bmatrix} + \omega \cdot \left[\begin{array}{c|c} \mathbf{H}_{dq} & \\ \hline & \end{array}\right] \cdot \begin{bmatrix} \boldsymbol{\psi}_{dq} \\ \hline \boldsymbol{\psi}_{fk} \end{bmatrix} \tag{27}$$

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \hline \psi_f \\ \psi_{kd} \\ \psi_{kq} \end{bmatrix} = \begin{array}{c} \begin{matrix} d & q & f & kd & kq \end{matrix} \\ \left[\begin{array}{cc|ccc} L_d & & L_{ad} & L_{ad} & \\ & L_q & & & L_{aq} \\ \hline L_{ad} & & L_f & L_{ad} & \\ L_{ad} & & L_{ad} & L_{kd} & \\ & L_{aq} & & & L_{kq} \end{array}\right] \end{array} \cdot \begin{bmatrix} i_d \\ i_q \\ \hline i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} \tag{28}$$

$$\downarrow$$

$$\begin{bmatrix} \boldsymbol{\psi}_{dq} \\ \hline \boldsymbol{\psi}_{fk} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{L}_{dq} & \mathbf{L}_{(dq)(fk)} \\ \hline \mathbf{L}_{(fk)(dq)} & \mathbf{L}_{fk} \end{array}\right] \cdot \begin{bmatrix} \mathbf{i}_{dq} \\ \hline \mathbf{i}_{fk} \end{bmatrix} \tag{29}$$

Fig.5 Basic synchronous motor equations: The platform for further algorithmic development.

The defined sub matrices of (26) and (28), directly show the content of the corresponding sub matrices of (27), respectively (29).

2. The flux model of the synchronous motor

The synchronous motor *flux model* comprises the differential equations that describe the flux linkages $\mathbf{\Psi}_{fk} = [\psi_f, \psi_{kd}, \psi_{kq}]^t$, and – if required –

also the equations that uncover «hidden» coil currents. See next.

Two sets of equations from Fig. 5 provide the basis for the analysis that follows; the lower set of equations from respectively (27) and (29):

$$\mathbf{e}_{fk} = \mathbf{r}_{fk}\cdot\mathbf{i}_{fk} + d\mathbf{\Psi}_{fk}/dt \tag{30}$$

$$\mathbf{\Psi}_{fk} = \mathbf{L}_{(fk)(dq)}\cdot\mathbf{i}_{dq} + \mathbf{L}_{fk}\cdot\mathbf{i}_{fk} \tag{31}$$

It is chosen to retain the flux variables $\mathbf{\Psi}_{fk}$ as state variables, while eliminating the currents $\mathbf{i}_{fk}$ from the «surface» of analysis: Solving $\mathbf{i}_{fk}$ from (31) and inserting the expression for it into (30), yields;

$$d\mathbf{\Psi}_{fk}/dt = \mathbf{e}_{fk} + (-\,\mathbf{r}_{fk}\cdot\mathbf{L}_{fk}^{-1})\cdot\mathbf{\Psi}_{fk} + (\mathbf{r}_{fk}\cdot\mathbf{L}_{fk}^{-1}\cdot\mathbf{L}_{(fk)(dq)})\cdot\mathbf{i}_{dq} \tag{32}$$

Flux linkages and currents are referred to the model machine's local d-q axes. The flux linkages will conveniently be kept locally referenced, while the main motor current should be described relative to the chosen synchronous global reference phasor.

The shift from global to local description is given by the following transformation:

$$\mathbf{i}_{dq} = \mathbf{T}\cdot\mathbf{i}_{DQ} \text{ where ; } \mathbf{T} = \begin{bmatrix} \cos\beta_{SM} & -\sin\beta_{SM} \\ \sin\beta_{SM} & \cos\beta_{SM} \end{bmatrix} \tag{33}$$

Here small letters (dq) signal locally referenced currents, and capital letters (DQ) globally referenced. β_{SM} is the angular displacement of the local motor axes relative to the stated synchronous global ones.

The expression for $\mathbf{i}_{dq}$ from (33) is inserted into (32). At the same time new flux variables $\mathbf{\phi}_{fk} = \omega_o\cdot\mathbf{\Psi}_{fk}$ are introduced. The form of (32) then becomes;

$$d\mathbf{\phi}_{fk}/dt = \omega_o\cdot\mathbf{e}_{fk} - (\mathbf{r}_{fk}\cdot\mathbf{L}_{fk}^{-1})\cdot\mathbf{\phi}_{fk} + (\omega_o\cdot\mathbf{r}_{fk}\cdot\mathbf{L}_{fk}^{-1}\cdot\mathbf{L}_{(fk)(dq)}\cdot\mathbf{T})\cdot\mathbf{i}_{DQ} \tag{34}$$

Inserting into (34) the appropriate sub matrices from Fig. 5, doing some further reductions, and introducing specific synchronous machine parameter terms where-ever appropriate, the *Flux model of the Synchronous Motor* appears as shown in Fig. 6. The pu scaling factor K_f follows from the defining equation $e_f = K_f\cdot E_f$ of (36), where $E_{f(o)}$ is pu field voltage read from the machine's phasor diagram for the initial operating state.

For completeness, a summary set up is included of the interrelationships between «external» machine parameters $(X_d, X'_d, X''_d, X_q, X''_q, T'_{do}, T''_{do}, T''_{qo})$ and «internal» (model) parameters $(X_{a\sigma}, X_{ad}, X_{aq}, X_{f\sigma}, X_{kd\sigma}, X_{kq\sigma}, r_a, r_f, r_{kd}, r_{kq})$. See (39).

As termination of present step 2 on the modeling of the SM flux linkages, the equations that uncover the field- and damper currents are given:

$$d\mathbf{\phi}_{SM}/dt = \omega_o\cdot(\mathbf{e}_{SM} + \mathbf{F}_{SMi}\cdot\mathbf{i}_{SM} + \mathbf{F}_{SM\phi}\cdot\mathbf{\phi}_{SM}) \tag{35}$$

$\mathbf{i}_{SM} = [i_D, i_Q]^t$; SM current, global reference.
$\mathbf{\phi}_{SM} = [\phi_f, \phi_{kd}, \phi_{kq}]^t$; SM flux linkages, local reference

$$\mathbf{e}_{SM} = \begin{array}{c} \begin{bmatrix} K_f\cdot E_f \\ 0 \\ 0 \end{bmatrix} \end{array}\begin{array}{l} f \\ kd \\ kq \end{array} \qquad \begin{array}{lcl} E_f & = & (E_{fo} + \Delta E_f) = \text{field voltage} \\ K_f & = & (\sqrt{2}/(\omega_o\cdot T'_{do}))\cdot X_{ad}/(X_d - X'_d) \\ \Delta E_f & = & \text{voltage control response} \end{array} \tag{36}$$

$$\mathbf{F}_{SMi} = \omega_o\cdot\begin{array}{cc} \quad D & \quad Q \\ \left[\begin{array}{c} F_{SMi}(f,D) \\ F_{SMi}(kd,D) \\ F_{SMi}(kq,D) \end{array}\right. & \left.\begin{array}{c} F_{SMi}(f,Q) \\ F_{SMi}(kd,Q) \\ F_{SMi}(kq,Q) \end{array}\right] \end{array}\begin{array}{l} \\ f \\ kd \\ kq \end{array} \tag{37}$$

$F_{SMi}(f,D) = (1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'_{ad})\cdot X''_{ad}\cdot\cos\beta_{SM}$
$F_{SMi}(f,Q) = -\,(1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'_{ad})\cdot X''_{ad}\cdot\sin\beta_{SM}$
$F_{SMi}(kd,D) = (1/(\omega_o\cdot T''_{do}))\cdot X'_{ad}\cdot\cos\beta_{SM}$
$F_{SMi}(kd,Q) = -\,(1/(\omega_o\cdot T''_{do}))\cdot X'_{ad}\cdot\sin\beta_{SM}$
$F_{SMi}(kq,D) = (1/(\omega_o\cdot T''_{qo})\cdot X_{aq}\cdot\sin\beta_{SM}$
$F_{SMi}(kq,Q) = (1/(\omega_o\cdot T''_{qo})\cdot X_{aq}\cdot\cos\beta_{SM}$

$$\mathbf{F}_{SM\phi} = \omega_o\cdot\begin{array}{ccc} f & kd & kq \\ \left[\begin{array}{c} F_{SM\phi}(f,f) \\ F_{SM\phi}(kd,f) \\ 0 \end{array}\right. & \begin{array}{c} F_{SM\phi}(f,kd) \\ F_{SM\phi}(kd,kd) \\ 0 \end{array} & \left.\begin{array}{c} 0 \\ 0 \\ F_{SM\phi}(kq,kq) \end{array}\right] \end{array}\begin{array}{l} \\ f \\ kd \\ kq \end{array} \tag{38}$$

$F_{SM\phi}(f,f) = -\,(1/(\omega_o\cdot T'_{do}\cdot X'_{ad}))\cdot[(X_{ad}/X'_{ad})\cdot(X'_d - X''_d) + X''_{ad}]$
$F_{SM\phi}(f,kd) = (1/(\omega_o\cdot T'_{do}\cdot X'_{ad}))\cdot(X_{ad}/X'_{ad})\cdot(X'_d - X''_d)$
$F_{SM\phi}(kd,f) = (1/(\omega_o\cdot T''_{do}\cdot X_{ad}))\cdot(X_d - X'_d)$
$F_{SM\phi}(kd,kd) = -\,1/(\omega_o\cdot T''_{do})$
$F_{SM\phi}(kq,kq) = -\,1/(\omega_o\cdot T''_{qo})$

Fig. 6 Flux model of the Synchronous Motor (SM). (Subscript 'SM' applied to identify component).

$$\begin{array}{l} X_d = X_{a\sigma} + X_{ad} \quad X_f = X_{f\sigma} + X_{ad} \quad X_{kd} = X_{kd\sigma} + X_{ad} \\ X_q = X_{a\sigma} + X_{aq} \quad X_{kq} = X_{kq\sigma} + X_{aq} \\ X'_d = X_{a\sigma} + X'_{ad} \text{ where } 1/X'_{ad} = (1/X_{ad}) + (1/X_{f\sigma}) \\ X''_d = X_{a\sigma} + X''_{ad} \text{ where } 1/X''_{ad} = (1/X_{ad}) + (1/X_{f\sigma}) + (1/X_{kd\sigma}) \\ \qquad\qquad\qquad\qquad\qquad\quad = (1/X'_{ad}) + (1/X_{kd\sigma}) \\ X''_q = X_{a\sigma} + X''_{aq} \text{ where } 1/X''_{aq} = (1/X_{aq}) + (1/X_{kq\sigma}) \\ T'_{do} = L_f/r_f = X_f/(\omega_o\cdot r_f) \text{ (Open stator: Seen from field side)} \\ T''_{do} = L/r_{kd} = X/(\omega_o\cdot r_{kd}) \text{ (Open stator: Seen from 'kd' side)} \\ \qquad\qquad X = X_{kd\sigma} + X'_{ad} \\ T''_{qo} = L_{kq}/r_{kq} = X_{kq}/(\omega_o\cdot r_{kq}) \text{ (Open stator: Seen from 'kq' side)} \end{array} \tag{39}$$

At any time during integration the field- and damper currents $\mathbf{i}_{fk}$ (= $\mathbf{i}_{SMr}$ of Figure 7) may be derived from equation (31), after introducing $\mathbf{i}_{dq} = \mathbf{T}\cdot\mathbf{i}_{SM}$ and $\mathbf{\phi}_{fk} = \omega_o\cdot\mathbf{\Psi}_{fk}$ (=$\mathbf{\phi}_{SM}$):

$$\mathbf{i}_{SMr} = [\, i_f,\ i_{kd},\ i_{kq}]^t = \mathbf{X}^{-1}_{fk}\cdot(\,\phi_{SM} - \mathbf{X}_{DQr}\cdot\mathbf{i}_{SM}) \qquad (40)$$

where;

$$\mathbf{X}_{fk} = \begin{array}{c|c|c|c} & f & kd & kq \\ \hline f & X^2_{ad}/(X_d - X'_d) & X_{ad} & \\ kd & X_{ad} & X_{ad} + X''_{ad}\cdot X'_{ad}/(X'_d - X''_d) & \\ kq & & & X^2_{aq}/(X_q - X''_q) \end{array}$$

$$\mathbf{X}_{DQr} = \mathbf{X}_{(fk)(dq)}\cdot\mathbf{T} = \begin{array}{c|c|c} & D & Q \\ \hline f & X_{ad}\cdot\cos\beta_{SM} & -X_{ad}\cdot\sin\beta_{SM} \\ kd & X_{ad}\cdot\cos\beta_{SM} & -X_{ad}\cdot\sin\beta_{SM} \\ kq & X_{aq}\cdot\sin\beta_{SM} & X_{aq}\cdot\cos\beta_{SM} \end{array}$$

Fig. 7 Locally referenced currents $\mathbf{i}_{fk} = [\, i_f,\ i_{kd},\ i_{kq}]^t$ determined from (locally referenced) motor flux linkages and (globally referenced) motor currents $\mathbf{i}_{DQ}$ $(= \mathbf{i}_{SM})$.

3. *The electrical circuit model of the synchronous motor*

In the context of power network analysis the task at hand is that of equivalencing the synchronous motor model of Fig. 5, by an oriented, standardized d–q axis series circuit comprising an **R**-term, an inductive **X**-term, and an e.m.f. $\Delta\mathbf{E}$. See Fig.1 with associated text in Part 1 of the paper.

Three sets of equations from Fig. 5 form the basis for the ensuing analysis,– namely the upper set from (27), and both sets from (29):

$$\mathbf{e}_{dq} = \mathbf{r}_a\cdot\mathbf{i}_{dq} + d\mathbf{\Psi}_{dq}/dt + \omega\cdot\mathbf{H}_{dq}\cdot\mathbf{\Psi}_{dq} \qquad (41)$$

$$\mathbf{\Psi}_{dq} = \mathbf{L}_{dq}\cdot\mathbf{i}_{dq} + \mathbf{L}_{(dq)(fk)}\cdot\mathbf{i}_{fk} \qquad (42)$$

$$\mathbf{\Psi}_{fk} = \mathbf{L}_{(fk)(dq)}\cdot\mathbf{i}_{dq} + \mathbf{L}_{fk}\cdot\mathbf{i}_{fk} \qquad (43)$$

The expression found for $\mathbf{i}_{fk}$ from (43) is inserted into (42), which then describes $\mathbf{\Psi}_{dq}$ as a function of $\mathbf{i}_{dq}$ and $\mathbf{\Psi}_{fk}$. The expression thus found for $\mathbf{\Psi}_{dq}$ is inserted into (41), yielding finally the applied motor voltage as function of the motor's state variables $\mathbf{i}_{dq}$ and $\mathbf{\Psi}_{fk}$. Introducing the new flux linkage variables $\mathbf{\phi}_{fk} = \omega_o\cdot\mathbf{\Psi}_{fk}$, and $\mathbf{\phi}_{dq} = \omega_o\cdot\mathbf{\Psi}_{dq}$, one finds as a result from this process;

$$\begin{aligned}\mathbf{e}_{dq} = \ & \mathbf{r}_a\cdot\mathbf{i}_{dq} + (\mathbf{L}_{dq} - \mathbf{L}_{(dq)(fk)}\cdot\mathbf{L}_{fk}^{-1}\cdot\mathbf{L}_{(fk)(dq)})\cdot d\mathbf{i}_{dq}/dt \\ & + \omega\cdot\mathbf{H}_{dq}\cdot(\mathbf{L}_{dq} - \mathbf{L}_{(dq)(fk)}\cdot\mathbf{L}_{fk}^{-1}\cdot\mathbf{L}_{(fk)(dq)})\cdot\mathbf{i}_{dq} \\ & + (1/\omega_o)\cdot\mathbf{L}_{(dq)(fk)}\cdot\mathbf{L}_{fk}^{-1}\cdot d\mathbf{\phi}_{fk}/dt \\ & + \Omega\cdot\mathbf{H}_{dq}\cdot\mathbf{L}_{(dq)(fk)}\cdot\mathbf{L}_{fk}^{-1}\cdot\mathbf{\phi}_{fk} \end{aligned} \qquad (44)$$

where $\Omega = (\omega/\omega_o)$ = pu angular speed of rotating part. It remains to replace the locally referenced motor voltage $\mathbf{e}_{dq}$ and motor current $\mathbf{i}_{dq}$, by their globally referenced counterparts $\mathbf{e}_{DQ}$ and $\mathbf{i}_{DQ}$, respectively.

It is next pointed to a few premises and rules that are crucial to the process of shifting from local to global reference (or vice versa):

For motor voltage and current the following holds true:

$$\mathbf{e}_{dq} = \mathbf{T}\cdot\mathbf{e}_{DQ} \qquad \mathbf{i}_{dq} = \mathbf{T}\cdot\mathbf{i}_{DQ} \qquad (45)$$

Definition-wise, for angle & speed of rotating part ;

$$\beta = (\omega_o\cdot t - \theta) \rightarrow d\beta/dt = (\omega_o - \omega) = \omega_o\cdot(1 - \Omega) \qquad (46)$$

From mathematics;

$$d\mathbf{i}_{dq}/dt = d(\mathbf{T}\cdot\mathbf{i}_{DQ})/dt = (d\mathbf{T}/dt)\cdot\mathbf{i}_{DQ} + \mathbf{T}\cdot d\mathbf{i}_{DQ}/dt \qquad (47)$$

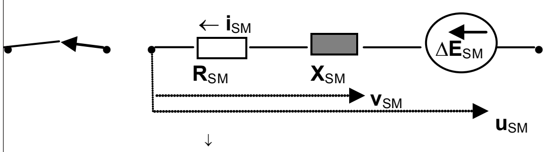
From mathematics and (46);

$$d\mathbf{T}/dt = (d\beta/dt)\cdot(d\mathbf{T}/d\beta) = \omega_o\cdot(1 - \Omega)\cdot d\mathbf{T}/d\beta \qquad (48)$$

Applying (35) and (45) to (44), observing the stated premises and rules from above, while abiding with the adopted definitions associated with *the electrical circuit model* of Fig. 1, – one arrives after some straightforward but tedious elaborations at the sought electrical circuit model of Fig. 8.

4. *The electromechanical model of the synchronous motor*

The task at hand is to describe the performance of the remaining two synchronous motor state variables, – namely *pu angular speed* Ω of its rotating part, and the *electrical angle* β associated with the rotating field set up by the motor's main 3-phase winding. For oversight reasons the set of algorithms to apply in this context is denoted «the electromechanical model». See model description of Fig. 9.



↓

$\underline{v_{SM} = R_{SM}\cdot i_{SM} + (1/\omega_o)\cdot X_{SM}\cdot di_{SM}/dt}$ & $\underline{v_{SM} - \Delta E_{SM} = u_{SM}}$

$i_{SM} = [i_{SM(D)}\ ,\ i_{SM(Q)}]^t$ = SM current, global reference
$u_{SM} = [u_{SM(D)}\ ,\ u_{SM(Q)}]^t$ = SM bus voltage, global reference

$\Delta E_{SM} = (V_{SM} + H_{SM}\cdot\phi_{SM})$ = synchronous motor e.m.f. (49)

$$V_{SM} = \begin{array}{cc} \begin{bmatrix} C_f\cdot E_f\cdot\cos\beta_{SM} \\ -C_f\cdot E_f\cdot\sin\beta_{SM} \end{bmatrix} & \begin{array}{c} D \\ Q \end{array} \end{array} \qquad (50)$$

$E_f = (E_{fo} + \Delta E_f)$ = field voltage. E_{fo} is initial value. ΔE_f is AVC response. $C_f = (\sqrt{2}/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})$

$$R_{SM} = \begin{array}{cc} \begin{array}{cc} D & Q \end{array} & \\ \begin{bmatrix} R_{SM(D,D)} & R_{SM(D,Q)} \\ R_{SM(Q,D)} & R_{SM(Q,Q)} \end{bmatrix} & \begin{array}{c} D \\ Q \end{array} \end{array} \qquad (51)$$

$\Delta\Omega_{SM} = (\Omega_{SM} - 1)$ = deviation from synchr. speed of rotating part (pu)

$R_{SM(D,D)} = (r_a + \hat{X}''_r) + (1+2\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin 2\beta_{SM} + \bar{X}''_r\cdot\cos 2\beta_{SM}$

$R_{SM(D,Q)} = -\hat{X}'' + (1+2\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos 2\beta_{SM} - \bar{X}''_r\cdot\sin 2\beta_{SM}$

$R_{SM(Q,D)} = \hat{X}'' + (1+2\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos 2\beta_{SM} - \bar{X}''_r\cdot\sin 2\beta_{SM}$

$R_{SM(Q,Q)} = (r_a + \hat{X}''_r) - (1+2\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin 2\beta_{SM} - \bar{X}''_r\cdot\cos 2\beta_{SM}$

$\hat{X}'' = ½\cdot(X''_d + X''_q)$ & $\bar{X}'' = ½\cdot(X''_d - X''_q)$

$\hat{X}''_r = ½\cdot(X''_{rd} + X''_{rq})$ & $\bar{X}''_r = ½\cdot(X''_{rd} - X''_{rq})$

X''_d = d-axis subtransient reactance
X''_q = q-axis subtransient reactance
$X''_{rd} = X_{ad}\cdot[(f_1/(\omega_o T'_{do}))\cdot(X''_{ad}/X'_{ad}) + (f_3/(\omega_o\cdot T''_{do}))\cdot(X'_{ad}/X_{ad})]$
$X''_{rq} = -X_{aq}\cdot(f_5/(\omega_o\cdot T''_{qo}))$

$$X_{SM} = \begin{array}{cc} \begin{array}{cc} D & Q \end{array} & \\ \begin{bmatrix} \hat{X}''+\bar{X}''\cdot\cos 2\beta_{SM} & -\bar{X}''\cdot\sin 2\beta_{SM} \\ -\bar{X}''\cdot\sin 2\beta_{SM} & \hat{X}''-\bar{X}''\cdot\cos 2\beta_{SM} \end{bmatrix} & \begin{array}{c} D \\ Q \end{array} \end{array} \qquad (52)$$

$$H_{SM} = \begin{array}{cc} \begin{array}{ccc} f & kd & kq \end{array} & \\ \begin{bmatrix} H_{SM(D,f)} & H_{SM(D,kd)} & H_{SM(D,kq)} \\ H_{SM(Q,f)} & H_{SM(Q,kd)} & H_{SM(Q,kq)} \end{bmatrix} & \begin{array}{c} D \\ Q \end{array} \end{array} \qquad (53)$$

$H_{SM(D,f)} = \Omega_{SM}\cdot f_1\cdot\sin\beta_{SM} + f_2\cdot\cos\beta_{SM}$
$H_{SM(D,kd)} = \Omega_{SM}\cdot f_3\cdot\sin\beta_{SM} + f_4\cdot\cos\beta_{SM}$
$H_{SM(D,kq)} = \Omega_{SM}\cdot f_5\cdot\cos\beta_{SM} + f_6\cdot\sin\beta_{SM}$
$H_{SM(Q,f)} = \Omega_{SM}\cdot f_1\cdot\cos\beta_{SM} - f_2\cdot\sin\beta_{SM}$
$H_{SM(Q,kd)} = \Omega_{SM}\cdot f_3\cdot\cos\beta_{SM} - f_4\cdot\sin\beta_{SM}$
$H_{SM(Q,kq)} = -\Omega_{SM}\cdot f_5\cdot\sin\beta_{SM} + f_6\cdot\cos\beta_{SM}$

$f_1 = (X_d - X'_d)\cdot X''_{ad}/(X_{ad}\cdot X'_{ad})$ $\quad f_3 = (X'_d - X''_d)/X'_{ad}$
$f_5 = -(X_q - X''_q)/X_{aq}$ $\quad f_6 = f_5/(\omega_o\cdot T''_{qo})$
$f_2 = f_1\cdot[(X''_{rd}/X''_{ad}) - (X_{ad}/X'_{ad})\cdot(1/(\omega_o\cdot T'_{do}))]$
$f_4 = f_3\cdot[(X_{ad}/X'_{ad})\cdot(f_1/(\omega_o\cdot T'_{do})) - 1/(\omega_o\cdot T''_{do})]$

Fig. 8 The electrical circuit model of the Synchronous Motor (SM). d-q axis frame of reference.

The algorithm that governs motor speed variation is the torque equation emanating from Newton's second law. The power supplied to the motor is definition-wise, see Fig. 5;

$$P_{(el)} = ½\cdot\mathbf{e}_{dq}^{\ t}\cdot\mathbf{i}_{dq} \qquad (54)$$

Setting in for $\mathbf{e}_{dq}$ and $\mathbf{i}_{dq}$ from (26)–(27), replacing the currents $\mathbf{i}_{dq}$ by their globally referenced counterparts in accordance with (33), and observing that $\boldsymbol{\phi}_{dq} = \omega_o\cdot\boldsymbol{\Psi}_{dq}$, one gets

$$P_{(el)} = \underbrace{½\cdot r_a\cdot(i_D^2+i_Q^2)}_{\text{Losses in main winding}} + \underbrace{½\cdot\Omega\cdot\mathbf{i}_{DQ}^{\ t}\cdot\mathbf{T}_1\cdot\boldsymbol{\phi}_{dq}}_{\text{Airgap power}} + \underbrace{k\cdot\mathbf{i}_{DQ}^{\ t}\cdot\mathbf{T}^t\cdot d\boldsymbol{\phi}_{dq}/dt}_{\text{Oscillating power (zero power over time)}} \qquad (55)$$

where $k=0.5/\omega_o$, and $\mathbf{T}$ and $\mathbf{T}_1$ are as given in Fig. 9. The *electrical torque* is found by dividing the expression for *airgap power* by Ω. The flux linkage vector $\boldsymbol{\phi}_{dq}$ is given as a function of $\mathbf{i}_{DQ}$ (denoted $\mathbf{i}_{SM}$ in Fig. 9) and $\boldsymbol{\phi}_{fk}$ (denoted $\boldsymbol{\phi}_{SM}$) from equations (42), (43), (33).

Torque equation:

$$\underline{d\Omega_{SM}/dt = J_{SM}\cdot(T_{SMel} - T_{SMmec})} \qquad (56)$$

Ω_{SM} = pu angular speed of rotating part
J_{SM} = inertia constant given by e.g. H-constant or acceleration time constant T_a.
T_{SMel} = $½\cdot i_{SM}{}^t\cdot T_{SM1}\cdot\phi_{dq}$ = pu electrical motor torque, where
$\phi_{dq} = X''_{SM}\cdot T_{SM}\cdot i_{SM} + f_{SM}\cdot\phi_{SM}$
i_{SM} = SM current, see Fig. 8
ϕ_{SM} = SM flux linkages, see Fig. 6

$$T_{SM1} = \begin{bmatrix} \sin\beta_{SM} & -\cos\beta_{SM} \\ \cos\beta_{SM} & \sin\beta_{SM} \end{bmatrix}$$

$$T_{SM} = \begin{bmatrix} \cos\beta_{SM} & -\sin\beta_{SM} \\ \sin\beta_{SM} & \cos\beta_{SM} \end{bmatrix} \quad \text{(from (33))}$$

$$X''_{SM} = \begin{bmatrix} X''_d & \\ & X''_q \end{bmatrix}$$

$$f_{SM} = \begin{bmatrix} f_1 & f_3 & \\ & & -f_5 \end{bmatrix} \quad \text{(for } (f_1,f_3,f_5)\text{, see Fig. 8)}$$

Motor mode of operation:
T_{SMmec} = $T_{SMmec(o)}\cdot\Omega_{SM}{}^{\kappa}$ = mechanical load torque. (κ=1.5-3.5)
- If the SM is running at t=-0: $T_{SMmec(o)} = T_{SMel(o)}$ = electrical motor torque from initial load flow.
- If the SM is to be started as an asynchronous motor: $T_{SMmec(o)}$ = (variable) coefficient to model mechanical friction, air resistance, etc., during startup. (Likely coefficient range; 0.01 - 0.06)

Generator mode of operation:
T_{SMmec} = $(T_{SMel(o)} + \Delta T_{mec})$ = mechanical torque (negative). ΔT_{mec} is the response of the *power control system*. It is considered outside the scope of this paper to delve into sub models yielding the incremental control response ΔT_{mec} , or ΔE_f of Fig. 6.

Electrical angle equation:

$$\underline{d\beta_{SM}/dt = \omega_o\cdot(1 - \Omega_{SM})} \qquad (57)$$

ω_o = $2\cdot\pi\cdot f_o$, where f_o is nominal frequency.
β_{SM} = electrical angle of axes of field set up by 3-phase winding, relative to synchronous reference phasor.

Fig. 9 The electromechanical model of the Synchronous Motor (SM).

After some elaborations the motor's torque balance becomes modeled in terms of (56) and adjoining definitions.

The algorithm that governs motor angle variation has already been established, see (46). The equation is copied into (57) to form the remaining part of the electromechanical model of the synchronous motor.

E. The Asynchronous Motor (AM)

Compared to the normal synchronous machine the traditional asynchronous machine lacks the field winding, and symmetry prevails regarding the electromagnetic effects of its rotor circuits.

For the synchronous machine it was presumed suitable to base «default» mathematical modeling on a *five-coil, salient pole generalised machine*. See Fig. 4.

In view of the availability of machine data and the desirability of generally retaining much the same level of precision within machine modeling , it appears reasonable to specify a *four-coil, cylindrical pole generalised machine* for modeling of the asynchronous motor/generator. Thus, the machine diagram of Fig. 4 and the implied modeling of the SM, can serve as proper development basis, when observing the following special interpretations:

– The «pseudo stationary» d- and q coils equivalence in the same way as outlined for the synchronous machine, the electromagnetic effects of the main three-phase winding of the asynchronous machine.
– There is *one* superfluous coil in the d axis. For ease of further adaptations it is chosen to eliminate the 'kd' coil. Thus, the remaining 2 coils labelled respectively 'f' and 'kq', take on the function of equivalencing the secondary (rotor) winding of the asynchronous machine.
– Since magnetic saliency is absent, the former 'f'- and 'kq'-coil now become identical in their new roles. Index 'r' for 'rotor' is in the following assigned to both coils. Inductances and flux linkages may be written as follows, see two first lines of (39), and also (28) – for comparison with the synchronous machine:

For the d-axis: $L_d = L_{a\sigma} + L_m \qquad L_{rd} = L_{r\sigma} + L_m$
For the q-axis: $L_q = L_{a\sigma} + L_m \qquad L_{rq} = L_{r\sigma} + L_m$

(58)

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_{rd} \\ \psi_{rq} \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} d & q & rd & rq \end{array} \\ \begin{bmatrix} L_d & & L_m & \\ & L_q & & L_m \\ L_m & & L_{rd} & \\ & L_m & & L_{rq} \end{bmatrix} \end{array} \begin{bmatrix} i_d \\ i_q \\ i_{rd} \\ i_{rq} \end{bmatrix} \qquad (59)$$

Parameter-wise, further implications for the «simplified SM model» are as follows:

$$\begin{array}{lll}
X''_d & = X'_d & \text{(there is no third circuit in the d-axis)} \\
X''_{ad} & = X'_{ad} & \text{''} \\
X_d & = X_q = (X_{a\sigma} + X_m) & \text{(see (58) above)} \\
X_{ad} & = X_{aq} = X_m & \text{''} \\
X_{rd} & = X_{rq} = X_r = \omega_o \cdot L_r = (X_{r\sigma} + X_m) & \text{''} \\
X'_{ad} & = X'_m = 1/((1/X_m) + (1/X_{r\sigma})) & \text{(see (39))} \\
X''_{aq} & = X'_{ad} = X'_m & \text{''} \\
X'_M & = X'_d = (X_{a\sigma} + X'_m) & \text{''} \\
T'_{do} & = T'_{ro} = T''_{qo} = L_r/r_r = X_r/(\omega_o \cdot r_r) & \text{''}
\end{array} \qquad (60)$$

The four-coil representation decided on, implies 4 state variables for describing the electrical performance of the asynchronous motor, whether singly- or doubly-fed. As state variables are chosen the coil currents (i_d,i_q) and the flux linkages associated with respectively the 'r_d'- and 'r_q'-coil defined above.

The further elaboration to produce the set of component sub models required for modeling of the asynchronous motor, is presented in three steps: Step 1 develops the *electrical circuit model*, Step 2 the *flux model*, and Step 3 the *electromechanical model*. See Table I of Part 1 for overview.

1. The electrical circuit model of the asynchronous motor

Applying the above premises to the model of Fig. 8, the following «asynchronous motor version» is found for key parameters of Fig. 8:

$$\hat{X}'' = X'_M = (X_{a\sigma} + X'_m) \qquad \bar{X}'' = 0$$

$$\hat{X}''_r = r_r \cdot (X_m/X_r)^2 \qquad \bar{X}''_r = 0$$

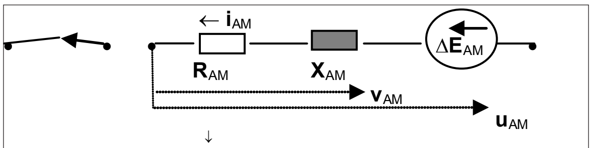
$$f_1 = (X_m/X_r) \qquad f_2 = - r_r \cdot (X_m/X_r^{\,2}) \qquad f_3 = 0$$

$$f_4 = 0 \qquad f_5 = - f_1 \qquad f_6 = f_2 \qquad (61)$$

The defined e.m.f. for the synchronous motor is given as $\Delta \mathbf{E}_{SM} = (\mathbf{V}_{SM} + \mathbf{H}_{SM} \cdot \boldsymbol{\phi}_{SM})$, see Fig. 8. In modifying this equation to cover the asynchronous motor, it is to be observed that:

– There is no separate emf. associated with the traditional asynchronous machine. Thus $\mathbf{V}_{SM} \rightarrow \mathbf{V}_{AM} = 0$. See (49).
– Because coil 'kd' is presumed removed, column no. 2 of the (2x3) matrix $\mathbf{H}_{SM}$ is deleted to produce a preliminary version $\mathbf{H}_{AMprelim}$ of $\mathbf{H}_{AM}$.
– The flux linkages $\boldsymbol{\phi}_{SM} = [\phi_f, \phi_{kd}, \phi_{kq}]^t$ of the SM are referenced the local motor axes (d,q), whereas the flux linkages of the AM $\boldsymbol{\phi}_{AM} = [\phi_{rd}, \phi_{rq}]^t$, most conveniently should be globally (D,Q) referenced. The shift of description from global to local reference is generally given by $\boldsymbol{\phi}_{local} = \mathbf{T}\cdot\boldsymbol{\phi}_{global}$, see (33). Interpreting now $\boldsymbol{\phi}_{AM}$ as being the globally referenced flux linkages, one arrives at the following expression for the asynchronous motor emf.: $\Delta\mathbf{E}_{AM} = (\mathbf{H}_{AMprelim}\cdot\mathbf{T})\cdot\boldsymbol{\phi}_{AM} = \mathbf{H}_{AM}\cdot\boldsymbol{\phi}_{AM}$.

Applying all the foregoing simplifying / modifying observations to the content of Fig. 8, the following *electrical circuit model* is established for the traditional asynchronous motor, see Fig. 10.



$\mathbf{v}_{AM} = \mathbf{R}_{AM}\cdot\mathbf{i}_{AM} + (1/\omega_o)\cdot\mathbf{X}_{AM}\cdot d\mathbf{i}_{AM}/dt$ & $\mathbf{v}_{AM} - \Delta\mathbf{E}_{AM} = \mathbf{u}_{AM}$

$\mathbf{i}_{AM} = [i_{AM(D)}, i_{AM(Q)}]^t$ = AM current, global reference
$\mathbf{u}_{AM} = [u_{AM(D)}, u_{AM(Q)}]^t$ = AM bus voltage, global reference

$\Delta\mathbf{E}_{AM} = (\mathbf{H}_{AM}\cdot\boldsymbol{\phi}_{AM})$ = asynchronous motor e.m.f. (62)

$$\mathbf{R}_{AM} = \begin{array}{c c} \begin{array}{cc} D & Q \end{array} & \\ \begin{bmatrix} r_a + r_r\cdot(X_m/X_r)^2 & -X'_M \\ X'_M & r_a + r_r\cdot(X_m/X_r)^2 \end{bmatrix} & \begin{array}{c} D \\ Q \end{array} \end{array} \quad (63)$$

$$\mathbf{X}_{AM} = \begin{array}{c c} \begin{array}{cc} D & Q \end{array} & \\ \begin{bmatrix} X'_M & \\ & X'_M \end{bmatrix} & \begin{array}{c} D \\ Q \end{array} \end{array} \quad (64)$$

$$\mathbf{H}_{AM} = (X_m/X_r)\cdot \begin{array}{c c} \begin{array}{cc} rd & rq \end{array} & \\ \begin{bmatrix} -(r_r/X_r) & -\Omega_{AM} \\ \Omega_{AM} & -(r_r/X_r) \end{bmatrix} & \begin{array}{c} D \\ Q \end{array} \end{array} \quad (65)$$

		(example data)
$X_{a\sigma}$	= stator leakage reactance	(0.08pu)
$X_{r\sigma}$	= rotor leakage reactance	(0.08pu)
X_m	= magnetizing reactance	(2.50pu)
r_a	= stator resistance	(0.03pu)
r_r	= rotor resistance	(0.03pu)

$X_r = (X_{r\sigma} + X_m)$
$X'_M = (X_{a\sigma} + X'_m) = X_{a\sigma} + (X_m\cdot X_{r\sigma}/X_r)$
$T'_{ro} = L_r/r_r = X_r/(\omega_o\cdot r_r)$
Ω_{AM} = pu speed of rotating part
s_{AM} = pu motor slip = $(1 - \Omega_{AM})$

Fig. 10 The electrical circuit model of the Asynchronous Motor (AM). D–Q axis reference frame.

2. The flux model of the asynchronous motor

The source for model reduction is the *flux model* of the synchronous motor given in Fig. 6.

To have the appropriate algorithmic platform, a preliminary «asynchronous motor version» of the matrices ($\mathbf{e}_{SM}$, $\mathbf{F}_{SMi}$, $\mathbf{F}_{SM\phi}$) of Fig. 6 are first evaluated. To this end one brings to bear the implications that were ascertained via equations (61) and their associated text. The following follows readily, when noting that row and column associated with the 'kd'-coil are to be deleted:

$$\mathbf{e}_{SM} \rightarrow e_{AM} = 0$$

$$\mathbf{F}_{AMi(prelim)} = (r_r\cdot X_m/X_r)\cdot \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \quad (66)$$

$\mathbf{F}_{AM\phi(prelim)} = -(r_r/X_r)\cdot\mathbf{1}_{(2x2)}$ (where $\mathbf{1}_{(2x2)}$ is a 2x2 unit matrix)

With the «AM versions» (e_{AM}, $\mathbf{F}_{AMi(prelim)}$, $\mathbf{F}_{AM\phi(prelim)}$) to replace ($e_{SM}$, $\mathbf{F}_{SMi}$, $\mathbf{F}_{SM\phi}$), one gets the following tentative expression for the flux model of the asynchronous motor, – when taking into account that the flux linkages $\boldsymbol{\phi}_{AM}$ for the AM are presumed globally referenced:

$$d(\mathbf{T}\cdot\boldsymbol{\phi}_{AM})/dt = \omega_o\cdot\mathbf{F}_{AMi(prelim)}\cdot\mathbf{i}_{AM} + \omega_o\cdot\mathbf{F}_{AM\phi(prelim)}\cdot(\mathbf{T}\cdot\boldsymbol{\phi}_{AM})$$

Processing the matrix product and rearranging the equation, the following initial version of the flux model is found:

$$d\boldsymbol{\phi}_{AM}/dt = \omega_o\cdot(\mathbf{T}^{-1}\cdot\mathbf{F}_{AMi(prelim)})\cdot\mathbf{i}_{AM} + \omega_o\cdot[\mathbf{T}^{-1}\cdot\mathbf{F}_{AM\phi(prelim)}\cdot\mathbf{T} - (1/\omega_o)\cdot(d\beta/dt)\cdot\mathbf{T}^{-1}\cdot d\mathbf{T}/d\beta]\cdot\boldsymbol{\phi}_{AM} \quad (67)$$

Further development of this equation yields the sought flux model of the traditional asynchronous motor:

$d\boldsymbol{\phi}_{AM}/dt = \omega_o\cdot(\mathbf{F}_{AMi}\cdot\mathbf{i}_{AM} + \mathbf{F}_{AM\phi}\cdot\boldsymbol{\phi}_{AM})$ (68)

$\mathbf{i}_{AM} = [i_D, i_Q]^t$; AM current, global reference
$\boldsymbol{\phi}_{AM} = [\phi_{rD}, \phi_{rQ}]^t$; AM flux linkages, global reference

$$\mathbf{F}_{AMi} = \begin{array}{c c} \begin{array}{cc} D & Q \end{array} & \\ \begin{bmatrix} (r_r\cdot X_m/X_r) & \\ & (r_r\cdot X_m/X_r) \end{bmatrix} & \begin{array}{c} rD \\ rQ \end{array} \end{array} \quad (69)$$

For parameter interpretation, see Fig. 10

$$\mathbf{F}_{AM\phi} = \begin{array}{c c} \begin{array}{cc} D & Q \end{array} & \\ \begin{bmatrix} -(r_r/X_r) & (1-\Omega_{AM}) \\ -(1-\Omega_{AM}) & -(r_r/X_r) \end{bmatrix} & \begin{array}{c} rD \\ rQ \end{array} \end{array} \quad (70)$$

Fig. 11 *Flux model* of the Asynchronous Motor (AM).

3. *The electromechanical model of the asynchronous motor*

The asynchronous motor state variable to be described here is pu angular speed Ω_{AM} of the motor's rotating part. The set of expressions developed in this context is denoted *«the electromechanical model»*. See Fig. 12.

Suitable basis for development is the upper main part of Fig. 9, relating to the SM's torque equation. Applying the stated AM premises, the sought sub model becomes:

Torque equation:

$$d\Omega_{AM}/dt= J_{AM}\cdot(T_{AMel} -T_{AMmec}) \quad (71)$$

Ω_{AM} = pu angular speed of rotating part

J_{AM} = inertia constant given by e.g. H-constant or acceleration time constant T_a.

T_{AMel} = $½\cdot(X_m/X_r)\cdot(\mathbf{1}_c\cdot\mathbf{i}_{AM})^t\cdot\phi_{AM}$ = pu electrical motor torque

1c = 2x2 skew integer matrix, see Fig. 3.

Motor mode of operation:

–If the AM is running at t=-0:

T_{AMmec} = $T_{AMmec(o)}\cdot(\Omega_{AM}/\Omega_{AM(o)})^{\kappa}$ = mechanical load torque.

κ = (say) 1.5-3.5, depending on type of load.

$T_{AMmec(o)}$ = $T_{AMel(o)}$ = el. motor torque from initial load flow.

–If the AM is to be started from stillstand:

T_{AMmec} = $T_{AMmec(o)}\cdot\Omega_{AM}^{\kappa}$ = mechanical load torque.

$T_{AMmec(o)}$ = (variable) coefficient to model mech. friction, air resistance, etc., during start-up. Likely coeff. range ; 0.01 – 0.06. κ = (say) 1.5-3.5

Generator mode of operation:

T_{AMmec} = $(T_{AMel(o)} +\Delta T_{mec})$ = mechanical torque (negative). ΔT_{mec} is the response of the *power control system*. ΔT_{mec} is not further dealt with here, see comment to control responses in Fig.9.

Fig. 12 Electromechanical model of the Asynchronous Motor (AM).

Neglecting stator transients (by presuming $d\mathbf{i}_{AM}/dt=0$) and replacing the flux linkages ϕ_{AM} of (68) by an equivalent motor emf. formulation, the model description of Fig. 10 and 11 of the singly-fed AM, becomes identical to the third order model presented in [10]. See equations (15) and (14) there. Further: If three phase voltage supply is introduced also to the AM rotor circuit of the model described by Fig. 10 and 11 above, the stated model simplification/ modification leads to a model for the doubly-fed asynchronous motor that is identical to the one presented by equations (21) and (20) of [10].

2. Detailed analysis of unbalanced conditions

A. Overview observations

In the detailed analysis of one or a very few synchronous machines, complete solutions for the principal unbalanced short-circuit conditions can be obtained. Solution may be afforded by the use of different formulations, like e.g. Laplace transform, equivalent two-phase (α,β) quantities, or direct determination of three phase currents and voltages. Reason for holding back in applying d–q axis formulation relates evidently in part to the fact that coefficients of the equations now include functions of time, and to problems with using ordinary operational solution methods to the analysis task.

In the practical analysis of complex multi-machine power systems it is normally deemed suitable to use the method of symmetrical components,- which involves the assumption that all harmonics can be neglected. Thus symmetrical components is primarily a tool for analyzing the behaviour of *fundamental* current and voltage components. As a consequence of the inherent lack of stringency in applying symmetrical components to the study of unbalanced conditions, the system analyst is faced also with the specific task of defining the most appropriate sequence component models for the particular type of non-symmetric analysis at hand.

B. Analysis of unbalanced conditions in the d–q axis frame of reference

Based on the presented methodology of analysis it is illustrated how detailed analysis of unbalanced conditions suitably can be performed within the d–q axis frame of reference. This is somewhat contrary to the above citation, which stems from a time of less computing power at the fingertips.

Problem solution is afforded by introducing into the network at the location of (sudden) unbalance, a *special electrical circuit model* that accounts for the unbalance at hand.

Bringing forth this special circuit model is afforded in two steps; first the power network model to equivalence the unbalance is defined in the 3-phase frame of reference, next the model is transformed into the d–q axis frame of reference via the Park transformation.

The result from the electrical analysis – whether balanced or unbalanced – appears in terms of *instantaneous* traces of currents and voltages. Corresponding *rms.* traces are readily generated from the instantaneous records. Occurring harmonics may produce some influence on registered rms. traces.

To illustrate somewhat more in depth on processes of unbalanced analysis, two different fault cases are dealt with next; forced opening of one of the three phases of a power transmission, and phase-to-phase short circuit at the terminals of a synchronous machine.

Forced opening of one of the three phases of a power transmission

It is assumed that phase 'R' of the three phases ('RST') of the considered power transmission is suddenly and temporarily opened between points ('p,q'). The points are located infinitely close to each other along the said transmission.

In the three phase frame of reference the following *resistive serial component* is introduced in between 'p' and 'q' to account for opening of phase 'R':

$$^{RST}\mathbf{r} = \begin{array}{c} \begin{array}{ccc} R & S & T \end{array} \\ \left[\begin{array}{ccc} r_R & & \\ & r & \\ & & r \end{array}\right] \end{array} \begin{array}{c} \\ R \\ S \\ T \end{array} \qquad (72)$$

r_R is the *series* resistance of phase 'R'. At desired point in time this resistance should in principle increase to infinity, to observe the fact that phase 'R' is being opened. The *series* resistance of phase 'S' and 'T' are here set equal and denoted r. r is to be set to zero.

The resistance matrix of (72) is next to be transformed into $^{DQ0}\mathbf{r}$ which is the model of the resistive component in the D–Q axis frame of reference:

$$^{DQ0}\mathbf{r} = \mathbf{P} \cdot {}^{RST}\mathbf{r} \cdot \mathbf{P}^{-1} \qquad (73)$$

where **P** is the Park transformation, see (2). The result:

$$^{DQ0}\mathbf{r} = \begin{array}{c} \begin{array}{ccc} D & Q & 0 \end{array} \\ \left[\begin{array}{ccc} r+(r_R-r)(1+\cos 2\theta)/3 & -(r_R-r)\cdot\sin(2\theta)/3 & (r_R-r)\cdot\cos\theta\cdot(2/3) \\ -(r_R-r)\cdot\sin(2\theta)/3 & r+(r_R-r)(1-\cos 2\theta)/3 & -(r_R-r)\sin\theta\cdot(2/3) \\ (r_R-r)\cdot\cos\theta\cdot(1/3) & -(r_R-r)\cdot\sin\theta\cdot(1/3) & r+(r_R-r)/3 \end{array}\right] \end{array} \qquad (74)$$

where $\theta = \omega_o \cdot t$, see comment to equation (17). As no zero sequence variables are involved in this example case, 3rd row and column can be deleted from (74). Setting furthermore r= 0, one gets the following *special electrical circuit model* to insert in between 'p' and 'q' of the transmission to be faulted:

$$^{DQ}\mathbf{r} = \begin{array}{c} \begin{array}{cc} D & Q \end{array} \\ \left[\begin{array}{cc} (r_R/3)\cdot(1+\cos 2\theta) & -(r_R/3)\cdot\sin 2\theta \\ -(r_R/3)\cdot\sin 2\theta & (r_R/3)\cdot(1-\cos 2\theta) \end{array}\right] \end{array} \begin{array}{c} \\ D \\ Q \end{array} \qquad (75)$$

Opening of phase 'R' is then simulated by quickly increasing the «arc-resistance» r_R from zero to such a large value that phase 'R' becomes open in the course of an appropriately short period of time, – say 0.02s which is the duration of one 50 Hz fundamental cycle.

Line-to-line short circuit of a synchronous machine

It is assumed that solid short circuit suddenly is applied between phases 'S' and 'T' at the teminals of a synchronous generator in isolated and idle operation.

In the three phase frame of reference the following *set of Δ - connected resistances* (r_{RS},r_{ST},r_{TR}) is introduced at the fault site, to account for the *phase to phase short circuit*:

$r_{ST} = r_{fault}$ = fault resistance located between phase 'S' and 'T' at fault location

$r_{RS} = r_{TR} = r_{large}$ = resistance to attain large value. (76)

Transforming the set of delta-connected resistances into its equivalent set of star-connected resistances (r_R,r_S,r_T), it is found that;

$r_R = (r_{RS} \cdot r_{TR})/N \rightarrow ½ \cdot r_{large}$ (since $r_{large} >> r_{fault}$)

$r_S = (r_{RS} \cdot r_{ST})/N \rightarrow ½ \cdot r_{fault}$ ”

$r_T = (r_{TR} \cdot r_{ST})/N \rightarrow ½ \cdot r_{fault}$ ” (77)

where $N = (r_{RS} + r_{ST} + r_{TR})$.

From a formal viewpoint (77) is identical to (72). Thus the transformed description (74) must be valid also for the case of phase-to-phase short circuit provided $r_R = ½\cdot r_{large}$ and $r = ½\cdot r_{fault}$.

As no zero sequence variables are present in this fault case either, 3rd row and column are again deleted from (74). Setting furthermore $r = ½\cdot r_{fault} = 0$, one arrives at the following *special electrical circuit model* to connect as shunt at the bus to be faulted:

$$ {}^{DQ}\mathbf{r} = \begin{array}{ccc} D & Q & \\ \begin{array}{|c|} \hline (r_{large}/6)\cdot(1+\cos 2\theta) \\ \hline -\,(r_{large}/6)\cdot\sin 2\theta \\ \hline \end{array} & \begin{array}{c|} \hline -\,(r_{large}/6)\cdot\sin 2\theta \\ \hline (r_{large}/6)\cdot(1-\cos 2\theta) \\ \hline \end{array} & \begin{array}{c} D \\ Q \end{array} \end{array} \qquad (78) $$

Line-to-line short circuit is then simulated by connecting the shunt resistance ${}^{DQ}\mathbf{r}$ at the point in time when the fault occurs.

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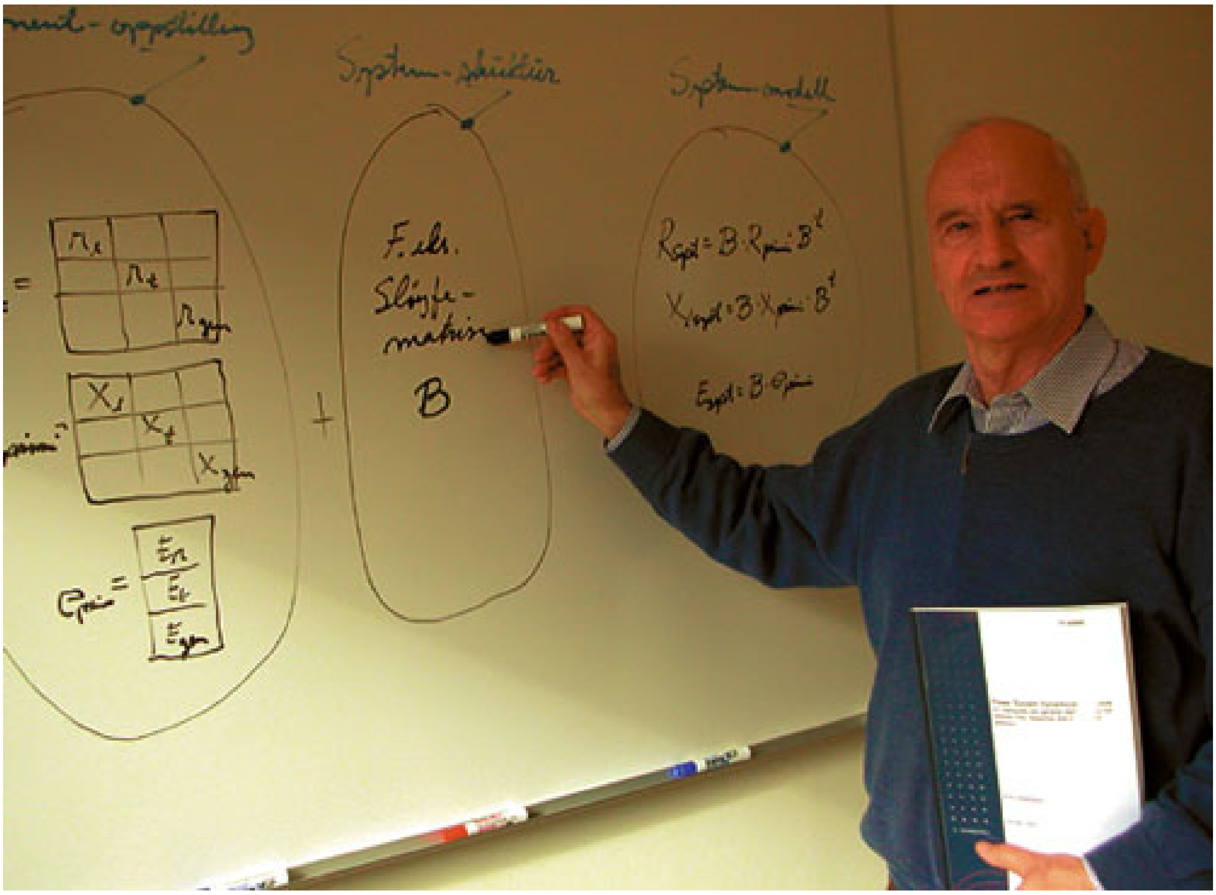

Photo: Harald Danielsen

Arne Johannesen received his M.Sc. in Electrical Engineering from the Norwegian Institute of Technology (NTH) in 1956. Following assignments as assistant professor at NTH, he worked for two years with prof. J.A. Harle at Dept. of Electrical Engineering, University of Alberta, Edmonton, Canada, on a post-doctorate fellowship granted by National Research Council of Canada. Since 1961 he has worked with The Norwegian Electric Power Research Institute (EFI), which merged into SINTEF Energy Research in 1998 (www.sintef.no/home/SINTEF-Energy-Research). Main positions there; division manager, senior/chief research officer, and presently senior consultant. From 1976 to 1998 he served part-time as professor II; with NTH until 1996, and since then with The Norwegian University of Science and Technology (NTNU, www.ntnu.no) into which NTH merged.

From 1961 Arne Johannesen has contributed to building up a strong power system research group at EFI (today SINTEF Energy Research) with main focus on developing and applying practical tools for optimizing the design and market driven utilization of complex power systems – the latter optionally comprising extensive hydro-dominated production subsystems. Today this research group consists of about 70 scientists that do mission-based research for industry and government and are active in the international research community. In 1996 EFI established a spin-off company, Powel (www.powel.com), based on proprietary software systems. The company has been a success and delivers software to power companies on the international market.

An Intelligible and Practicable Methodology for Power System Dynamic Analysis

Electrical power systems all over the world are steadily being tied more closely together by strengthening of local national connections, as well as more ties across borders to neighboring countries.

It is a challenge both in design and operation of an expanding interconnected power system, to ensure that geographically distributed power supply and demand becomes matched in such a way that agreed-upon qualities of delivery conditions are met.

Power system dynamic analyses have to be conducted as part of the processes of initially defining proper power quality constraints, and next following up by checking quality conditions during operation.

This report deals with power system dynamic analysis. Central to the presented methodology is the development of a stock of compact sub models for modelling of power system components. Formulation and solution-wise, problem complexity becomes thereby largely confined to local component level rather than overall system level.

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