

STF90 A06030 – Unrestricted

REPORT

Information Flow Property Preserving Transformation of UML Interaction Diagrams

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SINTEF ICT

Cooperative and Trusted Systems

July 2006



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SINTEF REPORT

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Research Council of Norway (Norges forskningsråd)

REPORT NO. SFT90 A06030	CLASSIFICATION Unrestricted	CLIENTS REF. 152839/220	
CLASS. THIS PAGE Unrestricted	ISBN 82-14-02852-3	PROJECT NO. 40332800	NO. OF PAGES/APPENDICES 32/2
ELECTRONIC FILE CODE N/A	PROJECT MANAGER (NAME, SIGN.) Ketil Stølen <i>Ketil Stølen</i>	CHECKED BY (NAME, SIGN.) Jan Øyvind Agedal <i>J.O. Agedal</i>	
FILE CODE N/A	DATE 2006-03-29	APPROVED BY (NAME, POSITION, SIGN.) Bjørn Skjellaug, Research director <i>[Signature]</i>	

ABSTRACT
We present an approach for secure information flow property preserving refinement and transformation of UML inspired interaction diagrams. The approach is formally underpinned by trace-semantics. The semantics is sufficiently expressive to distinguish underspecification from explicit nondeterminism. A running example is used to introduce the approach and to demonstrate that it is of practical value.

KEYWORDS	ENGLISH	NORWEGIAN
GROUP 1	ICT, modelling	IKT, modellering
GROUP 2	Design,	Design
SELECTED BY AUTHOR	MDA, Information flow security,	UML, Sikker informasjonsflyt,
	Refinement, Transformation, UML	Raffinering, Transformasjon, UML



Information Flow Property Preserving Transformation of UML Interaction Diagrams

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July 17, 2006

Abstract

We present an approach for secure information flow property preserving refinement and transformation of UML inspired interaction diagrams. The approach is formally underpinned by trace-semantics. The semantics is sufficiently expressive to distinguish underspecification from explicit nondeterminism. A running example is used to introduce the approach and to demonstrate that it is of practical value.

1 Introduction

Security is an important attribute of many software systems. Nevertheless, careful engineering of security into overall design is often neglected. Security features are typically built into an application in an ad-hoc manner or are only integrated during the final phases of system development [24]. It is, however, a common view in the security field that security mechanisms should be taken into account and built into the system during early phases of system development at a higher level of abstraction than the level of implementation.

The Object Management Group (OMG) advocates a framework for system development, Model Driven Architecture (MDA) [27], that aims to raise the level of abstraction of the programming environment by supporting (1) a model-driven development process, (2) a clear separation of abstract, platform independent models (PIMs) and refined, platform dependent models (PSMs), and (3) transformations between PIMs and PSMs.

By integrating security into MDA, security documentation can be specified and analyzed at a high level of abstraction during system development, thereby reducing the need for ad hoc integration of security mechanisms after system implementation. Moreover, the advantage of analyzing PIMs rather than PSMs is that analysis is more feasible at high level of abstractions as opposed to levels closer to implementation which may include too much detail to make analysis practical. Also, it is well known that the earlier an error is discovered, the easier and cheaper it is to fix.

In order to contribute towards a formal foundation of security within the MDA framework, we restrict the notion of security to a property of *secure information flow*. Secure information flow properties in general, provide a way

of specifying security requirements by selecting a set of domains, i.e. abstractions of real system entities such as users or files, and then restricting allowed flow of information between these domains. There are numerous definitions of information flow properties [25]. The property that we consider is based on a definition given in [20]. Informally, the property holds if an observer communicating with a system, based on its observations and its knowledge of the system specification, cannot infer that another observer has communicated with the same system.

The relationship between information flow security and refinement has been researched for a fairly long time. In 1989 it was shown by Jacob [20] that the derivation of secure systems from security specifications can be practically infeasible. It has later been observed e.g. in [17, 21, 30], that the problem originates in the inability of most specification languages, UML 2.0 [13] included, to distinguish between underspecification (potential nondeterminism) and explicit nondeterminism¹. An exception in this respect is STAIRS [14, 15, 16], an approach that provides a formal semantics for UML 2.0 inspired interaction diagrams where the distinction of potential and explicit nondeterminism is made. The approach presented in this report extends the theory of STAIRS wrt. refinement and information flow. It also provides the first steps towards a formal foundation for model-driven security in the setting of UML interactions. In particular, the contributions of this report are:

- A formal definition of a notion of secure information flow in STAIRS. Thereby we provide a foundation for analysis of UML 2.0 interaction diagrams wrt. secure information flow. The only work that we are aware of that deals with information flow security in a proposed semantics of UML is [21], but only a simplified version of UML 1.5 [12] interaction diagrams is considered in this work.
- Showing that the STAIRS notion of refinement preserves our notion of secure information flow. The most notable recent theoretical works that we are aware of that address refinement and secure information flow are [5, 26]. In [5], conditions for *checking* that a given refinement is information flow preserving are presented, and [26] proposes a way of *modifying* refinement operations such that they remain secure information flow preserving. These approaches are clearly different from ours. We prove that *all* refinements preserve our notion of secure information flow, thus there is no need to check or modify given refinements as in [5, 26].
- A formal definition of a notion of *transformation* that preserves our notion of secure information flow. The notion of refinement is a relation between two specifications and it is in general not subject for automation. In contrast, a transformation takes a specification and delivers another specification as output in the sense of a compiler. There are no related works that we are aware of that address secure information flow with respect to this notion of transformation.
- A generalization of the notion of refinement into a more general notion of refinement modulo transformation. This more general concept may be

¹Also termed *unpredictability* [21] and *probabilistic nondeterminism* [30].

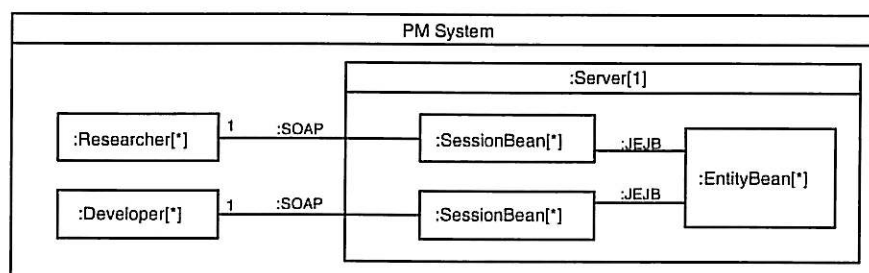


Figure 1: Overview of the Project Management System

understood as a kind of data refinement [19].

The report is structured as follows: In Sect. 2 we use STAIRS to specify a PIM of an example system which is sufficiently realistic to suggest that our approach is of practical value. In Sect. 3 we define an information flow property in terms of the STAIRS semantics, and explain why our PIM is secure with respect to this property. Sect. 4 presents a formalization of refinement in the STAIRS semantics, and explains why the information flow property is preserved under refinement. In Sect. 5 we introduce a notion of transformation which is information flow property preserving. We also integrate this notion of transformation and the classical notion of refinement from STAIRS into a more general concept. In Sect. 6 we show how our PIM can be transformed to PSMs in such a way that the information flow property is preserved. Sect. 7 describes related work. Sect. 8 provides conclusions and suggests directions for future work. The appendix provides formal definitions of auxiliary operators used in this report.

2 The Project Management System

Consider a large software developing company that aims to develop a distributed system, the project management system (the PM system), in order to centralize all storage of software development projects. Software developers should be able to retrieve projects from a server to their local machines, edit or add files to the project, and upload any changes back to the server. Projects stored on the server should be versioned, and whenever a developer updates a project, only changes in the project with respect to the developer's local copy should be updated.

Currently, the company has no unified development method, and developers working on different projects are to a large degree given flexibility in the method they choose to adopt. The company wants to assess the different methods in order to recommend improvements, and possibly to introduce a unified development process. This task is assigned to a group of researches. The researchers are to pick a set of sample projects, and assess each project thoroughly with respect to progress, quality of code etc. For convenience, the PM system should be augmented slightly such that the researchers will be able to retrieve projects from the server over the Internet on a regular basis. This additional functionality is not of high priority, thus it will be implemented with little resources. The researcher will not use the same versioning system that the developers are

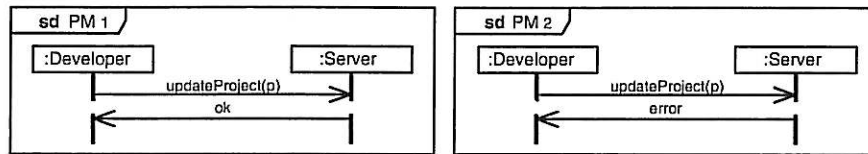


Figure 2: Simple interaction scenario of the PM system

using, thus for simplicity, the whole project is copied to the researchers for every retrieval.

To make the assessment as realistic as possible, the software developers of the company should not know which projects are sampled for the assessment. We ensure this by requiring that the developers should not, at any given moment in time, be able to deduce whether a researcher has retrieved a project from the server. This means that the developers are *ignorant* of the researchers.

The company decides that the server where all projects are stored shall expose two web-service endpoints, one for the developers and one for the researchers as illustrated in Fig. 1. Also, the server implementation will be based on J2EE technology, where so-called Java session beans are responsible for handling SOAP-message communication between the server and the clients, and the Java entity beans are responsible for handling persistence.

2.1 Capturing Behavior Using STAIRS

The interaction diagrams in Fig. 2 specify two simple interactions of the PM system in which a developer updates a project by sending an update message to the server, whereupon the server responds by returning an ok or an error message depending on whether or not the update was successful.

In the graphical diagrams, vertical dashed lines correspond to so-called *lifelines*, and the signals of messages correspond to labels decorating the arrows between the lifelines.

Definition 1 A message is a triple (tr, re, si) , consisting of a transmitter tr , a receiver re , and a signal si .

Example The two messages in the left most diagram of Fig. 2 are represented by the triples (D, S, u) and (S, D, o) . For conciseness we refer to the lifelines and the signals in graphical diagrams by their first letters if this can be done unambiguously.

In the trace-semantics of STAIRS [15], the transmission and the reception of a message m is represented by a *transmission event* (denoted $(!, m)$) and a *reception event* (denoted $(?, m)$), respectively. For conciseness, an event $(!, (tr, re, si))$ is usually denoted $!si$ if the transmitter and the receiver can be deduced from the context. The same convention applies for reception events.

A *trace* is a sequence of events representing a single run. A trace is causal and weakly sequenced. Causality means that a message must be transmitted before it is received. Weak sequencing [13] means that events are ordered by their vertical position with respect to each lifeline. For example, if two messages

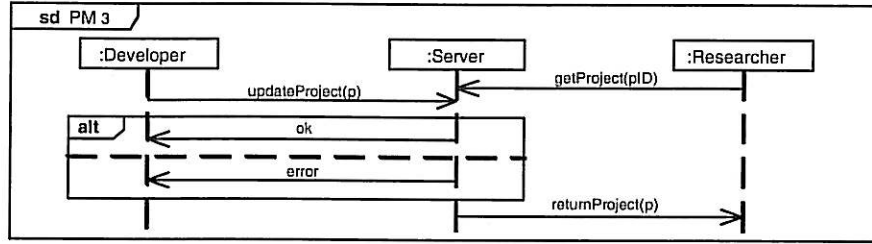


Figure 3: Potential nondeterministic alternatives

are transmitted from a lifeline l_1 to a lifeline l_2 , then the first message is not necessarily received by l_2 before the second message is transmitted from l_1 .

Definition 2 An event is a pair, (k, m) , consisting of a kind $k \in \{!, ?\}$ and a message m . A trace is a sequence of events. We let \mathcal{H} denote the set of all traces.

Example The diagrams PM_1 and PM_2 in Fig. 2 describe the traces $\langle !u, ?u, !o, ?o \rangle$ and $\langle !u, ?u, !e, ?e \rangle$, respectively.

STAIRS distinguishes between *positive* and *negative* traces. Positive traces represent desired or acceptable behavior, while negative traces represent undesired or unacceptable behavior. The remaining traces are *inconclusive* meaning that their status is not decided or that they are irrelevant. The semantics of a diagram in STAIRS is set of pairs of positive and negative traces. Such pairs are referred to as *interaction obligations*. Each interaction obligation represents an explicit nondeterministic alternative. Underspecification (also referred to as potential nondeterminism) is represented by allowing the implementer to choose between different alternative behaviors within a single interaction obligation.

Definition 3 An interaction obligation is a pair, (p, n) , where p is a set of positive traces and n is a set of negative traces. The semantics of a diagram d in STAIRS, written $\llbracket d \rrbracket$, is a set of interaction obligations.

Unless otherwise indicated, traces of interaction diagrams are interpreted as positive.

Example Diagram PM_1 and PM_2 of Fig. 2 describe the interaction obligations $(\{\langle !u, ?u, !o, ?o \rangle\}, \emptyset)$ and $(\{\langle !u, ?u, !e, ?e \rangle\}, \emptyset)$, respectively. The set of negative traces in each interaction obligation is empty since traces of diagrams are interpreted as positive by default.

2.2 Potential Nondeterminism

The diagram in Fig. 3 specifies the interaction in which a developer tries to update a project before it is retrieved by a researcher who has requested the same project prior to the developer's update attempt. The updated project cannot be stored on the server while it is being read, but whether or not the update will have to be resubmitted (upon reception of an error message) by the developer is a postponed design decision.

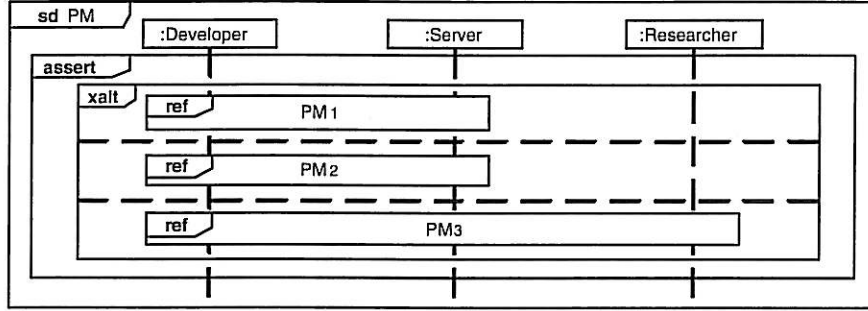


Figure 4: Explicit nondeterministic alternatives

In interaction diagrams as interpreted in STAIRS, underspecification in the form of alternative design decisions is explicitly expressed by the so-called **alt**-operator as illustrated in Fig. 3. Interactions that are separated by the **alt**-operator are interpreted as potential nondeterministic trace alternatives. In Fig. 3 the topmost operand of **alt** specifies the design alternative in which an ok message is transmitted back to the developer. The second alternative specifies the alternative where the developer receives an error message.

Definition 4 The **alt**-operator specifies potential nondeterminism by leaving the choice between its operands open. The semantics is the inner union of each point-wise selection of interaction obligations from its operands:

$$\llbracket \mathbf{alt}[d_1, \dots, d_n] \rrbracket \stackrel{\text{def}}{=} \{ \biguplus \{ o_1, \dots, o_n \} \mid \forall i \in [1, n] : o_i \in \llbracket d_i \rrbracket \}$$

The inner union of interaction obligations is defined as:

$$\biguplus_{i \in [1, n]} (p_i, n_i) \stackrel{\text{def}}{=} \left(\bigcup_{i \in [1, n]} p_i, \bigcup_{i \in [1, n]} n_i \right)$$

Example The diagram PM_3 in Fig. 3 describes an interaction obligation (p, \emptyset) where p includes the traces $\langle !g, ?g, !u, ?u, !o, ?o, !r, ?r \rangle$ and $\langle !g, ?g, !u, ?u, !e, ?e, !r, ?r \rangle$ due to the **alt**-operator. Note that p will also contain other traces due to weak sequencing. E.g. p will also contain the trace $\langle !g, !u, ?g, ?u, !e, ?e, !r, ?r \rangle$, where both the messages get project ($!g$) and the update project ($!u$) are transmitted by the researcher and the developer, respectively, before the corresponding messages ($?g$ and $?u$) are received by the server.

2.3 Explicit Nondeterminism

In STAIRS we may use the **xalt**-operator to specify explicit nondeterminism. This is illustrated in the interaction diagram of Fig. 4. It states that any correct implementation must offer the choice between the operands PM_1 , PM_2 , and PM_3 . Each alternative specified by an **xalt**-operator represents an explicit nondeterministic choice that must be preserved by any correct refinement of the diagram.

Definition 5 The **xalt**-operator specifies explicit nondeterminism by requiring all operands to be reflected in any proper implementation. The semantics is the union of the sets of interaction obligations characterized by the operands:

$$\llbracket \mathbf{xalt}[d_1, \dots, d_n] \rrbracket \stackrel{\text{def}}{=} \bigcup_{i \in [1, n]} \llbracket d_i \rrbracket$$

Assume now for the sake of the running example, that the diagram in Fig. 4 describes all the relevant interactions of the PM system. We may then encapsulate the interactions of the diagram using the **assert**-operator (as shown in Fig. 4), which means that all traces that are not described within the scope of the assert, i.e. the inconclusive traces, are to be interpreted as negative. This is of course an incomplete specification, but interaction diagrams are often used to capture incomplete aspects of a system.

Definition 6 The **assert**-operator makes all inconclusive traces negative. The sets of positive and negative traces are left unchanged:

$$\llbracket \mathbf{assert} d \rrbracket \stackrel{\text{def}}{=} \{(p, n \cup (\mathcal{H} \setminus p)) \mid (p, n) \in \llbracket d \rrbracket\}$$

Example The STAIRS-semantics interprets the diagram PM of Fig. 4 as $\{(p_1, \mathcal{H} \setminus p_1), (p_2, \mathcal{H} \setminus p_2), (p_3, \mathcal{H} \setminus p_3)\}$ where the trace sets p_1 , p_2 , and p_3 are described by the diagrams PM_1 , PM_2 , and PM_3 , respectively.

A diagram is *well-defined* if its semantic interpretation does not contain an interaction obligation (p, n) such that $p \cap n \neq \emptyset$. We denote by \mathcal{D} the set of all well-defined diagrams that can be constructed by the syntactic operators presented in this report.

3 Why Our Specification Ensures Ignorance

A requirement of the PM system is that the developers should be ignorant of the researchers. In this section we formalize this notion. For this purpose, we distinguish between *observers* and the *system* under observation. Each observer is only able to observe its own interaction with the system, and we assume that each observer has complete knowledge of the diagram describing the system they interact with. Based on this knowledge, an observer can construct the set of all system behaviors which are *compatible* with a given observation, the so-called *inference set*, and try to deduce confidential information from this set. The secure information flow property that we formalize (the ignorance property) demands that an observer o must not, based on the inferences it can make, deduce with certainty that another observer o' has communicated with the system under observation.

3.1 Systems, Behavior and Observations

In this section we define the notions of system, observer, behavior, and observation in terms of the STAIRS semantics.

Definition 7 *Systems and observers are represented by sets of lifelines.*

Example In our running example, we require that developers should be ignorant of researchers. Thus developers and researchers are observers. With respect to the diagram PM (of Fig. 4), let D and R denote the observers $\{ : Developer \}$ and $\{ : Researcher \}$, respectively. Both the developer and the researcher communicate with the same server. Hence, the server is the system under observation. We let S denote the system $\{ : Server \}$. This applies throughout the rest of our examples.

STAIRS does not prohibit an implementation of a diagram to produce traces that are not described by the diagram (the inconclusive traces). The set of traces allowed by an interaction obligation is therefore its set of non-negative traces (i.e. the union of its positive and inconclusive traces). Such traces are called *behavior traces*. The set of all behavior traces that are derived from exactly one interaction obligation is called a *behavior alternative*.

In general, an interaction diagram may describe the behavior of several systems or observers. We are usually interested in reasoning about the behavior of a specific system. A behavior alternative of a system s can be obtained from an interaction obligation a by restricting all behavior traces in a to the lifelines in s .

Definition 8 *The behavior alternative of system s in interaction obligation (p, n) , written $\nabla((p, n), s)$, is defined:*

$$\nabla((p, n), s) \stackrel{\text{def}}{=} \{ E.s \odot h \mid h \in \mathcal{H} \setminus n \}$$

Here $E.s$ is the set of events that may occur on the lifelines in s . Furthermore, the function $A \odot h$ yields the trace obtained from trace h by filtering away all events in h that are not in the set of events A (see App. A).

We lift the function to yield the behavior alternatives of a system s as described in a diagram d .

Definition 9 *The behavior of a system s in diagram d , written $\nabla(d, s)$, is defined:*

$$\nabla(d, s) \stackrel{\text{def}}{=} \bigcup_{a \in [d]} \{ \nabla(a, s) \}$$

Example Let PM be the diagram specified in Fig. 4. The behavior of system S in PM is given by the behavior traces that can occur on the lifeline in S : $\nabla(PM, S) = \{ b_1, b_2, b_3 \}$, where $b_1 = \{ \langle ?u, !o \rangle \}$ (corresponding to PM₁ in Fig. 2), $b_2 = \{ \langle ?u, !e \rangle \}$ (corresponding to PM₂ in Fig. 2), and $b_3 = \{ \langle ?g, ?u, !o, !r \rangle, \langle ?g, ?u, !e, !r \rangle \}$ (corresponding to PM₃ in Fig. 3).

An observer may observe its own interaction with the system under observation. The set of *observation traces* that an observer can make of a behavior alternative b is obtained from b by filtering away the events that are not transmitted to or from the observer.

Definition 10 *The set of all observation traces that an observer o can make of behavior alternative b , written $o \triangleright b$, is defined:*

$$o \triangleright b \stackrel{\text{def}}{=} \{ RT.o \odot h \mid h \in b \}$$

$\text{RT}.o$ yields the set of events that can be transmitted to or from the observer o (see App. A).

We lift the observation function to diagram-system pairs.

Definition 11 *The set of all observation traces that an observer o can make of system s in diagram d , written $o \triangleright (d, s)$, is defined:*

$$o \triangleright (d, s) \stackrel{\text{def}}{=} \bigcup_{b \in \nabla(d, s)} o \triangleright b$$

The definition implies that an observer may observe liveness properties (such as termination) because neither the set of behavior traces nor the set of observations traces have to be prefix-closed [3].

Example Based on Fig. 4, we have that $D \triangleright (PM, S)$ yields the observation set $\{\langle ?u, !o \rangle, \langle ?u, !e \rangle\}$, and $R \triangleright (PM, S)$ yields the set $\{\langle ?g, !r \rangle\}$.

3.2 Inference and Ignorance

We assume that observers have full access to the specification of the system, i.e. the observers have complete knowledge of the behavior alternatives of the system under observation. An observer may then, based on a behavior alternative b , construct an *inference set* consisting of all traces in b that are compatible with a given observation h .

Definition 12 *The inference set that observer o can construct from behavior alternative b , based on observation trace h , written $o \triangleleft_h b$, is defined:*

$$o \triangleleft_h b \stackrel{\text{def}}{=} \{h^b \in b \mid h = (\text{RT}.o \otimes h^b)\}$$

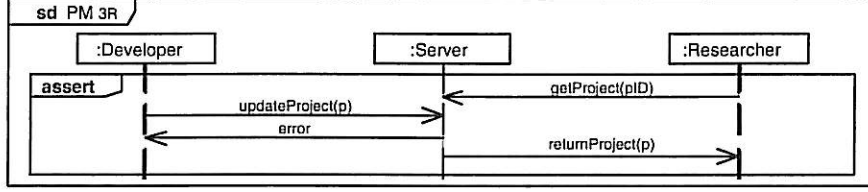
Example Continuing the previous examples, let h_1 and h_2 be observation traces $\langle ?u, !o \rangle$ and $\langle ?u, !e \rangle$ of observer D , respectively, and let behavior alternative b be the set $\{\langle ?g, ?u, !o, !r \rangle, \langle ?g, ?u, !e, !r \rangle\}$. We have that observer D , based on h_1 can infer the following set of behavior traces from b : $D \triangleleft_{h_1} b = \{\langle ?g, ?u, !o, !r \rangle\}$. Similarly we have that $D \triangleleft_{h_2} b = \{\langle ?g, ?u, !e, !r \rangle\}$.

If an inference set that an observer o can construct has a trace that *compromise* another observer o' , meaning that the trace contains an event that is sent to or received from o' , then o may deduce that o' has done something. Conversely, if none of the traces in the inference set compromise o' , then o cannot deduce that o' has done something. We say that a behavior alternative from which such an inference set can be constructed *hides* o' .

Definition 13 *Behavior alternative b hides observer o' from observer o wrt. observation h , written $o \triangleright_{b, h} o'$, iff*

$$\forall h^b \in o \triangleleft_h b : \text{RT}.o' \otimes h^b = \langle \rangle$$

Example Consider diagram PM_3 (Fig. 3). Let $b \in \nabla(\text{PM}_3, S)$ and $h \in D \triangleright (\text{PM}_3, S)$, such that behavior alternative b equals $\{\langle ?g, ?u, !o, !r \rangle, \langle ?g, ?u, !e, !r \rangle\}$ and observation $h = \langle ?u, !o \rangle$. The set of traces that D can infer from b based on h equals $D \triangleleft_h b = \{h^b\}$ (where $h^b = \langle ?g, ?u, !o, !r \rangle$). Since the trace h^b contains events that compromises observer R ($\text{RT}.R \otimes h^b = \langle ?g, !r \rangle$), we have that b does

Figure 5: Refinement of diagram PM₃

not hide R from D wrt. h , i.e. $D \not\lambda_{b,h} R$ yields false.

Observer o is ignorant of observer o' if o cannot infer with certainty that o' has communicated with the system under observation. More precisely, this means that if o , based on an observation h and a behavior alternative b_1 , is able to infer a trace that compromises o' , then there must exist another behavior alternative b_2 that hides o' from o wrt. the same observation h .

Definition 14 An observer o is ignorant of observer o' wrt. diagram-system pair S_d , written $o \lambda^{S_d} o'$, iff

$$\forall h \in o \triangleright S_d : \forall b_1 \in \nabla S_d : \neg(o \lambda_{b_1,h} o') \Rightarrow \exists b_2 \in \nabla S_d : o \lambda_{b_2,h} o' \wedge b_2 = o \triangleleft_h b_2$$

The last condition ($b_2 = o \triangleleft_h b_2$) ensures that the ignorance property is preserved during refinement. This will be explained in Sect 4.

Example D is ignorant of R wrt. S in diagram PM (Fig. 4). To see this, note that for each observation trace h that D can make of S , we have that for all traces that compromise R that D can infer based on h , there exists a behavior alternative that hides R from D wrt. h . E.g. as described in the previous example, D can infer the compromising trace $\langle ?g, ?u, !o, !r \rangle$ based on observation $h = \langle ?u, !o \rangle$. However, there also exists a behavior alternative $b \in \nabla(\text{PM}, S)$ that hides R wrt. h . Specifically, $b = \{\langle ?u, !o \rangle\}$ (corresponding to the interaction obligation described by diagram PM₁ in Fig. 2). We have that D , based on h , can infer the trace $\langle ?u, !o \rangle$ from b . Since this trace does not compromise R ($\text{RT}.R \otimes \langle ?u, !o \rangle = \langle \rangle$), all traces in b (since there is only one trace in b) hide R from D wrt. h .

4 Why the Ignorance Property is Preserved by Refinement

Refinement means to add information to a specification such that the specification becomes more complete. This may be achieved by categorizing inconclusive traces as either positive or negative, or by reducing the set of positive traces. Negative traces always remain negative. We first define refinement of interaction obligations formally. This definition is then lifted to interaction diagrams.

Definition 15 An interaction obligation (p_c, n_c) is a refinement of an interaction obligation (p_a, n_a) , written $(p_a, n_a) \rightsquigarrow (p_c, n_c)$, iff

$$n_a \subseteq n_c \wedge p_a \subseteq p_c \cup n_c$$

Definition 16 An interaction diagram d_c is a refinement of an interaction diagram d_a , written $d_a \rightsquigarrow d_c$, iff

$$(\forall a \in \llbracket d_a \rrbracket : \exists c \in \llbracket d_c \rrbracket : a \rightsquigarrow c) \wedge (\forall c \in \llbracket d_c \rrbracket : \exists a \in \llbracket d_a \rrbracket : a \rightsquigarrow c)$$

Example The diagram PM_{3R} (Fig. 5) is a refinement of diagram PM_3 (in Fig. 3). That is, $\llbracket \text{PM}_3 \rrbracket = \{(p_a, \emptyset)\}$ and $\llbracket \text{PM}_{3R} \rrbracket = \{(p_c, \mathcal{H} \setminus p_c)\}$ such that $p_c \subset p_a$. PM_3 differs from PM_{3R} in that some positive traces in PM_3 and all inconclusive traces have been made negative in PM_{3R} .

Theorem 1 The ignorance property is preserved by refinement, formally:

$$o \uparrow^{(d_a, s)} o' \wedge d_a \rightsquigarrow d_c \Rightarrow o \uparrow^{(d_c, s)} o'$$

The proof of the theorem relies on the fact that the ignorance property is defined in terms of behavior alternatives as opposed to individual traces which is how secure information flow properties are usually defined in the literature [25]. That is, instead of demanding that there exists a trace that fulfills a certain criterion, we demand that there exists a behavior alternative such that *all* its traces fulfill a certain criterion. In Def. 14, this criterion is expressed by the condition $o \downarrow_{b_2, h} o'$ which requires that none of the traces in b_2 that are compatible with observation h must compromise observer o' . The condition $b_2 = o \triangleleft_h b_2$ demands that all traces in b_2 must be compatible with observation h . This ensures that all traces in b_2 hide o' from o . Since *all* (not just some) traces in b_2 must fulfill this requirement, the criterion holds regardless of how b_2 is refined. Note that we do not have to worry about empty behavior alternatives, since no well-defined diagram can yield such a behavior alternative given the syntactic constructs presented in this report.

5 Ignorance Preserving Transformation

The notion of refinement addressed above is a binary relation on specifications that formalizes the process of stepwise development by removal of underspecification. This process depends on human intuition and is in general not subject for automation. A transformation on the other hand, is an executable function that takes a syntactic specification and produces another specification (e.g. a PIM to a PSM). Semantically, we represent a syntactic transformation by a set of functions mapping traces over abstract events to traces over concrete events. The reason why a single syntactic transformation is represented by a set of functions is that transformations often introduce a finer granularity in the sense that one abstract trace may correspond to a set of concrete traces. Alternatively, we could have used a function mapping traces to sets of traces.

5.1 Transformations from a Semantic Perspective

We denote by \mathcal{H}_a a set of traces over an abstract set of events, and by \mathcal{H}_c a set of traces over a concrete set of events. Hence, contrary to earlier, we now have an abstract universe \mathcal{H}_a and a concrete universe \mathcal{H}_c . Transformations are semantically represented by a set of functions

$$F \subseteq \mathcal{H}_a \rightarrow \mathcal{H}_c$$

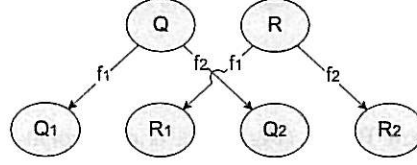


Figure 6: Example of a transformation

that maps abstract traces to concrete traces.

In order to ensure that transformations preserve our security property, we impose two requirements on each function $f \in F$.

First, we require f to be transmitter and receiver preserving in the following sense:

$$f(E_{l_1, l_2} \otimes h_a) = E_{l_1, l_2} \otimes f(h_a) \quad (1)$$

Here h_a is universally quantified over \mathcal{H}_a . Similarly, l_1 and l_2 range over the set of all lifelines. The function E_{l_1, l_2} yields the set of events that can occur on lifeline l_1 whose message is sent from l_1 to l_2 or from l_2 to l_1 .

Condition (1) is just states that an abstract trace whose events all have the same transmitter and receiver, say t and r , must be transformed to a concrete trace whose events also have the same transmitter and receiver t and r . Note that condition (1) may be weakened; e.g. by allowing lifelines to be decomposed into a set of lifelines.

The second condition requires each $f \in F$ to be *homomorphic* wrt. concatenation of traces:

$$f(h_1) \wedge f(h_2) = f(h_1 \wedge h_2) \quad (2)$$

The traces h_1 and h_2 are universally quantified over \mathcal{H}_a . Condition (2) essentially states that f is a function of *events* rather than trace histories.

Note that requirements (1) and (2) ensure that one abstract observation is transformed to one and only one concrete observation. We let \mathcal{F} denote the set of all functions that abide to requirements (1) and (2) above.

5.2 Ignorance Preserving Transformation of Interaction Diagrams

In the following, d_a and d_c are interaction diagrams with traces in \mathcal{H}_a and \mathcal{H}_c , respectively. Moreover, T is a (syntactic) transformation on interaction diagrams whose semantics $\llbracket T \rrbracket \subseteq \mathcal{F}$ is a transformation as defined above.

We first define what it means to translate an interaction obligation. Then we lift this notion of translation to interactions diagrams.

Definition 17 *Interaction obligation* (p_c, n_c) is a translation of *interaction obligation* (p_a, n_a) with respect to function $f \in \mathcal{F}$, written $(p_a, n_a) \xrightarrow{f} (p_c, n_c)$, iff

$$\mathcal{H}_c \setminus n_c = \{f(h_a) \mid h_a \in (\mathcal{H}_a \setminus n_a)\} \wedge p_c = \{f(h) \mid h \in p_a\}$$

The first condition states that each non-negative trace of the concrete interaction obligation must be a translation of a non-negative trace of the abstract obligation. The second condition ensures that a positive trace in the abstract

obligation is transformed to a positive (not inconclusive) trace in the concrete obligation.

Definition 18 *An interaction diagram d_c is a translation of an interaction diagram d_a with respect to transformation T , written, $d_a \hookrightarrow_T d_c$, iff*

$$\llbracket d_c \rrbracket = \{c \mid a \hookrightarrow_f c \wedge a \in \llbracket d_a \rrbracket \wedge f \in \llbracket T \rrbracket\}$$

If $F_1, F_2 \subseteq \mathcal{F}$ then $F_1 \circ F_2$ is understood as the set of functions obtained by functional point-to-point composition of all functions in F_1 and F_2 . That is,

$$F_1 \circ F_2 \stackrel{\text{def}}{=} \{f_1 \circ f_2 \mid f_1 \in F_1 \wedge f_2 \in F_2\}$$

If T_1 and T_2 are syntactic transformations, we write $T_1 \circ T_2$ instead of $\llbracket T_1 \rrbracket \circ \llbracket T_2 \rrbracket$.

Lemma 1 \hookrightarrow is transitive, formally:

$$d_1 \hookrightarrow_{T_1} d_2 \wedge d_2 \hookrightarrow_{T_2} d_3 \Rightarrow d_1 \hookrightarrow_{T_2 \circ T_1} d_3$$

Theorem 2 *The ignorance property is preserved by transformation, formally*

$$o \uparrow^{(d_a, s)} o' \wedge d_a \hookrightarrow_T d_c \Rightarrow o \uparrow^{(d_c, s)} o'$$

The proof relies on two facts. First, all transformations are interpreted by a set of functions where each function by definition abides to conditions (1) and (2) that together ensure that one observation at the abstract level is transformed to one and only one observation at the concrete level. Second, each function that interprets a transformation is by Def. 18 applied to interaction obligations in the manner illustrated in Fig 6. That is, each function translates each interaction obligation in the abstract diagram into a concrete interaction obligation. This ensures that additional granularity introduced at the concrete level is in the form of explicit nondeterminism as opposed to potential nondeterminism because the cardinality of an obligation (i.e. the number of traces in the obligation that may provide potential nondeterminism) is never increased during transformation.

The notions of refinement and transformation may be combined into a more general notion of refinement.

Definition 19 *The interaction obligations o_a and o'_c are in a refinement relation wrt. function $f \in \mathcal{F}$, written $o_a \rightsquigarrow_f o'_c$, iff*

$$\exists o_c \in \mathcal{O} : o_a \hookrightarrow_f o_c \wedge o_c \rightsquigarrow o'_c$$

Definition 20 *Two interaction diagrams d_a and d'_c are in a refinement relation wrt. transformation T , written $d_a \rightsquigarrow_T d'_c$, iff*

$$\llbracket d'_c \rrbracket = \{o'_c \in \mathcal{O} \mid o_a \rightsquigarrow_f o'_c \wedge o_a \in \llbracket d_a \rrbracket \wedge f \in \llbracket T \rrbracket\}$$

Lemma 2 \rightsquigarrow is transitive modulo translation.

$$d_1 \rightsquigarrow_{T_1} d_2 \wedge d_2 \rightsquigarrow_{T_2} d_3 \Rightarrow d_1 \rightsquigarrow_{T_2 \circ T_1} d_3$$

The results on preservation of the ignorance property carry over to the general notion of refinement.

Corollary 1 *The ignorance property is preserved by refinement modulo transformation, formally:*

$$o \uparrow^{(d_a, s)} o' \wedge d_a \rightsquigarrow_T d_c \Rightarrow o \uparrow^{(d_c, s)} o'$$

The proof is straightforward given Theorem 1 and 2.

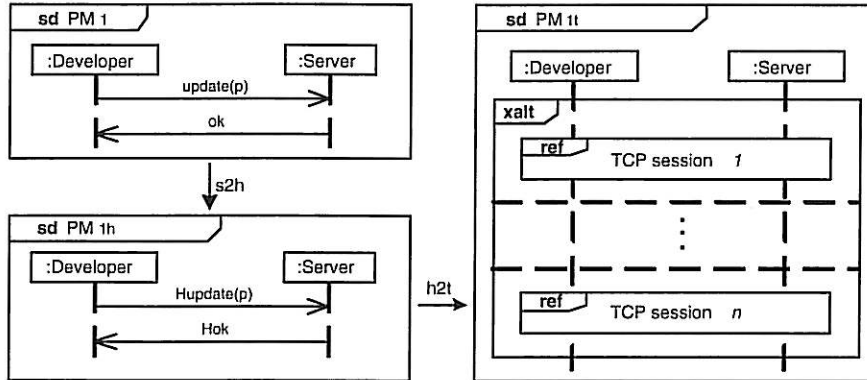


Figure 7: Example transformation

6 Transforming the Project Management System

In Sect. 5 we showed that our notion of transformation preserves the ignorance property. The purpose of this section is to show that our notion of transformation is not so strong that it cannot be used to semantically characterize transformations of practical value.

In our running example, the developers and the researchers of the PM system communicate with the server by SOAP messages. SOAP, however, is typically bound to HTTP which again typically runs over TCP, etc. In the following we demonstrate how the PM system based on SOAP can be transformed into a system based on TCP in such a way that the ignorance property is guaranteed to be preserved even when developers are able to observe TCP messages as opposed to SOAP messages. In MDA terminology, this can be seen as a transformation from a PIM to a PSM, where the platform is understood as a protocol.

6.1 SOAP to TCP

In practice, a transformation from SOAP to TCP typically works as follows: First each SOAP request and response message is encapsulated by a HTTP request and response header, respectively, then a new so-called TCP session is created for each HTTP request-response pair. That is, for each HTTP request-response pair, (1) a connection is established between the two sides of communication, (2) both the request and response are segmented, encapsulated by TCP frames and transmitted, (3) finally the connection is explicitly terminated. The TCP protocol must, among other things, handle message overtaking and message disappearance. Hence, a HTTP request-response pair may be translated into one of several potential TCP session alternatives.

Let $d_s \xrightarrow{s2h} d_h \xrightarrow{h2t} d_t$ be two transformations that translate diagrams at the SOAP level to diagrams at the HTTP and TCP levels. $s2h$ may be interpreted by the set $\llbracket s2h \rrbracket$ consisting of one bijective function that substitutes the signal si of each event in d_s with a signal that represents its encapsulated HTTP equivalent, e.g. by concatenating the letter “H” and si . $h2t$ can be interpreted

as the set $\llbracket h2t \rrbracket$ such that each function in $\llbracket h2t \rrbracket$ translates a HTTP request message and a HTTP response message into a TCP session alternative. The fact that the functions in $\llbracket s2h \rrbracket$ and $\llbracket h2t \rrbracket$ satisfy requirements (1) and (2) is almost immediate. (1) is satisfied because the transformation preserves the transmitters and receivers of events. (2) is satisfied because $s2h$ and $h2t$ can be expressed as a function of single events (as opposed to traces). Since translations are transitive by Lemma 2, $d_s \hookrightarrow_{s2h} d_h \hookrightarrow_{h2t} d_t$ is ignorance preserving by Theorem 2.

Example The two left most diagrams in Fig. 7 illustrate the case where $s2h$ is applied to diagram PM_1 (also specified in Fig. 2). Here, the SOAP level message names $update(p)$ and ok have been transformed to the HTTP level message names $Hupdate(p)$ and Hok in diagram PM_{1h} . Diagrams PM_{1h} and PM_{1t} illustrate the application of $h2t$ where the HTTP request and response messages are translated in a number of different TCP session alternatives. These alternatives are composed by the **xalt**, since by our definition of transformation, each function in $\llbracket h2t \rrbracket$ yields an explicit nondeterministic alternative.

7 Related Work

The work that is most related to ours is the work of Jürjens, largely summarized in the book [21]. Not only does he give a formal semantics for a fragment of UML, but he also identifies conditions under which a secure information flow property (among other properties) is preserved under refinement. There are appreciable differences between our work and his. First, he formalizes a fragment of UML in which only a simplified version of sequence diagrams of UML 1.5 is included. In other words, while his work is based on formalized parts of UML that we do not consider, our work is based on STAIRS which provides a more in depth formalization of UML 2.0 interactions than what is considered in Jürjens work. Second, his semantics is based on so-called UML machines which are a kind of state machines. The semantics of STAIRS is not based on state machines. Third, while Jürjens distinguishes between potential and explicit nondeterminism in order to define a refinement preserving property of confidentiality and integrity, he does not rely on this distinction in the definition of his information flow property. That is, a secure information flow property for UML machines is formalized such that each behavior refinement to a *deterministic* UML machine satisfies this property, i.e. he effectively closes the property under refinement without considering explicit nondeterminism. He himself notes this is “rather strong” and it is in fact exactly this kind of definition that we have been trying to avoid. Our experience suggests that information flow properties closed under refinement tend to be too strong for practical use if the distinction between potential and explicit nondeterminism is not taken into consideration. For example, if we had based our running example on a definition of ignorance similar to that which is given in [20] (where explicit nondeterminism is not considered), and then closed this property under refinement without considering explicit nondeterminism, then the property would have been so strong that the example would not have made any sense. Finally, Jürjens does not address MDA or transformations.

Other efforts that are comparable to ours on a general level in that they

address security in an MDA-setting and/or model-based security, can be roughly classified into (1) access control related works: [2, 4, 7, 8, 9, 18, 22, 24, 28, 29], (2) secure database development [10], and (3) specification of high-level security requirements [1, 6, 11, 31]. Although all these works are comparable to ours on a general level, they are in fact quite different in that they do not deal with a formalized notion of refinement, nor do they address information flow security.

There are a number of more theoretical papers related to information flow security and refinement. The work of Jacob [20] is related to ours in that our formalization of *ignorance* is based on Jacob's formalization of this property. Jacob's property is, however, not preserved under refinement. In fact, he was the first that we are aware of to show that the derivation of secure systems from security specifications can be practically infeasible. This became known as the *refinement paradox*. It has later been observed that this "paradox" is a manifestation of failing to clearly distinguish between potential and explicit nondeterminism. As far as we are aware of, this observation was first made in [30].

Two notable recent theoretic works that define refinement operators that preserve information flow properties under refinement are [5, 26]. Both papers investigate a number of information flow properties, but their approaches differ from ours. Specifically, [5] presents sufficient conditions with which to check that a given refinement (defined in terms of simulation in a processes calculus) preserves information flow properties. Similarly, [26] presents refinement operators that can be used to check or modify refinements such that security is preserved. Both papers differ from ours in that we address secure information flow preservation in a formalism that distinguishes between potential and explicit nondeterminism. By taking this distinction into consideration in our definition of refinement and the ignorance property, we show that *all* refinements preserve ignorance, thus there is no need to check each specific refinement.

The work of Heisel. et. al. [17] is similar to ours in that they distinguish between potential and explicit nondeterminism. The main differences between their work and ours are: (1) They work in a probabilistic setting in a different formalism than ours and they do not consider UML interaction diagrams. (2) They consider a notion of confidentiality based on low-level indistinguishability. This notion of security is strictly speaking not a secure information flow *property*. (3) Their approach is different from ours in that they build the condition of security preservation into their notion of confidentiality preserving refinement. Wrt. refinement, we take the dual approach of strengthening the notion of security, i.e. by strengthening the ignorance property wrt. its original proposal [20].

8 Conclusions and Future Work

On a general level, we have argued that by integrating security into MDA, security documentation can be specified and analyzed at a high level of abstraction during early phases of system development, thereby reducing the need of ad hoc integration of security mechanisms at the level of implementation. Moreover, analysis is more feasible at high levels of abstraction than at levels closer to implementation because specifications at these levels may be too detailed to make analysis practical.

Our main objective has been to contribute towards a formal foundation for this by presenting an approach where high-level UML inspired interaction diagrams can be analyzed with respect to a secure information flow property, and transformed, if desired, down to the level of sequences of bits in such a way that the property is preserved.

Our approach is based on STAIRS [16], and the contributions of this report are specifically (1) the formalization of the ignorance property (Sect. 3), (2) showing that the ignorance property is preserved by the STAIRS notion of refinement (Sect. 4), and (3) the formalization of ignorance preserving transformations and the general notion of refinement (Sect. 5). We are not aware of any other work that exploits the distinction of potential and explicit nondeterminism in order to define a notion refinement in such a way that all refinements preserve a secure information flow property. Moreover, we are not aware of any work that addresses secure information flow and a notion of transformation.

A natural direction of future work is to generalize our results to other information flow properties. We are also planning to consider syntactic transformations, and to implement a computerized tool that will automate the security analysis of transformations.

Acknowledgments

The research in this report has been funded by the Research Council of Norway through the project SECURIS (152839/220). We would like to thank Fabio Massacci and Jan Øyvind Aagedal for useful comments and suggestions.

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A Glossary of Symbols

A.1 Set notation

Set*		Meaning
$\mathbb{P}(A)$	$\stackrel{\text{def}}{=} \{X \mid X \subseteq A\}$	The power set of A .
A^ω	$\stackrel{\text{def}}{=}$	The set of all finite and infinite sequences over the set A .
$l \in \mathcal{L}$	$\stackrel{\text{def}}{=}$	The set of all lifelines.
$si \in \mathcal{SI}$	$\stackrel{\text{def}}{=}$	The set of all signals.
$m \in \mathcal{M}$	$\stackrel{\text{def}}{=} \mathcal{L} \times \mathcal{L} \times \mathcal{SI}$	The set of all messages.
\mathcal{K}	$\stackrel{\text{def}}{=} \{!, ?\}$	The set of all kinds.
$e \in \mathcal{E}$	$\stackrel{\text{def}}{=} \mathcal{K} \times \mathcal{M}$	The set of all events.
$h \in \mathcal{H}$	$\stackrel{\text{def}}{=} \mathcal{E}^\omega$	The trace universe.
p, n, b	$\subseteq \mathcal{H}$	A set of positive, negative, and behavior traces, respectively.
\mathcal{H}_a	$\subseteq \mathcal{H}$	The abstract trace universe.
\mathcal{H}_c	$\subseteq \mathcal{H}$	The concrete trace universe.
$f \in \mathcal{F}$	$\subseteq \mathcal{H}_a \rightarrow \mathcal{H}_c$	The set of functions that conform to (2) and (1) Sect. 5.1.
$a, c \in \mathcal{O}$	$\stackrel{\text{def}}{=} \mathbb{P}(\mathcal{H}) \times \mathbb{P}(\mathcal{H})$	The set of all interaction obligations.
$o, o', s \in \mathcal{S}$	$\stackrel{\text{def}}{=} \mathbb{P}(\mathcal{L})$	The set of all systems.

* By the notation $a \in A$ we understand that A is ranged over by a .

A.2 Traces

Notation	Meaning	Example
$\langle \rangle$	The empty trace	
$\langle !a, ?a \rangle$	The trace with two events	$!a$ then $?a$
$\#h$	The length of trace h	$\#\langle !a, ?a \rangle = 2$
$A \odot h$	The trace h restricted to set A	$\{\langle !a, ?a \rangle \odot \langle !a, !b, ?a, ?b \rangle = \langle !a, ?a \rangle$
$h_1 \sqsubseteq h_2$	h_1 is prefix or equal to h_2	$\langle !a, !b \rangle \sqsubseteq \langle !a, !b, ?a \rangle$ yields true $\langle !a, ?a \rangle \sqsubseteq \langle !a, !b, ?a \rangle$ yields false
$h[n]$	The event in trace h at index n	$\langle !a, ?a, !b, ?b \rangle[2] = ?a$
$h_1 \frown h_2$	Concatination of h_1 and h_2	$\langle !a, ?a \rangle \frown \langle !b, ?b \rangle$ yields $\langle !a, ?a, !b, ?b \rangle$

A.3 Auxiliar Functions

Function	$\mathsf{K}._ \in \mathcal{E} \rightarrow \mathcal{K}$
Def.	$\mathsf{K}.e \stackrel{\text{def}}{=} !$ if $e \in \{!\} \times \mathcal{M}$. ? otherwise.
Meaning	The kind (output or input) of event e .
Function	$\mathsf{TR}._ \in \mathcal{E} \rightarrow \mathcal{L}$
Def.	$\mathsf{TR}.e \stackrel{\text{def}}{=} t$, where $e \in \{t\} \times \mathcal{L} \times \mathcal{ST}$
Meaning	The transmitter of event e .
Function	$\mathsf{RE}._ \in \mathcal{E} \rightarrow \mathcal{L}$
Def.	$\mathsf{RE}.e \stackrel{\text{def}}{=} r$, where $e \in \mathcal{L} \times \{r\} \times \mathcal{ST}$
Meaning	The transmitter of event e .
Function	$\mathsf{E}._ \in \mathcal{S} \rightarrow \mathbb{P}(\mathcal{E})$
Def.	$\mathsf{E}.s \stackrel{\text{def}}{=} \{e \in \mathcal{E} \mid (\mathsf{K}.e = ! \wedge \mathsf{TR}.e \in s) \vee (\mathsf{K}.e = ? \wedge \mathsf{RE}.e \in s)\}$
Meaning	The set of events that can occur on the lifelines of system s .
Function	$\mathsf{RT}._ \in \mathcal{S} \rightarrow \mathbb{P}(\mathcal{E})$
Def.	$\mathsf{RT}.s \stackrel{\text{def}}{=} \{e \in \mathcal{E} \mid \mathsf{TR}.e \in s \vee \mathsf{RE}.e \in s\}$
Meaning	The set of events whose message can be sent to or from the lifelines in s .
Function	$\mathsf{E}_{-, -} \in \mathcal{L} \times \mathcal{L} \rightarrow \mathbb{P}(\mathcal{E})$
Def.	$\mathsf{E}_{l_1, l_2} \stackrel{\text{def}}{=} \mathsf{E}.\{l_1\} \cap \mathsf{RT}.\{l_2\}$
Meaning	The set of events on lifeline l_1 whose message can be sent from l_1 to l_2 or from l_2 to l_1 .

B Proofs

The proofs in this section are structured in a way similar to that of [23], where hierarchical structure is used to demonstrate how each proof step is justified.

B.1 Theorem 1: Information flow property preserving behavior refinement

Theorem 1: *The ignorance property is preserved by refinement.*

ASSUME: 1. $o \wr^{(d_a, s)} o'$

2. $d_a \rightsquigarrow d_c$

PROVE: $o \wr^{(d_c, s)} o'$

(1)1. ASSUME: 1.1 $h_o \in o \triangleright (d_c, s)$

1.2 $\beta_1 \in \nabla(d_c, s)$

1.3 $\neg(o \wr_{\beta_1, h_o} o')$

PROVE: $\exists \beta_2 \in \nabla(d_c, s) : o \wr_{\beta_2, h_o} o' \wedge \beta_2 = o \triangleleft_{h_o} \beta_1$

(2)1. Choose $\alpha_2 \in \nabla(d_a, s)$ such that $o \wr_{\alpha_2, h_o} o'$ and $\alpha_2 = o \triangleleft_{h_o} \alpha_1$

(3)1. Choose $a_1 \in \llbracket d_a \rrbracket$ and $c_1 \in \llbracket d_c \rrbracket$ such that $a_1 \rightsquigarrow c_1$ and $\beta_1 = \nabla(c_1, s)$

PROOF: By assumption 1.2, assumption 2 and definition of \rightsquigarrow (Def. 16) and ∇ (Def. 9).

(3)2. Choose $\alpha_1 \in \nabla(d_a, s)$ such that $\alpha_1 = \nabla(a_1, s)$

PROOF: By (3)1 and definition of ∇ (Def. 9).

(3)3. $\neg(o \wr_{\alpha_1, h_o} o')$

- ⟨4⟩1. Choose $h \in \beta_1$ such that $\text{RT}.o \otimes h = h_o$ and $\text{RT}.o' \otimes h \neq \langle \rangle$
 PROOF: By assumption 1.3 and definition of \wr (Def. 13).
 ⟨4⟩2. $h \in \alpha_1$
 PROOF: By ⟨3⟩1, ⟨3⟩2, ⟨4⟩1, and definition of \rightsquigarrow (Def. 15).
 ⟨4⟩3. Q.E.D.
 PROOF: By ⟨4⟩1, ⟨4⟩2, and definition of \wr (Def. 13).
 ⟨3⟩4. Q.E.D.
 PROOF: By ⟨3⟩3, assumption 1 and definition of \wr (Def. 14).
 ⟨2⟩2. Choose $a_2 \in \llbracket d_a \rrbracket$ and $c_2 \in \llbracket d_c \rrbracket$ such that $a_2 \rightsquigarrow c_2$ and $\alpha_2 = \nabla(a_2, s)$
 PROOF: By ⟨2⟩1 and assumption 2, and definition of ∇ (Def. 9).
 ⟨2⟩3. Choose $\beta'_2 \in \nabla(d_c, s)$ such that $\beta'_2 = \nabla(c_2, s)$
 PROOF: By ⟨2⟩2 and definition of ∇ (Def. 9).
 ⟨2⟩4. $o \wr_{\beta'_2, h_o} o'$
 ASSUME: (a) $\neg(o \wr_{\beta'_2, h_o} o')$
 PROVE: False
 ⟨3⟩1. $\neg(o \wr_{\alpha_2, h_o} o')$
 ⟨4⟩1. $\exists h' \in o \triangleleft_{h_o} \alpha_2 : \text{RT}.o' \otimes h' \neq \langle \rangle$
 ⟨5⟩1. Choose $h \in o \triangleleft_{h_o} \beta'_2$ such that $\text{RT}.o' \otimes h \neq \langle \rangle$
 PROOF: By assumption (a) and definition of \wr (Def. 13).
 ⟨5⟩2. $h \in o \triangleleft_{h_o} \alpha_2$
 PROOF: By ⟨2⟩2, ⟨2⟩3, ⟨5⟩1 and definition of \rightsquigarrow (Def. 15).
 ⟨5⟩3. Q.E.D.
 PROOF: Select h' such that $h' = h$, then ⟨4⟩1 holds by ⟨5⟩1 and ⟨5⟩2.
 ⟨4⟩2. Q.E.D.
 PROOF: By ⟨4⟩1 and definition of \wr (Def. 13).
 ⟨3⟩2. Q.E.D.
 PROOF: ⟨3⟩1 contradicts ⟨2⟩1, hence assumption (a) is false and ⟨2⟩4 holds.
 ⟨2⟩5. $\beta'_2 = o \triangleleft_{h_o} \beta'_2$
 β'_2 is a subset of α_2 by ⟨2⟩2, definition of \rightsquigarrow (Def. 15), and definition of ∇ (Def. 8), hence ⟨2⟩5 must hold by ⟨2⟩1 and definition of \triangleleft (Def. 12).
 ⟨2⟩6. Q.E.D.
 PROOF: Select β_2 such that $\beta_2 = \beta'_2$, then ⟨1⟩1 holds by ⟨2⟩4 and ⟨2⟩5.
 ⟨1⟩2. Q.E.D.
 PROOF: By ⟨1⟩1 and definition of \wr (Def. 14).

B.2 Lemma 1: Transformation is transitive

Lemma 1: \hookrightarrow is transitive

ASSUME: 1. $d \hookrightarrow_{T_1} d_{T_1}$

2. $d_{T_1} \hookrightarrow_{T_2} d_{T_2}$

PROVE: $d \hookrightarrow_{T_2 \circ T_1} d_{T_2}$

- ⟨1⟩1. $\llbracket d_{T_2} \rrbracket = \{o_{T_2} \in \mathcal{O} \mid o \hookrightarrow_f o_{T_2} \wedge o \in \llbracket d \rrbracket \wedge f \in \llbracket T_2 \circ T_1 \rrbracket\}$
 ⟨2⟩1. $\exists (p_{T_2}, n_{T_2}) \in \llbracket d_{T_2} \rrbracket : (p, n) \hookrightarrow_f (p_{T_2}, n_{T_2})$ for arbitrary $(p, n) \in \llbracket d \rrbracket$
 and $f \in \llbracket T_2 \circ T_1 \rrbracket$
 ⟨3⟩1. Choose $f_1 \in \llbracket T_1 \rrbracket$ and $f_2 \in \llbracket T_2 \rrbracket$ such that $f = f_2 \circ f_1$
 PROOF: By definition of \circ .
 ⟨3⟩2. Choose $(p_{T_1}, n_{T_1}) \in \llbracket d_{T_1} \rrbracket$ such that $(p, n) \hookrightarrow_{f_1} (p_{T_1}, n_{T_1})$
 PROOF: By assumption 1 and definition of \hookrightarrow (Def. 17).

- ⟨3⟩3. Choose $(p_{T_2}, n_{T_2}) \in \llbracket d_{T_2} \rrbracket$ such that $(p_{T_1}, n_{T_1}) \hookrightarrow_{f_2} (p_{T_2}, n_{T_2})$
 PROOF: By assumption 2 and definition of \hookrightarrow (Def. 17).
- ⟨3⟩4. $(p, n) \hookrightarrow_{f_2 \circ f_1} (p_{T_2}, n_{T_2})$
 PROOF: By ⟨3⟩2, ⟨3⟩3, and Lemma 1.1 (Sect. B.2.1).
- ⟨3⟩5. Q.E.D.
 PROOF: By ⟨3⟩1 and ⟨3⟩4.
- ⟨2⟩2. $\exists (p, n) \in \llbracket d \rrbracket : \exists f \in \llbracket T_2 \circ T_1 \rrbracket : (p, n) \hookrightarrow_f (p_{T_2}, n_{T_2})$ for arbitrary
 $(p_{T_2}, n_{T_2}) \in \llbracket d_{T_2} \rrbracket$
- ⟨3⟩1. Choose $f_2 \in \llbracket T_2 \rrbracket$ and $(p_{T_1}, n_{T_1}) \in \llbracket d_{T_1} \rrbracket$ such that $(p_{T_1}, n_{T_1}) \hookrightarrow_{f_2}$
 (p_{T_2}, n_{T_2})
 PROOF: By assumption 2 and definition of \hookrightarrow (Def. 17).
- ⟨3⟩2. Choose $f_1 \in \llbracket T_1 \rrbracket$ and $(p, n) \in \llbracket d \rrbracket$ such that $(p, n) \hookrightarrow_{f_1} (p_{T_1}, n_{T_1})$
 PROOF: By assumption 1 and definition of \hookrightarrow (Def. 17).
- ⟨3⟩3. $(p, n) \hookrightarrow_{f_2 \circ f_1} (p_{T_2}, n_{T_2})$
 PROOF: By ⟨3⟩1, ⟨3⟩2, and Lemma 1.1 (Sect. B.2.1).
- ⟨3⟩4. $f_2 \circ f_1 \in \llbracket T_2 \circ T_1 \rrbracket$
 PROOF: By ⟨3⟩1, ⟨3⟩2, and definition of \circ .
- ⟨3⟩5. Q.E.D.
 PROOF: By ⟨3⟩3 and ⟨3⟩4.
- ⟨2⟩3. Q.E.D.
 PROOF: By ⟨2⟩1 and ⟨2⟩2.
- ⟨1⟩2. Q.E.D.
 PROOF: By definition of \hookrightarrow (Def. 17).

B.2.1 Lemma 1.1

ASSUME: 1. $(p, n) \hookrightarrow_{f_1} (p_{T_1}, n_{T_1})$

2. $(p_{T_1}, n_{T_1}) \hookrightarrow_{f_2} (p_{T_2}, n_{T_2})$

PROVE: $(p, n) \hookrightarrow_{f_2 \circ f_1} (p_{T_2}, n_{T_2})$

LET: $f \triangleq f_2 \circ f_1$

- ⟨1⟩1. $\mathcal{H}_{T_2} \setminus n_{T_2} = \{f(h) \mid h \in \mathcal{H} \setminus n\} \wedge p_{T_2} = \{f(h) \mid h \in p\}$
- ⟨2⟩1. $\forall h \in \mathcal{H} \setminus n : \exists h_{T_2} \in \mathcal{H}_{T_2} \setminus n_{T_2} : h_{T_2} = f(h)$
- ⟨3⟩1. $\exists h_{T_2} \in \mathcal{H}_{T_2} \setminus n_{T_2} : h_{T_2} = f(h)$ for arbitrary $h \in \mathcal{H} \setminus n$
- ⟨4⟩1. Choose $h_{T_1} \in \mathcal{H}_{T_1} \setminus n_{T_1}$ such that $h_{T_1} = f_1(h)$
 PROOF: By assumption 1 and definition of \hookrightarrow (Def. 17).
- ⟨4⟩2. Choose $h_{T_2} \in \mathcal{H}_{T_2} \setminus n_{T_2}$ such that $h_{T_2} = f_2(h_{T_1})$
 PROOF: By assumption 2 and definition of \hookrightarrow (Def. 17).
- ⟨4⟩3. Q.E.D.
 PROOF: By ⟨4⟩1 and ⟨4⟩2 and definition of f .
- ⟨3⟩2. Q.E.D.
 PROOF: By ⟨3⟩1.
- ⟨2⟩2. $\forall h_{T_2} \in \mathcal{H}_{T_2} \setminus n_{T_2} : \exists h \in \mathcal{H} \setminus n : h_{T_2} = f(h)$
- ⟨3⟩1. $\exists h \in \mathcal{H} \setminus n : h_{T_2} = f(h)$ for arbitrary $h_{T_2} \in \mathcal{H}_{T_2} \setminus n_{T_2}$
- ⟨4⟩1. Choose $h_{T_1} \in \mathcal{H}_{T_1} \setminus n_{T_1}$ such that $h_{T_2} = f_2(h_{T_1})$
 PROOF: By assumption 2 and definition of \hookrightarrow (Def. 17).
- ⟨4⟩2. Choose $h \in \mathcal{H} \setminus n$ such that $h_{T_1} = f_1(h)$
 PROOF: By assumption 1 and definition of \hookrightarrow (Def. 17).
- ⟨4⟩3. Q.E.D.
 PROOF: By ⟨4⟩1 and ⟨4⟩2 and definition of f .

- ⟨3⟩2. Q.E.D.
 PROOF: By ⟨3⟩1.
 ⟨2⟩3. $\forall h \in p : \exists h_{T_2} \in p_{T_2} : h_{T_2} = f(h)$
 ⟨3⟩1. $\exists h_{T_2} \in p_{T_2} : h_{T_2} = f(h)$ for arbitrary $h \in p$
 ⟨4⟩1. Choose $h_{T_1} \in p_{T_1}$ such that $h_{T_1} = f_1(h)$
 PROOF: By assumption 1 and definition of \hookrightarrow (Def. 17).
 ⟨4⟩2. Choose $h_{T_2} \in p_{T_2}$ such that $h_{T_2} = f_2(h_{T_1})$
 PROOF: By assumption 2 and definition of \hookrightarrow (Def. 17).
 ⟨4⟩3. Q.E.D.
 PROOF: By ⟨4⟩1 and ⟨4⟩2 and definition of f .
 ⟨3⟩2. Q.E.D.
 PROOF: By ⟨3⟩1.
 ⟨2⟩4. $\forall h_{T_2} \in p_{T_2} : \exists h \in p : h_{T_2} = f(h)$
 ⟨3⟩1. $\exists h \in p : h_{T_2} = f(h)$ for arbitrary $h_{T_2} \in p_{T_2}$
 ⟨4⟩1. Choose $h_{T_1} \in p_{T_1}$ such that $h_{T_2} = f_2(h_{T_1})$
 PROOF: By assumption 2 and definition of \hookrightarrow (Def. 17).
 ⟨4⟩2. Choose $h \in p$ such that $h_{T_1} = f_1(h)$
 PROOF: By assumption 1 and definition of \hookrightarrow (Def. 17).
 ⟨4⟩3. Q.E.D.
 PROOF: By ⟨4⟩1 and ⟨4⟩2 and definition of f .
 ⟨3⟩2. Q.E.D.
 PROOF: By ⟨3⟩1.
 ⟨2⟩5. Q.E.D.
 PROOF: By ⟨2⟩1 - ⟨2⟩4.
 ⟨1⟩2. Q.E.D.
 PROOF: By definition of \hookrightarrow (Def. 17).

B.3 Lemma 2: Refinement is transitive modulo translation

Lemma 2: \rightsquigarrow is transitive modulo translation.

ASSUME: 1. $d \rightsquigarrow_{T_1} d'_{T_1}$

2. $d'_{T_1} \rightsquigarrow_{T_2} d''_{T_2}$

PROVE: $d \rightsquigarrow_{T_2 \circ T_1} d''_{T_2}$

⟨1⟩1. $\exists d_{T_2} \in \mathcal{D} : d \hookrightarrow_{T_2 \circ T_1} d_{T_2} \wedge d_{T_2} \rightsquigarrow d''_{T_2}$

⟨2⟩1. Choose $d_{T_1} \in \mathcal{D}$ such that

(a) $d \hookrightarrow_{T_1} d_{T_1}$ and

(b) $d_{T_1} \rightsquigarrow d'_{T_1}$

PROOF: By assumption 1 and definition of \rightsquigarrow (Def. 20).

⟨2⟩2. Choose $d'_{T_2} \in \mathcal{D}$ such that

(a) $d'_{T_1} \hookrightarrow_{T_2} d'_{T_2}$ and

(b) $d'_{T_2} \rightsquigarrow d''_{T_2}$

PROOF: By assumption 2 and definition of \rightsquigarrow (Def. 20).

⟨2⟩3. Choose $d_{T_2} \in \mathcal{D}$ such that

(a) $d_{T_1} \hookrightarrow_{T_2} d_{T_2}$ and

(b) $d_{T_2} \rightsquigarrow d'_{T_2}$

PROOF: By ⟨2⟩1 (b), ⟨2⟩2 (a), and Lemma 2.1 (Sect. B.3.1).

⟨2⟩4. $d \hookrightarrow_{T_2 \circ T_1} d_{T_2}$

PROOF: By ⟨2⟩1 (a), ⟨2⟩3 (a), and Lemma 1 (Sect. B.2).

(2)5. $d_{T_2} \rightsquigarrow d''_{T_2}$

PROOF: By (2)3 (b), (2)2 (b), and transitivity of \rightsquigarrow (proved in [14]).

(2)6. Q.E.D.

PROOF: By (2)4 and (2)5.

(1)2. Q.E.D.

PROOF: By definition of \rightsquigarrow (Def. 20).

B.3.1 Lemma 2.1

ASSUME: 1. $d \rightsquigarrow d'$

2. $d' \hookrightarrow_T d'_T$

PROVE: $d \rightsquigarrow_T d'_T$

(1)1. $\llbracket d'_T \rrbracket = \{o'_T \in \mathcal{O} \mid o \rightsquigarrow_f o'_T \wedge o \in \llbracket d \rrbracket \wedge f \in \llbracket T \rrbracket\}$

(2)1. $\forall f \in \llbracket T \rrbracket : \forall o \in \llbracket d \rrbracket : \exists o'_T \in \llbracket d'_T \rrbracket : o \rightsquigarrow_f o'_T$

(3)1. $\exists o'_T \in \llbracket d'_T \rrbracket : o \rightsquigarrow_f o'_T$ for arbitrary $f \in \llbracket T \rrbracket$ and $o \in \llbracket d \rrbracket$

(4)1. Choose $o' \in \llbracket d' \rrbracket$ such that $o \rightsquigarrow o'$

PROOF: By assumption 1 and definition of \rightsquigarrow (Def. 15).

(4)2. Choose $o'_T \in \llbracket d'_T \rrbracket$ such that $o' \hookrightarrow_f o'_T$

PROOF: By assumption 2 and definition of \hookrightarrow (Def. 17).

(4)3. $o \rightsquigarrow_f o'_T$

PROOF: By (4)1, (4)2, and Lemma 2.2 (Sect. B.3.2).

(4)4. Q.E.D.

PROOF: By (4)3.

(3)2. Q.E.D.

PROOF: By \forall -rule.

(2)2. $\forall o'_T \in \llbracket d'_T \rrbracket : \exists f \in \llbracket T \rrbracket : \exists o \in \llbracket d \rrbracket : o \rightsquigarrow_f o'_T$

(3)1. $\exists f \in \llbracket T \rrbracket : \exists o \in \llbracket d \rrbracket : o \rightsquigarrow_f o'_T$ for arbitrary $o'_T \in \llbracket d'_T \rrbracket$

(4)1. Choose $o' \in \llbracket d' \rrbracket$ and $f \in \llbracket T \rrbracket$ such that $o' \hookrightarrow_f o'_T$

PROOF: By assumption 2 and definition of \hookrightarrow (Def. 17).

(4)2. Choose $o \in \llbracket d \rrbracket$ such that $o \rightsquigarrow o'$

PROOF: By assumption 1 and definition of \rightsquigarrow (Def. 15).

(4)3. $o \rightsquigarrow_f o'_T$

PROOF: By (4)2, (4)1, and Lemma 2.2 (Sect. B.3.2).

(4)4. Q.E.D.

PROOF: By (4)3.

(3)2. Q.E.D.

PROOF: By \forall -rule.

(2)3. Q.E.D.

PROOF: Trivial.

(1)2. Q.E.D.

PROOF: By (1)1 and definition of \rightsquigarrow (Def. 20).

B.3.2 Lemma 2.2

ASSUME: 1. $(p_a, n_a) \rightsquigarrow (p'_a, n'_a)$

2. $(p'_a, n'_a) \hookrightarrow_f (p'_c, n'_c)$

PROVE: $(p_a, n_a) \rightsquigarrow_f (p'_c, n'_c)$

(1)1. $\exists (p_c, n_c) \in \mathcal{O} : (p_a, n_a) \hookrightarrow_f (p_c, n_c) \wedge (p_c, n_c) \rightsquigarrow (p'_c, n'_c)$

- ⟨2⟩1. $\exists(p_c, n_c) \in \mathcal{O}$:
 $\mathcal{H}_c \setminus n_c = \{f(h) \mid h \in (\mathcal{H}_a \setminus n_a)\} \wedge$
 $p_c = \{f(h) \mid h \in p_a\} \wedge$
 $n_c \subseteq n'_c \wedge p_c \subseteq p'_c \cup n'_c$
- ⟨3⟩1. Choose $(p_c, n_c) \in \mathcal{O}$ such that $\mathcal{H}_c \setminus n_c = \{f(h) \mid h \in (\mathcal{H}_a \setminus n_a)\}$ and
 $p_c = \{f(h) \mid h \in p_a\}$
 PROOF: By definition of \mathcal{O} .
- ⟨3⟩2. $n_c \subseteq n'_c$
 ⟨4⟩1. $\mathcal{H}_a \setminus n'_a \subseteq \mathcal{H}_a \setminus n_a$
 PROOF: $n_a \subseteq n'_a$ by assumption 1 and definition of \rightsquigarrow (Def 15), hence
 ⟨4⟩1 must hold by definition of \mathcal{H}_a .
 ⟨4⟩2. $\{f(h') \mid h' \in (\mathcal{H}_a \setminus n'_a)\} \subseteq \{f(h) \mid h \in (\mathcal{H}_a \setminus n_a)\}$
 PROOF: By ⟨4⟩1 since f is a function and hence deterministic.
 ⟨4⟩3. $\mathcal{H}_c \setminus n'_c \subseteq \mathcal{H}_c \setminus n_c$
 PROOF: By ⟨4⟩2, assumption 2, ⟨3⟩1, and definition of \hookrightarrow (Def. 17).
 ⟨4⟩4. Q.E.D.
 PROOF: By ⟨4⟩3 and definition of \mathcal{H}_c .
- ⟨3⟩3. $p_c \subseteq p'_c \cup n'_c$
 ASSUME: 1.1 $p_c \not\subseteq p'_c \cup n'_c$
 PROVE: False
 ⟨4⟩1. $\exists h \in p_a : h \notin (p'_a \cup n'_a)$
 ⟨5⟩1. Choose $h_c \in p_c$ such that $h_c \notin (p'_c \cup n'_c)$
 PROOF: By assumption 1.1.
 ⟨5⟩2. Choose $h_a \in p_a$ such that $f(h_a) = h_c$
 PROOF: By ⟨5⟩1 and ⟨3⟩1.
 ⟨5⟩3. $h_a \notin p'_a$
 PROOF: By assumption 2, definition of \hookrightarrow (Def. 17), and modus tollens.
 ⟨5⟩4. $h_a \notin n'_c$
 PROOF: By assumption 2, definition of \hookrightarrow (Def. 17), and modus tollens.
 ⟨5⟩5. Q.E.D.
 PROOF: By ⟨5⟩2, ⟨5⟩3, and ⟨5⟩4.
- ⟨4⟩2. Q.E.D.
 PROOF: ⟨4⟩1 contradicts assumption 1 by definition of \rightsquigarrow (Def. 15).
 Thus assumption 1.1. is false, and ⟨3⟩3 must hold.
- ⟨3⟩4. Q.E.D.
 PROOF: By ⟨3⟩1, ⟨3⟩2, and ⟨3⟩3.
- ⟨2⟩2. Q.E.D.
 PROOF: By ⟨2⟩1, definition of \hookrightarrow (Def. 17), and definition of \rightsquigarrow (Def. 15).
- ⟨1⟩2. Q.E.D.
 PROOF: By definition of \rightsquigarrow (Def. 19).

B.4 Theorem 2: Ignorance preserving transformation

Theorem 2: *The ignorance property is preserved by transformation.*

- ASSUME: 1. $\sigma \{^{(d_a, s)} \sigma'$
 2. $d_a \hookrightarrow_T d_c$

PROVE: $o \wr^{(d_c, s)} o'$

(1)1. ASSUME: 1.1 $h_c \in o \triangleright (d_c, s)$

1.2 $c_1 \in \nabla(d_c, s)$

1.3 $\neg(o \wr_{c_1, h_c} o')$

PROVE: $\exists c_2 \in \nabla(d_c, s) : o \wr_{c_2, h_c} o' \wedge c_2 = o \triangleleft_{h_c} c_2$

LET: $\omega \triangleq (\text{RT}.o \cap \text{E}.s)$, and $\omega' \triangleq (\text{RT}.o' \cap \text{E}.s)$

(2)1. Choose $(p_c^1, n_c^1) \in \llbracket d_c \rrbracket$ such that $c_1 = \nabla((p_c^1, n_c^1), s)$

PROOF: By assumption 1.2 and definition of ∇ (Def. 9).

(2)2. Choose $(p_a^1, n_a^1) \in \llbracket d_a \rrbracket$ and $f \in \llbracket T \rrbracket$ such that $(p_a^1, n_a^1) \hookrightarrow_f (p_c^1, n_c^1)$

PROOF: By (2)1, assumption 2, and definition of \hookrightarrow (Def 18).

(2)3. Choose $\beta_1 \in \mathcal{H}_c \setminus n_c^1$ such that $h_c = \omega \otimes \beta_1$ and $\omega' \otimes \beta_1 \neq \langle \rangle$

(3)1. Choose $h_c^b \in o \triangleleft_{h_c} c_1 : \text{RT}.o' \otimes h_c^b \neq \langle \rangle$

PROOF: By assumption 1.3 and definition of \wr (Def. 13).

(3)2. Choose $\beta \in \mathcal{H}_c \setminus n_c^1$ such that $\text{E}.s \otimes \beta = h_c^b$

PROOF: By assumption 1.2, (2)1, (3)1 and definition of \triangleleft (Def. 12).

(3)3. $(\text{E}.s \cap \text{RT}.o) \otimes \beta = h_c$

(4)1. $\text{RT}.o \otimes h_c^b = h_c$

PROOF: By (3)1 and definition of \triangleleft (Def 12).

(4)2. Q.E.D.

PROOF: It is easy to see that $A \otimes (B \otimes h) = (A \cap B) \otimes h$ for arbitrary sets of events A and B and trace h . Thus (3)3 holds by (3)2 and (4)1.

(3)4. $(\text{E}.s \cap \text{RT}.o') \otimes \beta \neq \langle \rangle$

PROOF: By (3)1 and (3)2.

(3)5. Q.E.D.

PROOF: By (3)2, (3)3, (3)4 and definition of ω and ω' .

(2)4. Choose $\alpha_1 \in \mathcal{H}_a \setminus n_a^1$ such that $\beta_1 = f(\alpha_1)$

PROOF: By (2)2, (2)3, and definition of \hookrightarrow (Def. 17).

(2)5. $\omega' \otimes \alpha_1 \neq \langle \rangle$

ASSUME: (a) $\omega' \otimes \alpha_1 = \langle \rangle$

PROVE: False.

(3)1. $\omega' \otimes \beta_1 = \langle \rangle$

(4)1. $\omega' \otimes \beta_1 = f(\omega' \otimes \alpha_1)$

PROOF: By (2)4 and Lemma 2.1 (Sect. B.4.1).

(4)2. Q.E.D.

PROOF: By (4)1, assumption (a), and Lemma 2.3 (Sect. B.4.3).

(3)2. Q.E.D.

PROOF: Assumption (a) cannot hold since (3)1 contradicts (2)3, hence (2)5 must hold.

LET: $h_a \triangleq \omega \otimes \alpha_1$

(2)6. $h_c = f(h_a)$

PROOF: By (2)3, (2)4, definition of h_a and Lemma 2.1 (Sect. B.4.1).

(2)7. $h_a \in o \triangleright (d_a, s)$

PROOF: By definition of \triangleright (Def. 11) and ω .

(2)8. Choose $a_2 \in \nabla(d_a, s)$ such that $o \wr_{a_2, h_a} o' \wedge a_2 = o \triangleleft_{h_a} a_2$

(3)1. Choose $a_1 \in \nabla(d_a, s)$ such that $a_1 = \nabla((p_a^1, n_a^1), s)$

PROOF: By (2)2 and definition of ∇ (Def. 9).

(3)2. $\neg(o \wr_{a_1, h_a} o')$

(4)1. $\exists h \in o \triangleleft_{h_a} a_1 : \text{RT}.o' \otimes h \neq \langle \rangle$

(5)1. $(\text{E}.s \otimes \alpha_1) \in o \triangleleft_{h_a} a_1$

(6)1. $(E.s \otimes \alpha_1) \in a_1$

PROOF: By (2)4, (3)1 and definition of ∇ (Def. 9).

(6)2. $RT.o \otimes (E.s \otimes \alpha_1) = h_a$

PROOF: By definition of h_a .

(6)3. Q.E.D.

PROOF: By (6)1 and (6)2 and definition of \triangleleft (Def. 12).

(5)2. $RT.o' \otimes (E.s \otimes \alpha_1) \neq \langle \rangle$

PROOF: It is easy to see that $A \otimes (B \otimes h) = (A \cap B) \otimes h$ for arbitrary sets of events A and B and trace h . Hence (5)2 holds by (2)5 and definition of ω .

(5)3. Q.E.D.

PROOF: Select h such that $h = E.s \otimes \alpha_1$, then (4)1 holds by (5)1 and (5)2.

(4)2. Q.E.D.

PROOF: By (4)1 and definition of \wr (Def. 13).

(3)3. Q.E.D.

PROOF: By (3)2, (2)7, assumption 1, and definition of \wr (Def. 14).

(2)9. Choose $(p_a^2, n_a^2) \in \llbracket d_a \rrbracket$ such that $a_2 = \nabla((p_a^2, n_a^2), s)$

PROOF: By (2)8 and definition of ∇ (Def. 9).

(2)10. Choose $(p_c^2, n_c^2) \in \llbracket d_c \rrbracket$ such that $(p_a^2, n_a^2) \hookrightarrow_f (p_c^2, n_c^2)$

PROOF: By (2)9, assumption 2, and definition of \hookrightarrow (Def. 18).

(2)11. Choose $c_2 \in \nabla(d_c, s)$ such that $c_2 = \nabla((p_c^2, n_c^2), s)$

By (2)10 and definition of ∇ (Def. 9).

(2)12. $c_2 = o \triangleleft_{h_c} c_2$

ASSUME: (a) $\neg(c_2 = o \triangleleft_{h_c} c_2)$

PROVE: False.

(3)1. $a_2 \neq o \triangleleft_{h_a} a_2$

(4)1. $\exists h \in a_2 : (RT.o \otimes h) \neq h_a$

(5)1. Choose $h_c^b \in c_2$ such that $h_c \neq (RT.o \otimes h_c^b)$

PROOF: By assumption (a) and definition of \triangleleft (Def. 12).

(5)2. Choose $\beta \in \mathcal{H}_c \setminus n_c^2$ such that $E.s \otimes \beta = h_c^b$

PROOF: By (2)11, (5)1, and definition of ∇ (Def. 8).

(5)3. $\omega \otimes \beta \neq h_c$

PROOF: By (5)1, (5)2, and definition of ω and \otimes .

(5)4. Choose $\alpha \in \mathcal{H}_a \setminus n_a^2$ such that $\beta = f(\alpha)$

PROOF: By (2)10, (5)2, and definition of \hookrightarrow (Def. 17).

(5)5. $\omega \otimes \alpha \neq h_a$

(6)1. $f(\omega \otimes \alpha) \neq f(h_a)$

PROOF:

$$\omega \otimes \beta \neq h_c \quad [\text{By (5)3}]$$

$$f(\omega \otimes \alpha) \neq h_c \quad [\text{By (5)4 and Lemma 2.1}]$$

$$f(\omega \otimes \alpha) \neq f(h_a) \quad [\text{By (2)6}]$$

(6)2. Q.E.D.

$f(h_1) \neq f(h_2) \Rightarrow h_1 \neq h_2$ because f is a function.

(5)6. $(E.s \otimes \alpha) \in a_2$

PROOF: By (2)9, (5)4, and definition of ∇ (Def. 8).

(5)7. $RT.o \otimes (E.s \otimes \alpha) \neq h_a$

PROOF: By (5)5 and definition of ω and \otimes .

(5)8. Q.E.D.

- PROOF: Select h such that $h = (E.s \otimes \alpha)$, then $\langle 4 \rangle 1$ holds by $\langle 5 \rangle 6$ and $\langle 5 \rangle 7$.
- $\langle 4 \rangle 2$. Q.E.D.
PROOF: By $\langle 4 \rangle 1$ and definition of \triangleleft (Def. 12).
- $\langle 3 \rangle 2$. Q.E.D.
PROOF: Assumption (a) cannot hold since $\langle 3 \rangle 1$ contradicts $\langle 2 \rangle 8$. Hence $\langle 2 \rangle 12$ holds.
- $\langle 2 \rangle 13$. $o \wr_{c_2, h_c} o'$
- $\langle 3 \rangle 1$. $\forall h \in o \triangleleft_{h_c} c_2 : RT.o' \otimes h = \langle \rangle$
- $\langle 4 \rangle 1$. $RT.o' \otimes h = \langle \rangle$ for arbitrary $h \in o \triangleleft_{h_c} c_2$
- $\langle 5 \rangle 1$. Choose $\beta_2 \in \mathcal{H}_c \setminus n_c^2$ such that $E.s \otimes \beta_2 = h$
PROOF: By $\langle 2 \rangle 11$ and definition of \triangleleft (Def. 12).
- $\langle 5 \rangle 2$. Choose $\alpha_2 \in \mathcal{H}_a \setminus n_a^2$ such that $\beta_2 = f(\alpha_2)$
PROOF: By $\langle 2 \rangle 10$, $\langle 5 \rangle 1$, and definition of \hookrightarrow (Def. 17).
- $\langle 5 \rangle 3$. $\omega' \otimes \beta_2 = \langle \rangle$
- $\langle 6 \rangle 1$. $\omega' \otimes \alpha_2 = \langle \rangle$
PROOF: By $\langle 2 \rangle 8$, $\langle 2 \rangle 9$, $\langle 5 \rangle 2$, and definition of \wr (Def. 13) and ω' .
- $\langle 6 \rangle 2$. Q.E.D.
PROOF: By $\langle 5 \rangle 2$, $\langle 6 \rangle 1$ and Lemma 2.3 (Sect. B.4.3).
- $\langle 5 \rangle 4$. $RT.o' \otimes (E.s \otimes \beta_2) = \langle \rangle$
PROOF: By $\langle 5 \rangle 3$ and definition of ω' and \otimes .
- $\langle 5 \rangle 5$. Q.E.D.
PROOF: By $\langle 5 \rangle 1$, and $\langle 5 \rangle 4$.
- $\langle 4 \rangle 2$. Q.E.D.
PROOF: By $\langle 4 \rangle 1$ and \forall -rule.
- $\langle 3 \rangle 2$. Q.E.D.
PROOF: By $\langle 3 \rangle 1$ and definition of \wr (Def. 13).
- $\langle 2 \rangle 14$. Q.E.D.
PROOF: By $\langle 2 \rangle 11$, $\langle 2 \rangle 12$, and $\langle 2 \rangle 13$.
- $\langle 1 \rangle 2$. Q.E.D.
PROOF: By $\langle 1 \rangle 1$ and definition of \wr (Def. 14).

B.4.1 Lemma 2.1

ASSUME: 1. $\beta = f(\alpha)$ for $f \in \mathcal{F}$

PROVE: $\omega \otimes \beta = f(\omega \otimes \alpha)$ where $\omega \triangleq (RT.o \cap E.s)$ for arbitrary $s, o \in \mathcal{S}$

PROOF SKETCH: By induction over the length of α we show that $(1) \omega \otimes f(\alpha) = f(\omega \otimes \alpha)$ holds for the base case where $\#\alpha = 0$ and that (1) holds for $\#\alpha = n$ (induction hypothesis) implies that (1) holds for $\#\alpha = n + 1$.

$\langle 1 \rangle 1$. $\omega \otimes f(\alpha) = f(\omega \otimes \alpha)$

$\langle 2 \rangle 1$. CASE: $\#\alpha = 0$, i.e. $\alpha = \langle \rangle$ (base case).

PROOF:

$$\begin{aligned}
 \omega \otimes f(\alpha) &= f(\omega \otimes \alpha) \\
 \omega \otimes f(\langle \rangle) &= f(\omega \otimes \langle \rangle) \quad [\alpha = \langle \rangle] \\
 \omega \otimes f(\langle \rangle) &= f(\langle \rangle) \quad [\text{By definition of } \otimes.] \\
 \omega \otimes \langle \rangle &= \langle \rangle \quad [\text{By Lemma 2.3 (Sect. B.4.3).}] \\
 \langle \rangle &= \langle \rangle \quad [\text{By definition of } \otimes.]
 \end{aligned}$$

$\langle 2 \rangle 2$. CASE: $\#\alpha = n + 1$

ASSUME: $\omega \otimes f(\alpha_p) = f(\omega \otimes \alpha_p)$ where $\alpha_p \widehat{\langle e \rangle} = \alpha$ such that $e = \alpha[n + 1]$ (induction hypothesis).

PROOF:

$$\begin{array}{lll}
\omega \otimes f(\langle e \rangle) & = & f(\omega \otimes \langle e \rangle) & [\text{Lemma 2.2}] \\
\omega \otimes f(\alpha_p) \frown \omega \otimes f(\langle e \rangle) & = & f(\omega \otimes \alpha_p) \frown f(\omega \otimes \langle e \rangle) & [\text{Induct. hyp.}] \\
\omega \otimes f(\alpha_p) \frown \omega \otimes f(\langle e \rangle) & = & f((\omega \otimes \alpha_p) \frown (\omega \otimes \langle e \rangle)) & [*] \\
\omega \otimes (f(\alpha_p) \frown f(\langle e \rangle)) & = & f(\omega \otimes (\alpha_p \frown \langle e \rangle)) & [\text{Def. of } \otimes.] \\
\omega \otimes f(\alpha_p \frown \langle e \rangle) & = & f(\omega \otimes (\alpha_p \frown \langle e \rangle)) & [*] \\
\omega \otimes f(\alpha) & = & f(\omega \otimes \alpha) & [\text{Induct. hyp.}]
\end{array}$$

* These steps are valid because $f \in \mathcal{F}$, thus f fulfills requirement (2) (see Sect. 5.1) by definition of \mathcal{F} .

(2)3. Q.E.D.

PROOF: By (2)1 and (2)2.

B.4.2 Lemma 2.2PROVE: $\omega \otimes f(\langle e \rangle) = f(\omega \otimes \langle e \rangle)$ where $\omega \triangleq (\text{RT}.o \cap \text{E}.s)$ for some $o, s \in \mathcal{S}$ LET: $h \triangleq f(\langle e \rangle)$ (1)1. $\omega \otimes f(\langle e \rangle) = f(\omega \otimes \langle e \rangle)$ (2)1. $E_{l_s, l_o} \subseteq \omega$ for all $l_s \in s$ and $l_o \in o$ (3)1. $\omega = \bigcup_{l_s \in s, l_o \in o} E_{l_s, l_o}$ PROOF: $\omega = \text{RT}.o \cap \text{E}.s = \bigcup_{l_s \in s, l_o \in o} \text{RT}.l_o \cap \text{E}.l_s = \bigcup_{l_s \in s, l_o \in o} E_{l_s, l_o}$

(3)2. Q.E.D.

PROOF: $E_{l_s, l_o} \subseteq \bigcup_{l_s \in s, l_o \in o} E_{l_s, l_o}$ obviously holds for some $l_s \in s$ and $l_o \in o$, thus (2)1 holds by (3)1.(2)2. CASE: $e \in E_{l_s, l_o}$ for some $l_s \in s$ and $l_o \in o$ (3)1. $(E_{l_s, l_o} \otimes h = h) \Rightarrow (\omega \otimes h = h)$ PROOF: $E_{l_s, l_o} \otimes h = h$ implies that $h = \langle \rangle$ or that h only contains events in E_{l_s, l_o} . $\omega \otimes h = h$ trivially holds in the former case, and in the latter case because $E_{l_s, l_o} \subseteq \omega$ by (2)1.(3)2. $E_{l_s, l_o} \otimes f(\langle e \rangle) = f(E_{l_s, l_o} \otimes \langle e \rangle)$ PROOF: Holds since $f \in \mathcal{F}$ and all functions in \mathcal{F} must satisfy requirement (1) in Sect. 5.1 by definition of \mathcal{F} .

(3)3. Q.E.D.

PROOF:

 $E_{l_s, l_o} \otimes f(\langle e \rangle) = f(E_{l_s, l_o} \otimes \langle e \rangle)$ [Holds by (3)2] $E_{l_s, l_o} \otimes f(\langle e \rangle) = f(\langle e \rangle)$ [Holds since $e \in E_{l_s, l_o}$ ((2)2)] $E_{l_s, l_o} \otimes h = h$ [Def. of h] $\omega \otimes h = h$ [By (3)1] $\omega \otimes f(\langle e \rangle) = f(\langle e \rangle)$ [Def. of h] $\omega \otimes f(\langle e \rangle) = f(\omega \otimes \langle e \rangle)$ [$e \in \omega$ by (2)1 and (2)2.](2)3. CASE: $e \notin E_{l_s, l_o}$ for all $l_s \in s$ and $l_o \in o$ (3)1. $(E_{l_s, l_o} \otimes h = \langle \rangle) \Rightarrow (\omega \otimes h = \langle \rangle)$ PROOF: It is easy to see that the implication holds if $h = \langle \rangle$. If $h \neq \langle \rangle$, then the left side of the implication requires that h does not contain any events in E_{l_s, l_o} , but this entails that the right side of the implication holds because $E_{l_s, l_o} \subseteq \omega$ (by (2)1).(3)2. $E_{l_s, l_o} \otimes f(\langle e \rangle) = f(E_{l_s, l_o} \otimes \langle e \rangle)$ PROOF: Holds since $f \in \mathcal{F}$ and all functions in \mathcal{F} must satisfy requirement (1) in Sect. 5.1 by definition of \mathcal{F} .

⟨3⟩3. Q.E.D.

PROOF:

$$\begin{array}{ll}
 E_{l_a, l_a} \otimes f(\langle e \rangle) = f(E_{l_a, l_a} \otimes \langle e \rangle) & [\langle 3 \rangle 2] \\
 E_{l_a, l_a} \otimes f(\langle e \rangle) = f(\langle \rangle) & [\text{Case assumption } \langle 2 \rangle 3] \\
 E_{l_a, l_a} \otimes h = f(\langle \rangle) & [\text{Def. of } h] \\
 \omega \otimes h = f(\langle \rangle) & [\text{Lemma 2.3 (Sect. B.4.3) and } \langle 3 \rangle 1] \\
 \omega \otimes f(\langle e \rangle) = f(\langle \rangle) & [\text{By definition of } h] \\
 \omega \otimes f(\langle e \rangle) = f(\omega \otimes \langle e \rangle) & [\text{Case assumption } \langle 2 \rangle 3]
 \end{array}$$

⟨2⟩4. Q.E.D.

PROOF: By ⟨2⟩2 and ⟨2⟩3.

B.4.3 Lemma 2.3

ASSUME: 1. $f(\langle \rangle) = h$ where $f \in \mathcal{F}$

PROVE: $h = \langle \rangle$

⟨1⟩1. $E_{l_1, l_2} \otimes h = h$ for arbitrary $l_1, l_2 \in \mathcal{L}$.

PROOF:

$$\begin{array}{ll}
 E_{l_1, l_2} \otimes f(\langle \rangle) = f(E_{l_1, l_2} \otimes \langle \rangle) & [*] \\
 E_{l_1, l_2} \otimes f(\langle \rangle) = f(\langle \rangle) & [\text{By def. of } \otimes.] \\
 E_{l_1, l_2} \otimes h = h & [\text{By assumption 1.}]
 \end{array}$$

* Holds because f satisfies requirement (1) in Sect. 5.1.

⟨1⟩2. Q.E.D.

ASSUME: (a) $h \neq \langle \rangle$

PROVE: False.

⟨3⟩1. $\exists l, l' \in \mathcal{L} : \exists h' \in \mathcal{H} : E_{l, l'} \otimes h' \neq h'$

⟨4⟩1. Choose $l_a \in \mathcal{L}$ and $e \in \mathcal{E}$ such that $\text{TR}.e \neq l_a$

PROOF: Trivial.

⟨4⟩2. $\exists l_b \in \mathcal{L} : E_{l_a, l_b} \otimes \langle e \rangle = \langle \rangle$

PROOF: By ⟨4⟩1 and definition of E_{\cdot} .

⟨4⟩3. Q.E.D.

PROOF: Select l such that $l = l_a$ and h' such that $h' = \langle e \rangle$, then ⟨3⟩1 holds by ⟨4⟩2.

⟨3⟩2. Q.E.D.

PROOF: ⟨3⟩1 contradicts ⟨1⟩1, thus assumption (a) is false.

⟨2⟩1. Q.E.D.

PROOF: $\neg(h \neq \langle \rangle) \Rightarrow h = \langle \rangle$

