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TITLE

Industrial Aspects and Literature Survey: Fleet Composition and Routing

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ABSTRACT

The purpose of this paper is to describe industrial aspects of combined fleet composition and routing in maritime and road-based transportation, and to present the current status of research in the form of a comprehensive literature review. With a backdrop of industrial aspects, a categorized survey of relevant literature since the first published papers in the 1950's is given. First, the literature review discusses some early seminal and application-oriented papers, presents a classification of problems, and then focuses on a basic definition of combined fleet composition and routing: the fleet size and mix vehicle routing problem. Three basic mathematical formulations from the literature are presented and compared. Further, the literature of extended and related problems is described and categorized. Surveys of application oriented research in road-based and maritime transportation conclude the review. Finally, we contrast the literature with aspects of industrial applications from a critical, but constructive stance. Major issues for future work are suggested.

KEYWORDS	ENGLISH	NORWEGIAN
GROUP 1	Logistics, Information Technology	Logistikk, Informasjonsteknologi
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SELECTED BY AUTHOR	Transportation, Fleet composition	Transport, Flåtesammensetting
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1 Introduction

Efficient transportation is becoming more and more important to society. It is not unusual for transportation costs to account for 20% of the total cost of a product. In the EU, the transportation sector amounts to more than 10% of the gross domestic product (GDP) and employs 10 million people. Economic growth, increasing consumption, and globalization tend to increase the need for transportation. Strong competition between transportation providers and between goods owners, partly due to globalization, leads to higher demands on efficiency, customer service, timeliness, reactivity, and cost reduction in transportation.

The transportation industry and transportation in general face stronger competition. There is a strong pressure towards reducing movement costs, which again necessitates more frequent and qualified capacity adjustments. Recently, in addition to economy, climate change and other environmental concerns have become significant drivers towards more efficient transportation. Structural rationalization through mergers and acquisitions is a strong trend both for goods owners and transportation providers, and across transportation modalities. This trend also increases the general demand for both long-term and short horizon capacity adjustments in transportation, because such rationalization is often motivated by potential synergies related to capacity. In addition, there has been a strong increase in fleet capacity over the past decades, both in maritime and road-based goods transportation. Since 1980, the world maritime fleet has grown by some 25%. Productivity has increased by 12.5% in the same period, see for instance Christiansen et al. [27].

In the EU, road-based goods transportation increased by 37.9% in the period 1995-2005, whereas maritime goods transportation increased almost as much. The change in transportation volumes has closely followed the GDP. According to Eurostat's Panorama of Transport, Edition 2007 (Panorama of Transport, Edition 2007), the number of vehicles for road-based goods transportation reached 31.5 million in 2004, an increase by 46% relative to 1995. The increase in transportation volumes seems to continue, in parallel with economical growth and increasing globalization. The stronger growth in the number of freight vehicles than transportation volume on the road is interesting. It suggests a strong restructuring of fleets and their use in the EU over the past decade.

As indicated above, the industry faces fleet dimensioning challenges at all decision levels. For transportation providers and goods owners alike, a goal is to strike an optimal balance between owning and keeping a fleet and subcontracting transportation. At all levels of the decision hierarchy, market variables such as expected demand, transportation rates, and transportation costs are key factors in fleet dimensioning decisions. For a transportation provider, transportation rates determine revenue. Costs consist of vehicle acquisition and depreciation costs, driver costs, fuel, toll, and port costs. For goods owners with their own fleet, rates represent external costs for subcontracting, whereas transportation costs are similar to those of a transportation provider. Decisions regarding ownership or leasing, and how to deal with contingencies, are often an integral part of the goal. In general, there is a market for leasing missing capacity and subletting extra capacity. Important issues in fleet dimensioning are the value of overcapacity, and the risk of capacity shortage.

For several reasons that will be addressed later in this paper, transportation fleets are more often than not heterogeneous. Not only the total capacity, but also the size of individual vehicles, and an optimal composition of the fleet when taking all costs and revenues into consideration, are the goals of a fleet dimensioning and composition process. In the remainder of the paper, the term *fleet composition* will be used to cover both the determination of fleet size, and, in the case of heterogeneous fleet, how the fleet is composed of different types of vehicles.

There are typically computationally hard combinatorial optimization problems at the core of decisions related to design, composition, and operation of transportation systems. Examples are service network design, facility location problems, and routing problems. Humans can only produce high quality designs, plans, and decisions under realistic time and resource restrictions if the transportation system has a particularly simple structure, or if it is of very limited size. The inherently complex design and coordination problems in transportation that require good solutions to hard optimization problems can only to a certain extent be avoided through rationalization, i.e., simplification of system structure. Hence, for transportation operations of some size, humans need assistance through advanced decision support systems that are based on effective and computationally efficient methods for solving the relevant optimization problem. According to industrial experience, such decision support systems with functionality for optimization typically have a cost reduction potential of 5-30%, of course highly dependent on the type of decision, the type of application, and the skills of the human planners. In this survey paper, we will focus on planning that combines routing and fleet composition decisions. Our starting point is the classical vehicle routing problem (VRP) [147], where the routes only are restricted by the capacity of the vehicles. This problem is also denoted as the capacitated vehicle routing problem (CVRP). In the following, unless we explicitly state otherwise, we shall use the terms "vehicle routing problem" and "VRP" in the wide sense, i.e., not only referring to the specific CVRP but also its extensions. Since the classical papers by Dantzig and Fulkerson [34] and Dantzig and Ramser [35], operations research (OR) has developed quantitative models and methods for optimizing the operation of a fleet of vehicles in order to serve a transportation demand. The VRP holds a central place in quantitative methods in transportation management. The VRP belongs to the infamous class of NP-hard optimization problems, for which no computationally efficient algorithm is believed to exist. Literally thousands of papers have been written on the VRP. Over the past 50 years, our ability to produce high quality, if not optimal, solutions to instances of the VRP has increased tremendously. The advances are due to a combination of the general increase in computing power and a strong improvement of optimization methods for the VRP. Despite the methodological improvements, exact methods that guarantee to find an optimal solution cannot consistently solve instances of the CVRP with more than some 70 customers under realistic response time requirements. For larger problems, unless they have a particular structure, one has to give up the quest for optimality and resort to some form of approximation (in the wide sense) method for practical applications.

In line with much of the general criticism of OR, VRP research has been accused of being focused on theory and based on idealized models with assumptions that are non-realistic in practice. To a certain extent, this criticism is valid. The bulk of VRP research has been reductionistic in nature, with assumptions of Euclidean distances, deterministic and static travel times, deterministic demand, hard constraints, and a monolithic objective. Such assumptions are rarely warranted in industrial cases. The research community has defined the basic CVRP and extended it in a precise and step-wise fashion. In this way, a taxonomy of VRP variants has emerged, and this taxonomy is gradually being extended. The research community has gained considerable insight into the structure of each VRP variant. Specific algorithms have been devised for their resolution.

VRP research is regarded as one of the success stories of OR. Moreover, VRP research has proven to be industrially relevant. A software industry that provides routing tools to transportation planners has been established, and it is growing. Tools are based on methods developed by the scientific VRP community. The need for new scientific challenges, and an industrial demand for more powerful and versatile routing tools, has shifted the focus of VRP research to more complex, general, and larger size variants. Also, a trend towards a more holistic approach can be seen in the recent VRP literature [40, 64, 78, 116]. The term "Rich VRPs" is often used in the literature to denote VRP models that include many real-world aspects of routing problems. In the Rich VRPs line of research a general VRP model is the starting point and the goal is to devise an effective, uniform algorithmic approach. Several rich VRP models have been proposed and investigated in the literature.

One assumption that pervades the VRP literature is the one of a homogeneous fleet. As will be substantiated in Section 2, this assumption is not realistic in most industrial applications. Also, a focus on transportation costs rather than fleet costs is most common in the VRP models treated in the literature. Although capacity dimensioning aspects are found even in the title of the earliest VRP paper “Minimizing the number of tankers to meet a fixed schedule” by Dantzig and Fulkerson from 1954 [34], the first explicit treatment of fleet composition is arguably found in Kirby [83]. This paper describes a model for determining owned and hired wagons in a railway system. The so-called fleet size and mix vehicle routing problem (FSMVRP) was defined in a paper by Golden, Assad, Levy and Gheysens in 1984 [74].

The problem defines an extension of the classical VRP that accommodates a heterogeneous fleet and takes vehicle costs into consideration in addition to travel costs. In this survey paper, we will focus on OR that combines fleet composition and vehicle routing. In total, we have found some 120 scientific papers that address this combination. A survey paper considering fleet composition and routing is published by Baldacci et al. [2], but we have not found any paper that relates the literature on fleet composition and routing to industrial aspects in a general way.

With this background, our goal is twofold: to give an updated survey of OR literature on combined fleet composition and routing, and to contrast this literature with aspects of industrial applications. In this way, we document state-of-the-art, take a critical but constructive stance, and point to needs for future research. Regarding modalities, our scope is road-based and maritime goods transportation. Fleet composition aspects of airborne and rail transportation are somewhat different and certainly relevant to our overall goal. However, we argue that our focus on two modalities does not seriously limit insights, but it does limit paper extent to a reasonable level. We also point to similarities and differences between the two selected modi.

In general, the terms *route*, *tour* and *trip* will have the same meaning in this paper, i.e. a round trip performed by a vehicle starting and ending at a depot and visiting a specified number of customers in sequence. When describing an article, we will use the same term as used in that particular article.

The remainder of the paper is organized as follows. In Section 2, we will point to important industrial aspects related to fleet composition and routing. Three generic models of fleet composition and routing are presented in Section 3, followed by the survey of papers and state-of-the-art. In Section 4, we criticize the research conducted so far, point to industrial and scientific perspectives, and describe major issues for further research. Summary and conclusions are found in Section 5.

2 Industrial aspects of combined fleet composition and routing

We start this section regarding industrial aspects on combined fleet composition and routing by giving explanations and motivations for operating a heterogeneous rather than a homogeneous fleet. We then describe three main categories of vehicle attributes that may render a fleet heterogeneous. The categories are discussed, and aspects are exemplified both for maritime and road-based transportation. In Section 2.2, we present fleet composition tasks at the strategic, tactical, and operational levels, and motivate the integration of fleet composition and routing decisions. We give an account of industrial routing aspects in Section 2.3. Again, examples from road-based and maritime transportation are given. In Section 2.4, we summarize and accentuate differences between the two modalities regarding fleet composition and routing.

2.1 Heterogeneous fleets

In industry, a fleet of vehicles is rarely homogeneous. There are several reasons. A fleet is often acquired over a long period of time, and the vehicles will have different characteristics due to technological development and the market situation. Operating, maintenance, and depreciation costs will vary over the lifetime of a vehicle. Moreover, owners typically want to keep a diverse set of vehicle types in their fleets, both due to operational constraints and the inherent benefits of versatility.

We divide distinguishing aspects of vehicle types into three main categories:

- physical dimensions
- compatibility constraints
- costs

Physical dimensions such as the length, breadth, and height of a vehicle broadly determine its carrying capacity. The capacity may be offered through a single or several cargo holds. In the latter case, the cargo holds may have different capacities, equipment, and product compatibilities. In road-based transportation, physical dimensions and weight may constrain access to the road network. Notable examples are narrow roads in urban areas and old villages, and limited space at ramps for loading or unloading. Size and weight constraints may even vary over time, as exemplified by seasonal axle pressure limits due to spring thaw. Ships have similar physical dimension constraints, including draft restrictions that vary with tide, available berth space in ports, and canal restrictions on ship size. We may also regard vehicle speed as a physical dimension, or rather, a physical restriction. A lower speed vehicle may also have lower unit costs, but it may be impossible to use or give a less cost-efficient overall solution due to temporal constraints. In other cases, special, lower speed vehicles are needed due to equipment or environmental constraints.

Compatibility constraints beyond physical dimensions sometimes limit the product types that a vehicle can carry, and where the vehicle may travel. Often, customers need vehicles with special equipment for loading and unloading. Special certificates may be required for operation in certain areas. At sea, there are special zones that have particularly strong constraints on emission on sulphur. Similarly, and particularly in urban areas, there are now increasingly strong environmental restrictions on noise, gas and particle emissions that constrain the use of vehicles. Certain products such as corrosive chemicals require trucks or ships that have tanks with a special coating. Transportation of dangerous goods often requires special vehicles and certificates, and there can be route restrictions.

Costs are important distinguishing factors between types of vehicles. Large vehicles generally have a lower unit cost than smaller vehicles, given that capacity utilization is sufficiently high. Old vehicles have lower depreciation costs, but higher maintenance and environmental costs. Before deciding on long-term fleet investments, the organization must consider the strategic choice between ownership of transportation capacity and outsourcing. Expected revenues must be compared with expected costs, typically under considerable uncertainty. Over a shorter horizon, the goal is often to strike a balance between fixed fleet costs and contingency costs that accrue when demand exceeds capacity and external capacity must be bought, again a problem that may be compounded by high uncertainty. Extra capacity has a value because more flexibility may allow for better routing solutions.

Even if a fleet is dedicated to transporting a single product, and all vehicles are acquired at the same time, there may be good reasons for keeping a heterogeneous fleet. Transportation demand characteristics in volume, time, and geography may motivate the use of vehicles with different size, despite the fact that larger capacity vehicles are often less costly per unit. A heterogeneous fleet of vehicles is generally more flexible and cost effective towards demand variation. Moreover, there may be constraints that render some types of trucks or ships incompatible, as explained above.

2.2 Fleet composition tasks

Fleet composition, resizing, and allocation/assignment are tasks that fleet owners and managers are faced with across all transportation modalities. These tasks are found at all levels of the decision hierarchy. We will now discuss such tasks in more detail, with focus and examples from maritime and road-based goods transportation. First, we will give some general remarks.
A merger or acquisition between two transportation companies will require a strategic or tactical capacity adjustment, often, but not necessarily in the form of fleet downsizing. In other cases, it is

a question of repositioning the combined fleet and even acquisition of new capacity. Anyhow, capacity adjustment will involve selection of which vehicles to keep, which ones to sell or sublet, and a selection of the number and types of vehicles to buy or lease. Even in a strategic setting, decisions may involve what type of operation or trade each vehicle will be allocated to.

Unless routing and scheduling decisions are to a large degree predetermined, as they might be for instance in liner shipping or bus transportation, it is clear that there is generally a strong dependency between fleet composition and routing. By ignoring routing aspects, fleet composition decisions may be based on a too simplified view on transportation demand. Conversely, it is obvious that routing decisions are strongly dependent on the available fleet. Hence, in most cases, an integration of routing aspects in fleet composition decisions is warranted. Such integration is not without problems. The computational complexity of the integrated planning problem is higher. Particularly in a strategic setting, uncertainty in demand, travel times, and service times will be high, and it is not clear at which level of detail one should model routing aspects.

Despite our focus above, fleet composition and resizing are not only relevant in cases of a heterogeneous fleet. For a truly homogeneous fleet, fleet composition is reduced to determining the optimal number of vehicles. Operational fleet allocation in this case is reduced to considerations regarding vehicle status, typically based on position and load.

It is obvious that fleet composition and allocation decisions must be based on information about transportation demand, transportation costs, transportation income rates (for transportation service providers), as well as vehicle acquisition, depreciation, resale, and leasing prices. As will be described below, there are industrial fleet composition problems at all levels of the decision hierarchy, with resulting time horizons that vary between decades and minutes. The various transportation modalities, industrial sectors, and applications will have fleet composition cases with a wide variety of characteristics. Hence, it is hard to argue that one single problem formulation, unless it is very general, will be adequate for all real-life applications.

In particular, the uncertainty of information about which decisions are made will vary and typically be high for the strategic cases. For such problems, it may be irrelevant to bring detailed routing aspects into the problem model, because uncertainty is too high. In this case, routing costs must be addressed in a more aggregate way, but a detailed model of the various types of vehicle costs is called for.

2.2.1 Strategic fleet composition

At the strategic level, a shipping company or a goods owner may be faced with the challenge of acquiring transportation capacity through a fleet of ships to be used in a particular trade. The company may or may not have an existing fleet as a starting point. A current example is transportation of liquefied natural gas (LNG) between producing and consuming ports in the Northern Atlantic Ocean, where operators want to acquire capacity for, say, the next 20 years with no fleet to start with. Such decisions are critical to an organization, as they involve huge amounts of capital. An average LNG tanker easily costs 150 million USD (EIA 2007). Ownership or leasing/chartering of capacity is another critical question. The current average rate for long-term charters of LNG tankers is now (November 2007) between USD 55,000 and USD 65,000 per day (EIA 2007).

Regardless of modality, strategic fleet decisions involve considerable capital investment. Even over a few years, uncertainty in demand, costs, and revenues related to fleet operations is high. Hence, fleet composition problems at the strategic level contain important risk management aspects. Risk is typically reduced and flexibility increased through a mixture of long-term contracts combined with short-term spot cargos, if there is a spot market for the trade in question. A certain level of long-term capacity slack is often added to increase flexibility and to facilitate less costly routing solutions. Operators often acquire additional capacity over a shorter time at a higher cost to effectively cope with market fluctuations. Surplus capacity can sometimes be put into operation in other trades, if possible, or sublet. Although our example above may be an

extreme case from maritime transportation, similar schemes are found in road-based goods transportation. Even though the amount of capital involved will typically be several orders of magnitude lower, strategic fleet composition decisions for trucks and lorries are just as critical because competition is strong and profit margins small.
In strategic, long-term fleet management decision models it will typically not make sense to include routing aspects at a very detailed level, unless the transportation demand is highly predictable. On the other hand, all relevant revenues and costs related to the acquisition and operation of the fleet should be modeled as detailed as possible, taking possible long-term contracts and a spot markets into consideration. Chartering and subletting options must also be modeled. The motivation for somehow addressing uncertainty, if not by a fully-fledged stochastic model, is strong due to the characteristics of the decision type. If the company does have a fleet to start with, the model must also include the options of selling and subletting existing capacity.

2.2.2 Tactical and operational fleet composition

In a tactical setting of a few years or months (generally, time constants are longer in maritime than in road-based transportation), the problem is more the one of capacity adjustment, given an existing fleet. Uncertainty will be smaller. The rationale for adding routing aspects at a more detailed level will be stronger than in strategic fleet composition. Still, there is normally considerable uncertainty that should at least be carefully evaluated and analyzed. As in the strategic setting, main decisions are which new vehicles should be bought or chartered in, which existing vehicles should be sold or chartered out, and how to cope with demand fluctuations. However, tactical decisions focus more on adjustments of capacity and composition than in the strategic case. Moreover, tactical fleet composition includes detailed assignment of vehicles to routes, contracts, or types of operations. There is a “dual” problem in tactical fleet composition related to contract optimization: Given the fleet and a portfolio of contracts, which potential new contracts should one bid for. Integration of tactical fleet composition and contract optimization may prove beneficial.
The fleet size and mix vehicle routing problem (FSMVRP) is an extension of the classical VRP to a heterogeneous fleet and an extension of the objective to include vehicle acquisition and/or depreciation costs. It seems targeted at tactical and operational fleet composition challenges where it is possible to make detailed routing decisions. In our opinion, the FSMVRP definition is not applicable to all tactical fleet composition situations. The definition clearly indicates that it has originated from the vehicle routing community more than from an asset or portfolio management tradition.
An example of a tactical fleet composition situation where the FSMVRP is a relevant model is found in the first tier of newspaper distribution. Packages of newspapers must be transported by vans and smaller cars from the printing shop to kiosks and stores. For subscription newspapers, packages are also dropped off at several depots for the carriers. The fleet will typically be heterogeneous, both due to urban restrictions and to a need for flexibility. A new, detailed routing plan must be determined, say every 6 months, due to demand variations and new newspaper outlets. In this case, the optimal fleet composition together with a detailed routing plan must be found, while considering the existing fleet as well as acquisition of new vehicles. This is possible with a FSMVRP formulation. There might also be variation in newspaper demand that has to be considered.
At the operational level, a fleet allocation task of a transportation company generally consists of the integrated task of selecting a set of vehicles to accommodate today’s transportation orders, and at the same time determine the routing plan. The latter example may be extended to dynamic, minute-to-minute routing. The fleet to be considered may consist of vehicles with different characteristics. It can be fixed or flexible, in the latter case often both consisting of vehicles owned by the company as well as external capacity that is leased when demand is too high.

2.3 Industrial aspects of routing

By and large, the OR literature on routing has concentrated on idealized rather than rich and industrially adequate models. One symptom is the fact that the bulk of the literature is focused on homogeneous fleet models, whereas heterogeneous fleets are the most common in industry. The bulk of routing papers is independent of modality, but there is also literature that focuses on a specific modality. We observe a pronounced difference between modalities: the literature on ship routing is smaller, but focuses much more on industrial aspects than the literature on road-based goods transportation. The generic VRP literature seems much more influenced by road-based transportation than any other modality.

There are other important industrial aspects besides heterogeneous fleets. We will now give a brief account of such aspects. It is true that for many of them, new and more general routing variants have been defined and studied. Also, general, rich and industrially adequate routing models have been studied lately. For a more comprehensive treatment of industrial routing aspects, we refer to Hasle and Kloster [78].

Closely related to a vehicle is the driver, or more generally, the crew. The general assumption in the literature is that the vehicle and driver forms an inseparable unit: an equipage. In many applications, particularly for long distance operations, driving time restrictions will constrain driving. Allocation and exchange of drivers may then be important aspects of the problem.

In the classical VRP, there is a single depot for initial loading (delivery operations) or final unloading (pickup operations) where each vehicle starts and ends. Only a single tour per vehicle is allowed. In road-based goods transportation, there may be multiple depots. Drivers may start and end at home, and perform many tours per day. In ship routing there is typically a more general structure: ships move in a continuous fashion between calls at ports for loading and unloading, without any of the ports having a specific status as a depot. This is similar to the pickup and delivery problem (PDP), but without the central depot. In both modalities, one may also have service orders that do not require movement of goods, but for instance visits from service technicians or health personnel at specific locations. In contrast to the homogeneous situation of a demand that consists solely of pickup orders, delivery orders, pickup and delivery orders, or service orders, there are routing applications where the demand consists of all types of orders.

The generic VRP literature generally adopts a highly abstract model of distances, travel times, travel costs, and service times. Typically, distances between two nodes are assumed to be Euclidean. This is not so critical. However, speeds and costs are, with few exceptions, unrealistically modeled as constant and crisp values, with parts of the literature on stochastic VRPs as an exception. In real-life road-based transportation, there is a concrete road network that needs to be considered regarding calculation of distance, time, and travel cost. Particularly in urban areas, travel times are highly uncertain and speed may vary over time due to congestion. For maritime transportation, canals, coastal areas and even the oceans may be adequately modeled as a network with ports and waypoints as nodes. Ship speed, however, is adjustable and travel costs depend to a large degree on speed. Weather and currents influence strongly on speed and bunker consumption, but these effects may not be predictable except for planning over a few days. A proper consideration of uncertainty in travel times and travel costs is called for. In certain transportation applications, service times often constitute a significant part of the total time consumed. In the generic VRP literature, service times are predominantly assumed to be precise and constant. In practice, service time for a given load is dependent on the vehicle, the location, and the position of the load in the cargo hold. There is often considerable uncertainty. In the transportation of chemicals, certain sequences of products in a tank may require time-consuming and costly cleansing operations.

The hard capacity constraint and fixed order sizes we know from the OR literature is an abstraction that is often inadequate. In some applications overloading may be possible, but this can in some cases easily be remedied by adjusting the capacity constraint. If goods are discrete, filling the nominal capacity is sometimes not possible due to packing aspects. With fragile goods there can be stacking constraints. Packing may also influence service times and call for a sequencing of stops that takes variable service times into account. For bulk goods, order sizes are often somewhat flexible so adjustment to the available capacity is an option. If demand is measured in mass units and capacity is defined by volume, density and temperature will influence capacity. Such effects are seen in liquid bulk transportation, where there are typically also constraints on minimum filling levels due to problems with sloshing.

A more exotic capacity aspect is found in LNG transportation. Natural gas is liquefied and transported in tankers at a temperature of -163 °C under atmospheric pressure. There are no cooling facilities on board, so a significant part of the LNG evaporates during a long voyage. Luckily, the engines run on natural gas. Due to boil-off effects and the use of LNG as propellant, there must always be enough LNG left in the tanks to reach the next pickup port.

2.4 Modal differences

An important industrial aspect of combined fleet composition and routing is uncertainty. Heterogeneous fleets, either due to physical dimensions, compatibility constraints, or costs, are certainly the most common way of structuring capacity in both maritime and road-based transportation. We have also mentioned the presence of an existing fleet, the relation to contracts and spot markets, and the ability to hire or sublet capacity as important industrial aspects that need to be incorporated in adequate problem models for combined routing and fleet composition. As pointed out above, there are general differences between maritime and road-based operations. We give an overall picture, and it must be stressed that there is a large variety of routing applications within each modality.

Maritime operations typically have a continuous PDP structure with no depot. Road-based transportation more often has a classical VRP structure with a single depot and either pickups or deliveries only. As a counter-example, long distance, full truckload (FTL) operations on road often have a continuous PDP structure. In maritime transportation, time constants are generally longer than in road based transportation. Arguably, uncertainty in travel and service times is higher at sea, although there are uncertainty issues also in road-based transportation, for instance related to weather and urban congestion situations.

There are general differences that are more directly relevant to combined fleet composition and routing. The most striking difference between the road-based and maritime modalities is related to scale. Predominantly, vehicles are larger and the number of vehicles in the fleet is smaller in maritime transportation. Costs and revenues are typically much lower per vehicle in road-based transportation. Capital investment for a ship is very high, much higher than for the average lorry. Even when considering the total fleet, capital binding is generally much larger in maritime transportation. The lead time for acquisition is typically a few years, as opposed to a few months for road-based vehicles. Ship building is often one-of-a-kind, in contrast to the highly standardized manufacturing of trucks. The average life span of a ship is typically a few decades, whereas a truck normally gets replaced every few years. Maintenance is less standardized, takes longer time and costs more for a ship than a truck. Although there is a large variation between different types of shipping, ship loads are generally much larger than loads in road-based transportation, also relative to vehicle capacity. The modal differences, and also differences within the modalities, point to the need for a variety of models for combined fleet composition and routing.

3 Literature survey

This section presents a survey of papers regarding fleet composition and routing. Several other papers contain literature review of related research, and a survey about routing a heterogeneous fleet of vehicles is published by Baldacci et al. [2]. The survey gives an overview of papers on heterogeneous VRPs with fixed or variable fleet size, and focuses particularly on lower bounds and heuristic algorithms used for these problems.

Salhi and Rand [127] give an overview of early papers on fleet composition and state the shortage of literature combining fleet composition with routing of the vehicles. Osman and Salhi [111] summarize the papers regarding the FSMVRP up to that date. A survey paper on ship routing and scheduling is published by Christiansen et al. [26]. It refers to numerous papers considering fleet composition and routing in the maritime industry. Lee et al. [88] describe several papers concerning the Heterogeneous Fleet VRP in different variants, and Bräysy et al. [18] present an extensive literature review of papers regarding the FSMVRP with Time Windows (FSMVRPTW).

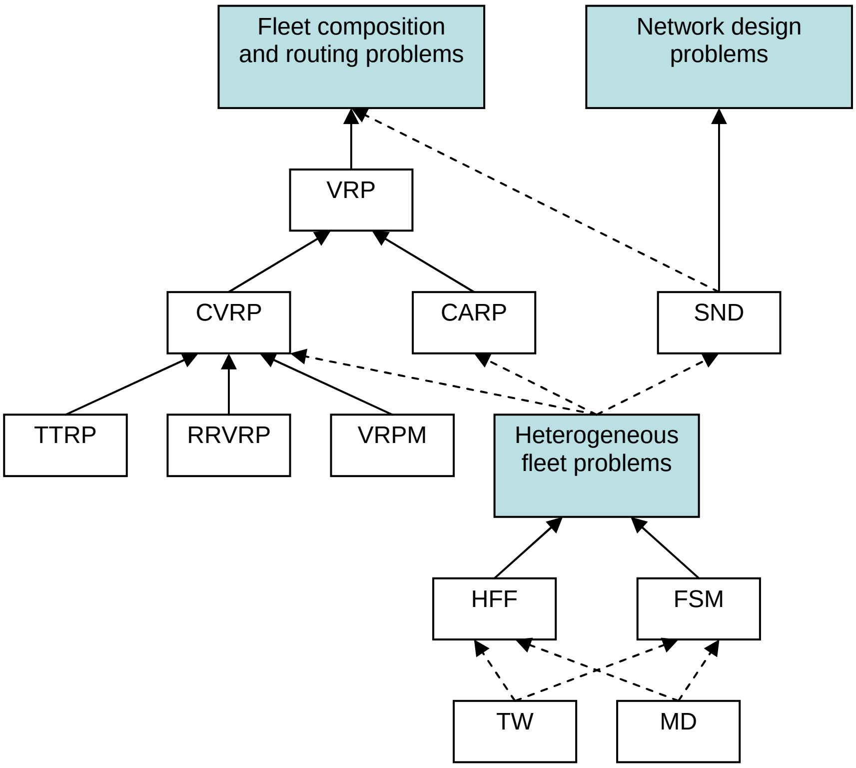

Figure 1. Different classes of routing problems

Figure 1 shows the different classes of routing problems discussed in this paper. The solid lines represent a direct connection where one problem class is derived from another. Hence, the dashed lines represent a possible connection where a class not necessary is derived from the other, but where there exists instances which fall into that category.

The vehicle routing problem (VRP) is a problem that belongs to a general class denoted as *fleet composition and routing problems*. The capacitated vehicle routing problem (CVRP) and the capacitated arc routing problem (CARP) are specific types of the VRP. Another general class is *network design problems* where the aim is to decide the selection of which arcs to open in a network rather than to construct routes departing and ending in a depot. Service network design (SND) problems consider decisions related to frequencies, modes, routes and schedules of services, combined with routing of freight. In recent studies, associated decisions on fleet composition and routing of vehicles are incorporated into SND problems. For all these classes of problems, there exist subclasses where heterogeneous fleets are allowed. Thus, both the heterogeneous fixed fleet (HFF) problem and the fleet size and mix (FSM) problem can be defined as extensions to the basic classes of routing problems. The problem definitions can also allow specific constraints or structural aspects such as time windows (TW) and multiple depots (MD). The truck and trailer routing problem (TTRP), the rollon-rolloff vehicle routing problem (RRVRP) and the VRP with multiple use of vehicles (VRPM) are in their standard definition derived directly from the CVRP.

This section is organized as follows. Section 3.1 starts with a discussion of three basic mathematical formulations for fleet composition and routing problems. Section 3.2 contains a summary of early application oriented papers up to the paper defining the Fleet Size and Mix VRP (FSMVRP). In Section 3.3, we present papers on the standard FSMVRP before discussing papers regarding the heterogeneous fixed fleet VRP (HFFVRP) in Section 3.4. In the latter problem, the fleet size is fixed or bounded by a maximum number of each vehicle type. Then some extensions to the FSMVRP are introduced; Section 3.5 presents the FSMVRP with time windows (FSMVRPTW). FSMVRP with multiple depots (FSMVRPMD) is discussed in Section 3.6. Section 3.7 presents fleet dimensioning in arc routing, and in Section 3.8 network optimization problems are discussed. Next, some related VRPs are presented, namely the truck and trailer VRP (TTVRP) the rollon-rolloff VRP (RRVRP), and the VRP with multiple use of vehicles (VRPM). These are similar problems that not necessarily consider fleet dimensioning. However, since the routing and the fleet composition in these problems clearly affect each other, they are included in this survey. The last subsections consider papers on industrial cases, covering road-based and maritime transportation in Section 3.10 and 3.11, respectively. Each subsection contains a list of the papers related to the topic considered.

The papers are classified with respect to three criteria; solution method, problem type, and modality. Some papers use exact methods on mathematical programming (MP) formulations, while others use construction heuristics or more advanced metaheuristics. The problem types used in this paper are *fleet sizing* where only the size of a homogeneous fleet is considered; *fleet composition* where the aim is to compose a fleet of different vehicle types, and *fleet composition and routing* in which fleet composition is combined with vehicle routing. Another problem type considered is *heterogeneous fixed fleet routing* where the problem is to route a fixed fleet of vehicles of different types. In addition, some related problems are treated. In *trucks and trailers* problems some customers can only be visited by a truck without a trailer and thus the truck has to drive subtours from the parked trailer. *Rollon-rolloff* denotes problems where tractors leave trailers at the customers and collect them later. In *multiple use of vehicles* problems, the common constraint that each vehicle can be used on only one route is relaxed. The last classification criterion is the modality of the problem. In this paper, we focus on road-based and maritime transportation. Many papers are generic, and not necessary related to a specific mode. However, most of these papers seem to be inspired by road-based problems.

3.1 Mathematical models

To make a precise statement of a problem, we need a mathematical formulation. There are several ways to describe fleet dimensioning and routing problems mathematically. Naturally, different variations of such problems require different formulations, but similar problems can also be formulated in dissimilar ways. The fleet size and mix vehicle routing problem (FSMVRP) is a typical and well-defined problem regarding fleet dimensioning and routing and we will use this problem as an example of how to model such problems. Thus, we will present three different mathematical formulations of the FSMVRP in this section, all taken from the literature. The formulations differ in various ways. For instance, some models include temporal issues while others do not. We can also see differences in how routes are represented. In order to increase the readability of the models and compare them, some notation is harmonized from the original versions. The formulations are used to describe the problem while most papers considering FSMVRP use heuristics to find solutions. Thus, the formulations are not necessary used to find exact solutions in practice.

3.1.1 Formulation A

Gheysens et al. [70] define the FSMVRP with the following mathematical formulation. The formulation is slightly modified with respect to the flow variable y compared to the first formulation described in Golden et al. [74]. The following notations are used:

n = number of customers,

K = number of vehicle types,

Q_k = capacity of a vehicle of type k ($Q_1 < Q_2 < \ldots < Q_K$),
f_k = fixed acquisition cost for a vehicle of type k ($f_1 < f_2 < \ldots < f_K$),
q_j = demand of customer j,
c_{ij} = cost of traveling from node i to *node j*.
Symmetric traveling costs are assumed and the costs are assumed to be independent of the vehicle type. Let $G=(N,A)$ be a graph where $N=\{0\}\cup\{1,\ldots,n\}\cup\{n+1\}$. $C=\{1,\ldots,n\}$ defines the set of customers and $\{0\}$ and $\{n+1\}$ represent the depot. $V=\{1,\ldots,K\}$ is the set of different vehicle types. $A\subseteq N\times N$ represents the travel possibilities between nodes, where $(i,i),(i,0),(n+1,i); i\in N$ are excluded. In addition, the following decision variables are used:
y_{ij}: flow of goods from i to j,
$x_{ij}^k = 1$ if a vehicle of type k travels directly from customer i to customer j, and 0 otherwise.

An infinite supply of each vehicle type is assumed. $\sum_{j=1}^{N} x_{0j}^k$ represents the number of vehicles of type k used.

$$\textit{Minimize} \quad \sum_{k\in V}\sum_{j\in N} f_k x_{0j}^k + \sum_{k\in V}\sum_{(i,j)\in A} c_{ij} x_{ij}^k, \tag{A1}$$

$$\text{subject to} \quad \sum_{k\in V}\sum_{i\in N} x_{ij}^k = 1, \qquad \forall j\in C, \tag{A2}$$

$$\sum_{i\in N} x_{ij}^k - \sum_{i\in N} x_{ji}^k = 0, \qquad \forall k\in V, \forall j\in C, \tag{A3}$$

$$\sum_{i\in N} y_{ij} - \sum_{i\in N} y_{ji} = q_j, \qquad \forall j\in C, \tag{A4}$$

$$y_{0j} \le \sum_{k\in V} Q_k x_{0j}^k, \qquad \forall j\in C, \tag{A5}$$

$$y_{ij} \le \sum_{k\in K} M_{ijk} x_{ij}^k, \qquad \forall (i,j)\in A, \tag{A6}$$

$$y_{ij} \ge 0, \qquad \forall (i,j)\in A, \tag{A7}$$

$$x_{ij}^k \in \{0,1\}, \qquad \forall k\in V, \forall (i,j)\in A. \tag{A8}$$

The first part of the objective function (A1) gives the total fixed cost of the vehicles used and the second part gives the total variable routing cost. Constraints (A2) state that each customer is visited exactly once, while constraints (A3) ensure that a vehicle of the same type as the one arriving at a customer will also leave the customer. Since constraints (A2) state that each customer only will be visited once, the type of vehicle arriving and leaving one particular customer has to be the same. Equations (A4) represent the movement of goods assuming that all customer demands must be satisfied, and in (A5), the total load on a trip y_{0j} is constrained not to exceed the capacity of the vehicle assigned to that trip. Constraints (A6) state that no goods are transported from i to j if no vehicle is serving the link between these nodes. M_{ijk} should be as small as possible, but still large enough to be redundant if a vehicle uses the arc. Golden et al. [74] define it to be the sum of all the customer demands. Constraints (A7) ensure that the flow is non-negative; while constraints (A8) define that each arc in the network has the value 1 if it is used and 0 if it is not used by a vehicle of type k.

3.1.2 Formulation A - continued

Salhi and Rand [127] extended the mixed integer programming (MIP) formulation by including the time dimension with parameters T_k as the maximum time a vehicle of type k can spend, t_{ij} as the time to travel link (i, j) and the continuous variable r_{ij} as the time a vehicle has left before reaching T_k after covering link (i, j). The following constraints are included in the model in addition to those used by Gheysens et al. [70].

$$\sum_{j\in N} y_{0j} = \sum_{i\in N} q_i \,, \tag{A9}$$

$$\sum_{i\in N} y_{i0} = 0 \,, \tag{A10}$$

$$y_{ij} \le \sum_{k\in V} Q_k x_{ij}^k \,, \qquad \forall (i,j)\in A \,, \tag{A11}$$

$$r_{ij} \le \sum_{k\in V} T_k x_{ij}^k \,, \qquad \forall (i,j)\in A \,, \tag{A12}$$

$$r_{0j} = \sum_{k\in V} T_k x_{0j}^k - \sum_{k\in V} t_{0j} x_{0j}^k \,, \qquad \forall j\in C \,, \tag{A13}$$

$$\sum_{i\in N} r_{ip} - \sum_{j\in N} r_{pj} = \sum_{k\in V}\sum_{j\in N} t_{pj} x_{pj}^k \,, \qquad \forall p\in C \,, \tag{A14}$$

$$y_{ii} = 0; r_{ij} \ge 0 \,, \qquad \forall (i,j)\in A \,. \tag{A15}$$

Constraints (A9) and (A10) ensure that the total quantity when leaving the depot is equal to the customer demands on the routes, and that nothing is returned to the depot. (A11) make sure that goods can travel from i to j only when there is a vehicle traveling from i to j, and that the total load on link (i, j) cannot exceed the capacity of the vehicle assigned to that trip. These constraints replace constraints (A5) and (A6) from the above formulation. (A12) denote that the spare time on link (i, j) is no more than the maximum time for the vehicle traveling the link, and (A13) ensure that the spare time after covering links leaving the depot does not exceed the maximum time minus the time required to travel to the first customer. Constraints (A14) state that every time a vehicle travels between two customers, the spare time is reduced by the time used on that connection.

3.1.3 Formulation B

Osman and Salhi [111] use a rather different formulation and introduce variable cost (α_k) and time factor (β_k) per distance unit for each vehicle type. In addition, service time for customers (δ_i) is included in their model.

R_k denotes the set of customers serviced by a vehicle k, and $\sigma(k)$ indicates the smallest vehicle type that can serve the customers in R_k. π_k shortest TSP-route which serves R_k including the depot and $\pi_k(i)$ indicates the position of customer i in π_k. $D(\pi_k)$ is the total distance of the route, $T(\pi_k)$ is the total travel time and $C(\pi_k)$ is the total variable and fixed cost of the route π_k. d_{ij} denotes the distance between customers i and j. The decision variable indicating the total number of mixed vehicles is denoted by v, and V is the set of desired vehicles of different types, $V = \{1,\ldots,v\}$. S is the feasible solution defined as $S = \{R_1,\ldots, R_v\}$, and Π is the set of all traveling salesman routes in S, $\Pi = \{\pi_1,\ldots, \pi_v\}$. $Q_{\sigma(p)}$ is the capacity of the smallest vehicle that can be used on to serve the customers in the set R_p. The following formulation expresses the optimization problem:

$$\underset{S,\Pi,v}{Minimize} \qquad C(S) = \sum_{p\in V} C(\pi_p) \,, \tag{B1}$$

$$\text{subject to} \qquad \bigcup_{p\in V} R_p = N \text{ and } R_p \cap R_q = \varnothing \,, \qquad \forall p \ne q \in V \,, \tag{B2}$$

$$\sum_{i\in R_p} q_i \le Q_{\sigma(p)} \,, \qquad \forall p\in V \,, \tag{B3}$$

$$D(\pi_p) = \sum_{i\in R\cup\{0\}} d_{i\pi_p(i)} \qquad \forall p\in V \,, \tag{B4}$$

$$T(\pi_p) = \beta_{\sigma(p)} D(\pi_p) + \sum_{i\in R_p} \delta_i \le T_{\sigma(p)} \,, \qquad \forall p\in V \,, \tag{B5}$$

$$C(\pi_p) = \alpha_{\sigma(p)} D(\pi_p) + F_{\sigma(p)}, \qquad \forall p \in V. \tag{B6}$$

The objective function (B1) is the total sum of costs in the solution to be minimized over all routes. Constraints (B2) ensure that each customer is supplied in exactly one route, while (B3) make sure that demand on each route does not exceed the capacity of the vehicle used for the route. The equations in (B4) represent the total distance of the shortest TSP-route between the customers included in each tour. However, since TSP is an NP-hard problem by itself, approximation methods may be used to estimate each route, π_p. (B5) guarantee that each tour does not exceed the maximum travel time. (B6) represent the total sum of costs in the solution to be minimized in the objective function.

3.1.4 Formulation C

Bräysy et al. [18] give a model of the FSMVRP with time windows (FSMVRPTW) defined by Liu and Shen [94]. The formulation is based on a vehicle flow arc formulation for the VRPTW. Let $G=(N,A)$ be a graph where $N=\{0\}\cup\{1,\ldots,n\}\cup\{n+1\}$ and both node 0 and node n+1 represent the depot. $C=\{1,\ldots,n\}$ is the set of customers, $A\subseteq N\times N$ represents the travel possibilities between nodes, while $V=\{1,\ldots,K\}$ is the set of alternative vehicles. One difference from the other formulations presented here is that this model defines each vehicle separately, while in the previous formulations only the vehicle types are defined. Vehicle k has acquisition and/or depreciation cost e_k and capacity q_k. The vehicle independent travel time between nodes is given by t_{ij}, and for each customer i, the time window $[a_i, b_i]$ defines the period within which the service must start. The service time at each customer is vehicle independent and given by s_i. The two types of variables in the formulation are x_{ij}^k which expresses whether vehicle k travels directly from node i to node j, and y_i^k that determines the exact start time of service at customer i if it is serviced by vehicle k. The formulation is:

Minimize
$$\sum_{k\in V}\sum_{j\in C}\left(e^k + y_{n+1}^k - y_0^k\right)x_{0j}^k, \tag{C1}$$

subject to
$$\sum_{k\in V}\sum_{j\in N} x_{ij}^k = 1, \qquad \forall i \in C, \tag{C2}$$

$$\sum_{i\in C}\sum_{j\in N} d_i x_{ij}^k \le q^k, \qquad \forall k \in V, \tag{C3}$$

$$\sum_{j\in N} x_{0j}^k = 1, \qquad \forall k \in V, \tag{C4}$$

$$\sum_{i\in N} x_{ij}^k - \sum_{i\in N} x_{ji}^k = 0, \qquad \forall k \in V, \forall j \in C, \tag{C5}$$

$$\sum_{i\in N} x_{i,n+1}^k = 1, \qquad \forall k \in V, \tag{C6}$$

$$x_{ij}^k\left(y_i^k + s_i + t_{ij} - y_j^k\right) \ge 0, \qquad \forall (i,j) \in A, \forall k \in V, \tag{C7}$$

$$x_{ij}^k\left(y_i^k - a_i\right) \ge 0, \qquad \forall (i,j) \in A, \forall k \in V, \tag{C8a}$$

$$x_{ij}^k\left(b_i - y_i^k\right) \ge 0, \qquad \forall (i,j) \in A, \forall k \in V, \tag{C8b}$$

$$x_{ij}^k \in \{0,1\}, \qquad \forall (i,j) \in A, \forall k \in V. \tag{C9}$$

The objective function (C1) adds the appropriately scaled fixed vehicle cost to the sum of time spent on the tour ("en route time") over all used vehicles. Constraints (C2) state that all customers are visited by exactly one vehicle, while constraints (C3) make sure that the capacity of the vehicle used for each tour is not exceeded. Constraints (C4) and (C6) enforce that each vehicle leaves and arrives at the depot exactly once, while constraints (C5) ensure that a vehicle arriving at a customer also leaves that customer. Constraints (C7) guarantee that the arrival time at two consecutive customers allow for service and travel time. (C8a) and (C8b) are the time window constraints that ensure that customer service starts during the defined interval. This formulation has non-linear travel time and time window constraints as well as objective function, but the authors present a linearized version of the formulation.

3.2 Early application oriented papers

The first referenced paper regarding fleet sizing is Dantzig and Fulkerson [34]. It investigates the problem of determining the minimum number of tankers required to meet a fixed schedule. Bellmore [9] treats a modification of the same problem where there are an insufficient number of vehicles and a utility associated with each vehicle delivery. Kirby [83] describes a model for determining the number of owned and hired wagons in a railway system. This paper is included because it is used as a reference for later papers related to fleet sizing. Wyatt [157] investigates a similar problem for optimizing a fleet of barges when further barges are available for hire and distinguishes between variable and fixed costs for company-owned vehicles. Gould [76] develops Wyatt's model further studying a case about size and composition of a vehicle fleet for road transport. He uses a linear programming (LP) formulation and introduces a heterogeneous vehicle fleet.

In their book on distributional management, Eilon et al. [50] analyze the case of minimum fleet size with homogeneous vehicles. They also consider the case where vehicles may have different capacity and present an integer programming (IP) formulation of the fleet composition problem. Mole [103] extends the work of Kirby [83], Wyatt [157] and Gould [76]. He uses a dynamic programming (DP) model for the more general case where the optimal fleet size is time dependent. The model can cater for vehicle obsolescence and the timing of investments in new vehicles, but it is limited to considering only a single type of vehicle. New [106] develops an LP model for planning the acquisition and disposal of equipment in a transportation fleet. The model is generally applicable to any problem of fleet planning under conditions of technological change with budgetary constraints. In Williams and Fowler [154], the size of a university motor pool is discussed, and a simulation model is used to estimate the minimum fleet size. Parikh [112] dealt with a practical application for a large transportation company with approximately 10.000 vehicles divided into 250 separate fleets, and developed a method based on queuing theory for solving a fleet sizing and allocation problem. Woods and Harris [155] present a simulation model for estimating production, orders and transportation of concrete. However, the model does not establish the best fleet composition by itself. Doll [44] rejects the heuristics developed so far, and shows a simple alternative procedure for approximating travel distance and estimating the schedules and the number of vehicles required. Etezadi and Beasley [54] developed an IP formulation for what they call the vehicle fleet composition problem. The model concerns the numbers and types of vehicles that the company should operate. They also suggest additional constraints to the model related to the expected distance traveled in a period.

Table 1. Early, application oriented papers

		Year	Method	Problem	Modality
1	Dantzig and Fulkerson [34]	1954	Linear programming	Fleet sizing	Maritime
2	Kirby [83]	1959	Analytic, statistical method	Fleet sizing	Rail
3	Wyatt [157]	1961	Analytic, statistical method	Fleet sizing	Maritime
4	Bellmore [9]	1968	Linear programming	Fleet sizing	Maritime
5	Gould [76]	1969	Linear programming	Fleet composition	Road-based
6	Eilon, Watson-Gandy and Christofides [50]	1971	Analytic, integer programming	Fleet composition	Road-based
7	Mole [103]	1975	Dynamic programming	Fleet sizing	Generic
8	New [106]	1975	Linear programming	Fleet composition	Generic
9	Parikh [112]	1977	Queuing theory	Fleet sizing	Road-based
10	Woods and Harris [155]	1979	Simulation	Fleet composition	Road-based
11	Doll [44]	1980	Statistical method	Fleet sizing	Road-based
12	Williams and Fowler [154]	1980	Simulation	Fleet composition	Road-based
13	Etezadi and Beasley [54]	1983	Integer programming	Fleet composition	Generic

3.3 Standard fleet size and mix vehicle routing problem

The fleet size and mix vehicle routing problem (FSMVRP) differs from the classical VRP by including the composition of the vehicle fleet in the problem definition. Thus, the objective is to minimize a total cost function that includes fixed costs for managing the vehicles in the fleet and variable routing costs. Several authors use benchmark instances to compare the results of different strategies and heuristics. Golden et al. [74] define 20 test instances with 12 to 100 nodes for the standard FSMVRP. The instances are based on VRP instances from [28], and [30]. Later papers use all (or a subset of) the same instances to show the quality of the proposed solution method. Baldacci et al. [2] show a comparison of computational results obtained from different heuristic algorithms on these benchmark instances.

The first published paper with reference to the FSMVRP is Golden et al. [74]. This paper defines the problem and presents a formulation, where the unit running cost is equal for all vehicle types and considered as a constant parameter with a value of one. Thus the variable costs c_{ij} in Formulation A are the same independent of the type of vehicle used on link (i, j). Golden et al. [74] suggest several heuristics for the problem. Some are based on the savings technique for the VRP presented by Clarke and Wright [30], while others are based on a giant TSP-tour that in turn is partitioned into subtours that fit the capacity of the vehicles. The superior algorithm is a multiple partition giant tour algorithm with a modified 3-opt improvement procedure (called Or-opt) as proposed by Or [109]. The authors also developed a procedure to calculate the lower bound that trades off the fixed costs against the routing costs. This lower-bound procedure is later used by Gheysens et al. [71] to create a separate heuristic. First, the heuristic uses a lower bound procedure in branch-and-bound (BB) to create the optimal fleet mix, and then a VRP is solved using that vehicle mix as the available fleet. The assignment of customers to vehicles is determined by solving the generalized assignment problem (GAP) due to the method proposed by Fisher and Jaikumar [62]. Gheysens et al. [70] show that this heuristic usually performs better than the savings and giant tour heuristics, but there is no guarantee to find a feasible solution and the running times are much larger for the lower bound heuristic.

An improved savings heuristic for the FSMVRP is presented by Desrochers and Verhoog [41] by extending their algorithm initially designed for the classical VRP. The method is a matching based savings algorithm using successive route fusion. In each iteration, the best fusion is selected by solving a weighted matching problem. The method yields inferior results when compared to the best results found by other published methods at the time, but on average, it is more stable in finding good results.

Salhi and Rand [127] developed a more advanced heuristic. The heuristic is based on a route perturbation procedure (RPERT) applied on the routes in order to improve the vehicle utilization of the whole fleet. The algorithm starts by solving a classical VRP with a given vehicle capacity to create a starting solution for the search. Then it checks if other vehicle types can be used on the routes and whether it is economical to remove a given route from the solution and insert the customers into other routes. Other refinement procedures like reallocation of customers from one route to another, swapping customers between routes, and combining or splitting routes are performed to check if it is possible to improve the solution. The whole algorithm is repeated several times with other vehicle capacities and starting solutions. The overall performance of this algorithm is better than the earlier algorithms. With the modified algorithm (MRPERT) suggested by Osman and Salhi [111] and the introduction of tabu search [73], the results are improved even more.

So far, the unit running cost has been considered as a constant parameter with a value of one, which means that the driving cost has been considered as equal irrespective of the size of the vehicle. However, Salhi et al. [129] present a MIP formulation for the FSMVRP where they introduce variable unit running costs. They show how simple modifications of some well-known methods can cater for variable running costs and also show the effect of neglecting such variability. The paper is referring to an earlier version of Salhi and Rand [127], and thus modifications are shown on the RPERT algorithm in addition to the savings and giant tour algorithms.

Taillard [137] presents a heuristic column generation (CG) method for solving the VRP with a heterogeneous fleet of vehicles. The paper addresses both the heterogeneous fixed fleet VRP (HFFVRP) with a diverse unit running costs for the different types of vehicles, and the traditional FSMVRP defined by Golden et al. [74]. In addition the diverse unit running costs are introduced even for the traditional FSMVRP. Thus, the variable costs c_{ij} in Formulation A are changed to c_{ijk} which means the cost of traveling from i to j using vehicle k. The method solves a homogeneous VRP for each of the vehicle types (without limitation on the number of available vehicles) by the adaptive memory procedure (AMP) of Rochat and Taillard [123]. The AMP generates a set of solutions to the VRP and extracts the single vehicle tours from these solutions. Each of these single vehicle tours then defines a column in a set partitioning problem that can be solved to optimality. The method outperforms the results from Osman and Salhi [111] for the eight largest instances in the test set. For the heterogeneous VRP with diverse unit running costs, new test instances are generated and solved. In a recent paper, Choi and Tcha [24] present a CG-based approach for solving the FSMVRP inspired by Taillard [137]. They present a tight IP model and the LP relaxation of the problem is solved by CG. A couple of dynamic programming schemes developed for the classical VRP are modified to efficiently generate feasible columns. The proposed algorithm was tested on benchmark instances with both fixed and variable costs. The results confirm the dominance of this algorithm, both in terms of quality and computation time.

The tabu search (TS) metaheuristic [73] has been the basis for several solution methods during the last years. Gendreau et al. [67] proposed a TS method based on the generalized insertion heuristic GENIUS by Gendreau et al. [66] and the AMP developed by Rochat and Taillard [123]. The method was compared to the results presented in Taillard [137] and shown to perform better on instances with variable unit running costs. However, the results were slightly poorer on instances where the routing costs were equal independent of vehicle type. Wassan and Osman [153] present a reactive tabu search metaheuristic with several neighborhood generation mechanisms and special data structures for efficiency. The method uses different deterministic moves for diversification instead of the classical approach with random moves. Variable neighborhood search (VNS) [102] is used and the paper shows how the strategic splitting of the neighborhood and the efficiency of the search are the major ingredients for a good result rather than neighborhood size. The test results are compared to results from earlier papers on the 20 test instances introduced by Golden et al. [74]. Wassan and Osman [153] provide the best results obtained so far on the instances with fixed unit running costs, and slightly poorer than Gendreau

et al. [67] on the instances with variable unit running costs. Lee et al. [88] present a heuristic based on tabu search and set partitioning for determining the composition of a fleet and the corresponding routes. Initial solutions are found using set partitioning on a giant tour found by a sweeping method [72]. Whenever the tabu search obtains a new solution, an optimal vehicle allocation is performed for the set of routes, which is constructed from the current solution by making a giant tour. Experiments were performed on the benchmark problems and for some of the instances new best solutions were found. Even better solutions are obtained by a tabu search heuristic developed by Brandão [15]. The algorithm is deterministic and uses three different methods to create initial solutions for the search. The neighborhood is defined with three different moves; single insertion, double insertion and swap, and the method uses intensification and diversification procedures and allows infeasible solutions with a penalty during the search.
Other methods to solve the FSMVRP are presented by Ochi et al. [107, 108], and Han and Cho [77]. Ochi et al. [107] developed a hybrid metaheuristic that uses genetic algorithms (GA) [80] and scatter search (SS) [85] coupled with a decomposition-into-petals procedure. The same method used with parallel genetic algorithms based on the island model is presented in Ochi et al. [108]. The method shows advantages on some instances when compared to Taillard [137], although Ochi et al. [107, 108] only present the test results by a graph and do not report the exact values. Han and Cho [77] introduce a generic intensification and diversification search metaheuristic that incorporates concepts from deterministic variants of simulated annealing like threshold accepting [48] and the great deluge algorithm [47], in addition to intensification and diversification strategies. Test results on the 20 instances from [74] indicate that the method performs well on the smaller instances, but compared to Taillard [137] and Gendreau et al. [67] the results on the larger instances are poorer. Renaud and Boctor [119] describe a sweep-based heuristic for the FSMVRP. The algorithm is an extension of the VRP algorithm by Renaud et al. [120]. It generates a large number of routes and the selection of which routes and vehicles to use is solved to optimality as a set-partitioning problem. The proposed heuristic outperforms the existing composite algorithms developed so far and is very close to, and sometimes better than, the best-known tabu search-based algorithms. Lima et al. [92] describe a memetic algorithm (MA) [104] for the FSMVRP, a hybrid of a genetic algorithm and a local search based on GENIUS and λ-interchange [110]. The authors tested the algorithm on the 20 instances from [74], and claim to have found new best solutions for eight of them. Engevall et al. [53] use concepts from cooperative game theory and formulate a cost-allocation problem as a vehicle routing game allowing the use of heterogeneous vehicles.
Teodorovic et al. [146] consider the FSMVRP with stochastic demand. The demand of each customer has a uniform distribution in the interval [0, b], and the probability of failure on a route in which the demand exceeds the vehicle capacity is computed. In case of a route failure, the vehicle has to return to the depot for unloading before continuing the route at the next planned customer. A heuristic procedure based on space-filling curves [6] is used to create a giant tour which is split into single routes and assigned to appropriate vehicles with an algorithm for finding several shortest paths in the network.
An exact approach to the FSMVRP is proposed by Yaman [159] when deriving formulations and valid inequalities to compute lower bounds to the problem. Six different formulations are developed; the first four are based on Miller-Tucker-Zemlin constraints [101] and the last two are based on flows. The computational results show that the solutions obtained from the strong formulations and the heuristic solutions in the literature are of good quality. Pessoa et al. [115] present another exact algorithm that introduces new powerful cuts that can be incorporated into a branch-cut-and-price algorithm. The authors solve instances up to 75 vertices to optimality. Baldacci et al. [3] present a MIP formulation for the FSMVRP with fixed unit running costs. They introduce two new classes of inequalities to improve the resulting bounds; new covering-type and fleet-dependent capacity inequalities. The authors state that the new cuts may contribute to the development of new overall consistent algorithms for this class of problems.

Table 2. Standard fleet size and mix vehicle routing problem

		Year	Method	Problem	Modality
1	Gheysens, Golden, and Assad [70]	1984	Integer programming, constructive heuristics	Fleet composition and routing	Generic
2	Golden, Assad, Levy and Gheysens [74]	1984	Integer programming, constructive heuristics	Fleet composition and routing	Generic
3	Gheysens Golden, and Assad [71]	1986	Constructive heuristics	Fleet composition and routing	Generic
4	Desrochers, and Verhoog [41]	1991	Constructive heuristics	Fleet composition and routing	Generic
5	Salhi, Sari, Saidi and Touati [129]	1992	Constructive heuristics	Fleet composition and routing	Generic
6	Salhi and Rand [127]	1993	Mixed integer programming, constructive heuristics	Fleet composition and routing	Generic
7	Teodorivic, Krcmar-Nozic and Pavkovic [146]	1995	Constructive heuristics, stochasticity	Fleet composition and routing	Generic
8	Osman and Salhi [111]	1996	Mixed integer programming, constructive heuristics, tabu search	Fleet composition and routing	Generic
9	Ochi, Vianna, Drummond, and Victor [107]	1998	Hybrid genetic algorithm	Fleet composition and routing	Generic
10	Ochi, Vianna, Drummond, and Victor [108]	1998	Hybrid genetic algorithm	Fleet composition and routing	Generic
11	Gendreau, Laporte, Musaraganyi and Taillard [67]	1999	Tabu search	Fleet composition and routing	Generic
12	Taillard [137]	1999	Heuristic column generation	Fleet composition and routing, Heterogeneous fixed fleet routing	Generic
13	Han and Cho [77]	2002	Generic intensification and diversification metaheuristic	Fleet composition and routing	Generic
14	Renaud and Boctor [119]	2002	Constructive heuristics, set partitioning	Fleet composition and routing	Generic
15	Wassan and Osman [153]	2002	Tabu search	Fleet composition and routing	Generic
16	Engevall, Göthe-Lundgren and Värbrand [53]	2004	Cooperative game theory	Fleet composition and routing	Road-based
17	Lima, Goldbarg and Goldbarg [92]	2004	Hybrid genetic algorithm	Fleet composition and routing	Generic
18	Lee, Kim, Kang and Kim [88]	2006	Tabu search, set partitioning	Fleet composition and routing	Generic
19	Yaman [159]	2006	Lower bound formulations	Fleet composition and routing	Generic
20	Baldacci, Battarra and Vigo [2]	2007	Survey paper	Fleet composition and routing, Heterogeneous fixed fleet routing	Generic
21	Baldacci, Battarra and Vigo [3]	2007	Mixed integer programming, lower bounds	Fleet composition and routing	Generic
22	Choi and Tcha [24]	2007	Integer programming, column generation	Fleet composition and routing	Generic
23	Pessoa, Poggi de Aragão and Uchoa [115]	2007	Branch-cut-and-price	Fleet composition and routing	Generic
24	Brandão [15]	2007	Tabu search	Fleet composition and routing	Generic

3.4 Heterogeneous fixed fleet vehicle routing problem

In the heterogeneous fixed fleet vehicle routing problem (HFFVRP), the fleet size is fixed or bounded by a maximum number, but unlike the classical VRP the vehicles can be of different size and have different fixed and variable costs. In contrast to the FSMVRP, the aim is not to construct an optimal fleet, but to utilize the different vehicles in a best possible way. Benchmark instances for the HFFVRP are similar to the eight largest instances of the set used for FSMVRP introduced by Golden et al. [74]. A comparison of different solution heuristics is shown in [2].

The HFFVRP is considered in several papers by Tarantilis and Kiranoudis. The first one [140] considers a real-world case about distributing fresh milk in Greece. The problem is modeled as a HFFVRP and solved with an algorithm called backtracking adaptive threshold accepting (BATA). The algorithm managed to provide practical solutions and the findings indicated considerable improvements in the operational performance of the company. Next, Tarantilis et al. [142] present a list based variant of the threshold accepting algorithm (LBTA). Here the threshold value used to decide whether to accept or decline a new solution is compared to the maximum value stored in a list. The list is of fixed size and contains the closest relative costs on new solutions found during the search compared to the best found solution. Tarantilis et al. [143] present a general version of the BATA algorithm, with the possibility to raise the value of the threshold when no feasible moves are found according to the existing threshold. Tests performed on the benchmark cases with variable unit running costs defined by Taillard [137] show competitive results for both algorithms, with BATA slightly better than LBTA. In Tarantilis and Kiranoudis [141] a more flexible adaptive memory-based algorithm for real-life transportation is shown. This two-phase construction heuristic is called GEROCA (generalized route construction algorithm). The first phase determines the appropriate pair of customers and vehicles. In the second phase the best insertion point is identified and the appropriate customer or sequence of customers is inserted. The method is tested on two case studies from the dairy and construction sectors which show that it outperforms the authors' previously published approaches to solve the HFFVRP. The method was also able to reduce the fleet size requirement and travel costs drastically compared to the current fleet scheduling practice.

A new intuitive algorithm for solving heterogeneous fixed fleet routing problems is developed by Gencer et al. [65]. The passenger pickup algorithm (PPA) uses the principle cluster-first, route-second which first groups the customers into suitable clusters and then finds the best route visiting all the customers in each cluster. Unlike most other algorithms it takes into account the possibility of vehicle lease when the number of available vehicles falls short. By this definition one can look at the problem as a fixed vehicle fleet problem with a variable fleet of leased vehicles. The algorithm also deals with the possibility of splitting the demand. It is compared with the BATA algorithm of Tarantilis and Kiranoudis [140], but fails to get a better overall result. However, PPA, generally finds solutions that utilize the vehicle capacity better, making it possible to reduce the number of vehicles needed. Li et al. [91] adapted their record-to-record travel algorithm for the VRP [90] to handle the HFFVRP. Like threshold accepting, the algorithm can be described as a deterministic variant of simulated annealing where the deviation from the best solution observed in the search will decide if a new solution is selected. This algorithm gives the best results so far on the eight benchmark instances defined by Taillard [137].

A split delivery variant of the HFFVRP is addressed by Tavakkoli-Moghaddam et al. [145]. The fleet cost is dependent on the number of vehicles used and the total unused capacity. To solve the problem the authors have developed a hybrid simulated annealing method which is tested on several new instances. Results from small instances are compared with the optimal solution found by exact methods, while the larger instances are compared with the lower bound found by solving a giant-tour TSP visiting all the customers. The results show that the proposed heuristic can find good solutions in reasonable time.

Table 3. Heterogeneous fixed fleet vehicle routing problem

		Year	Method	Problem	Modality
1	Tarantilis and Kiranoudis [140]	2001	Threshold accepting metaheuristic	Heterogeneous fixed fleet routing	Road-based
2	Tarantilis, Kiranoudis and Vassiliadis [142]	2003	Threshold accepting metaheuristic	Heterogeneous fixed fleet routing	Generic
3	Tarantilis, Kiranoudis and Vassiliadis [143]	2004	Threshold accepting metaheuristic	Heterogeneous fixed fleet routing	Generic
4	Gencer, Top and Aydogan [65]	2006	Constructive heuristics	Heterogeneous fixed fleet routing	Generic
5	Baldacci, Battarra and Vigo [2]	2007	Survey paper	Fleet composition and routing, Heterogeneous fixed fleet routing	Generic
6	Li, Golden and Wasil [91]	2007	Record-to-record travel algorithm	Heterogeneous fixed fleet routing	Generic
7	Tarantilis and Kiranoudis [141]	2007	Constructive heuristics, tabu search	Heterogeneous fixed fleet routing	Road-based
8	Tavakkoli-Moghaddam, Safaei, Kah and Rabbani [145]	2007	Simulated annealing	Heterogeneous fixed fleet routing, split service	Generic

3.5 Fleet composition and routing problem with time windows

A natural extension of the FSMVRP is to introduce time windows associated with each customer defining an interval wherein customer service has to start. This problem is denoted FSMVRP with time windows (FSMVRPTW). Time windows could be hard, where a solution not satisfying the time window constraints is defined as infeasible. Time windows could also be soft if an earlier or later service does not affect the feasibility of the solution, but is penalized in the objective function. This extension can also be used for fleet dimensioning and routing problems. Liu and Shen [94] introduced benchmark instances for the FSMVRPTW based on Solomon's [136] data sets for the classical VRPTW. Later papers on FSMVRPTW use the same instances as benchmarks.

Desrosiers et al. [42] consider the problem of finding the minimum number of homogeneous vehicles required to visit a set of customers subject to time windows constraints. An optimal solution approach using the augmented Lagrangian method is presented. Two Lagrangian relaxation methods are studied. In the first, time constraints are relaxed, and in the second, constraints requiring that each node must be visited. The case with a heterogeneous fleet is addressed by Ferland and Michelon [61]. They showed that the exact methods developed for the vehicle scheduling problem with time windows and a single vehicle type can be extended to the multiple vehicle type problem. They develop three different heuristics and two exact methods. The heuristics are based on discrete approximation, the assignment method and matching methods, while the exact methods use a column generation technique and time window constraint relaxation.

The standard fleet size and mix vehicle routing problem with time windows (FSMVRPTW) was defined by Liu and Shen [94]. The paper describes several insertion-based savings heuristics, and found that heuristics with the consideration of a sequential route construction parameter yields the best results. 168 new test instances were defined and computational results were reported. Test results on the 20 benchmarking instances for FSMVRP without time window constraints show that the heuristic performs quite well compared to the other heuristics for this problem. The idea of using sequential insertion-based heuristics for the FSMVRPTW was developed further by Dullaert et al. [49]. They present three heuristics, which are extensions of Solomon's [136] heuristic I1 with vehicle insertion savings based on Golden et al. [74]. Liu and Shen [94] do not specify distance and time coefficients, they are implicitly unitized, i.e. one unit of distance equals one unit of time. The solution cost is given by the total fixed costs of the used vehicles and the total schedule time, excluding the (constant) sum of service time at the customers. In the tests, however, Dullaert et al. [49] use only the total schedule time, i.e. both the service time at the customers and the vehicle fixed costs are excluded.

Dell'Amico et al. [39] present a solution approach based on a regret-based parallel insertion procedure and subsequent improvement by ruin and recreate for the FSMVRPTW defined in [94]. Computational experiments show that the method is robust and outperforms Liu and Shen [94] and Dullaert et al. [49] on the benchmark instances. Bräysy et al. [18] present a deterministic annealing metaheuristic for the same problem. The suggested metaheuristic comprises three phases. In the first phase, initial solutions are generated by means of a savings-based heuristic combining diversification strategies with learning mechanisms. In phase two, an attempt is made to reduce the number of routes in the initial solution with a local search procedure. In phase three, the solution from phase two is further improved by a set of four local search operators that are embedded in a deterministic annealing framework to guide the improvement process. Some new implementation strategies for efficient time window feasibility checks are also suggested. The computational experiments show the best results so far on the benchmark instances. The authors also propose two new variants of the FSMVRPTW.

The literature also considers other variants of the FSMVRPTW. Tavakkoli-Moghaddam et al. [144] present a variant where only the depot has a time window and the cost is independent of route length. Thus, the purpose of the depot time window is to restrict the route lengths. An LP model is developed and 18 small test problems are solved to optimality. The authors propose a hybrid simulated annealing algorithm based on the nearest neighborhood heuristic. The proposed method finds the optimal solution for small instances and results for ten larger test instances are reported. Calvete et al. [20] developed a MIP model for the VRP with hard and soft time windows, a heterogeneous fleet of vehicles and multiple objectives. To solve the problem they suggest a two-phase approach where the first phase enumerates the feasible routes and computes the total penalty incurred by each route due to deviations from targets. The second phase solves a set partitioning problem to get the best set of feasible routes. The methodology is tested on instances inspired by a real-life problem and other instances derived from some of the standard problems defined by Solomon [136].

Yepes and Medina [160] consider the case with soft time windows in a context of a heterogeneous fixed fleet of vehicles. They present a three-step hybrid local search algorithm based on a probabilistic variable neighborhood search [102] for solving the problem. The first step is an economic route construction based on GRASP [60] that builds a population of solutions. In step two, an evolution strategy based on so called extinctive selection is used to diversify the search and to select the best solution in the population, while step three is a post optimization method based on threshold accepting with restarts. Three new test instances based on one of the Solomon problems are defined, and results are reported.

Another approach for fleet dimensioning under time window constraints is given by Vis et al. [152] when they describe transportation between buffer areas and storage areas at a container terminal. The objective is to minimize the vehicle fleet size such that the transportation of each container starts within its time window. The problem is formulated as an IP model and simulation is used to estimate the fleet size.

Table 4. Fleet composition and routing problem with time windows

		Year	Method	Problem	Modality
1	Desrosiers, Sauve and Soumis [42]	1988	Lagrangian relaxation	Fleet sizing	Generic
2	Ferland and Michelon [61]	1988	Constructive heuristics, branch-and-bound, column generation	Heterogeneous fixed fleet routing	Generic
3	Liu and Shen [94]	1999	Constructive heuristics	Fleet composition and routing	Generic
4	Dullaert, Janssens, Sörensen and Vernimmen [49]	2002	Constructive heuristics	Fleet composition and routing	Generic
5	Vis, de Koster and Savelsbergh [152]	2005	Integer programming	Fleet sizing	Special
6	Tavakkoli-Moghaddam, Safaei and Gholipour [144]	2006	Hybrid simulated annealing	Fleet composition and routing	Generic
7	Yepes and Medina [160]	2006	Hybrid local search, threshold accepting	Heterogeneous fixed fleet routing	Generic
8	Bräysy, Dullaert, Hasle, Mester and Gendreau [18]	2007	Deterministic annealing, restart	Fleet composition and routing	Generic
9	Calvete, Gale, Oliveros and Sanchez-Valverde [20]	2007	Mixed integer goal programming, set partitioning	Fleet composition and routing	Generic
10	Dell'Amico, Monaci, Pagani and Vigo [39]	2007	Constructive heuristics, ruin and recreate algorithm	Fleet composition and routing	Generic

3.6 Fleet composition and routing problems with multiple depots

With multiple depots, the problem is even more complex than the standard FSMVRP. The problem is to determine which customers to be served by the different depots in addition to find the optimal composition of the fleet and the best possible routes for the vehicles. Salhi and Sari [128] address the FSMVRP with multiple depots (FSMVRPMD) by simultaneously allocating customers to depots, composing the vehicle fleet and constructing delivery routes. The authors propose a multi-level composite heuristic and design two reduction tests to enhance its efficiency. The heuristic consist of three levels. First, a starting solution is found by solving the FSMVRP for each of the depots and its natural customers. Borderline customers are then inserted to the existing routes. Second, a composite heuristic tries to improve the solutions found for each of the depots. Third, a composite heuristic that considers all depots simultaneously is used. Some of the refinement modules of the heuristic are taken from the RPERT procedure of Salhi and Rand [127]. The method is tested on 26 benchmark problems for the multiple depot VRP with competitive results. Salhi and Fraser [126] include the problem of deciding the number and location of depots to the FSMVRPMD. They present an iterative approach which combines two different heuristics for solving the location problem, the routing problem and the fleet composition problem simultaneously.

Time windows are introduced to the FSMVRPMD by Dondo and Cerda [45], when developing a three-phase approach for the problem. The first phase aims at identifying cost-effective clusters, while the second phase assigns the clusters to vehicles and sequences them on each route. The ordering of customers and scheduling of vehicle arrival times are performed in phase three by using a MIP model. The method is tested on some of Solomon's [136] benchmark problems. Irnich [82] presents a special type of a multiple depot pickup and delivery problem with a single hub and heterogeneous vehicles. The problem is to find a cost minimal set of routes, which realize all transportation requests when every request has an associated time window. All routes have to visit the hub once. They also need to be cycles starting and ending at one specific depot, but different routes can use different depots. The author presents a network model, computes lower bounds, and solves a set-partitioning problem to find solutions. The solutions are then compared by using different vehicle fleet scenarios.

Table 5. Fleet composition and routing problem with multiple depots

		Year	Method	Problem	Modality
1	Salhi and Fraser [126]	1996	Constructive heuristics	Fleet composition and routing	Generic
2	Salhi and Sari [128]	1997	Constructive heuristics	Fleet composition and routing	Generic
3	Irnich [82]	2000	Network model, lower bounds, set partitioning	Fleet composition and routing	Generic
4	Dondo and Cerda [45]	2007	Constructive heuristics	Fleet composition and routing	Generic

3.7 Fleet composition in arc routing

Arc routing problems (ARPs) described in [51, 52] is a well-established area of research related to the vehicle routing problem. The aim of an ARP is to determine a least-cost traversal of a specific arc subset of a graph. Typical examples of ARP applications are snow removal, while garbage collection and mail delivery are also often modeled as ARPs. The difference from a VRP is that in an ARP, the routes should traverse arcs instead of visiting nodes. The Chinese postman problem (CPP) is about determining a minimum length walk covering each arc in the graph at least once. However, when it is required to traverse only a subset of the arcs in the graph, the problem becomes a rural postman problem (RPP). Golden and Wong [75] introduced the capacitated arc routing problem (CARP) where each arc has a non-negative weight and all arcs with a positive weight must be traversed by a fleet of vehicles based at the depot.
Ulusoy [150] considers the fleet size and mix problem for capacitated arc routing. The solution procedure is closely related to the giant-tour algorithms explained in Section 3.3 for FSMVRP. First, a CPP or a RPP is solved to form a giant tour; second, the giant tour is partitioned into single vehicle subtours; third, a shortest path problem is solved on the new network, and fourth, the method tries to improve the solution by exchanging arcs among the vehicle tours. Another example of arc routing in fleet composition and routing problems can be found in Del Pia and Filippi [38], describing a capacitated arc routing problem for their truck and trailer problem about waste collection in Italy. Finally, Perrier et al. [113] use an arc routing model for plowing in their survey about winter road maintenance.

Table 6. Fleet composition in arc routing

		Year	Method	Problem	Modality
1	Ulusoy [150]	1985	Constructive heuristics	Fleet composition and routing	Generic
2	Del Pia and Filippi [38]	2006	Constructive heuristics, variable neighborhood search	Trucks and trailers	Road-based
3	Perrier, Langevin, and Campbell [113]	2007	Survey paper for winter road maintenance, simulation methods, heuristics	Fleet composition and routing	Road-based

3.8 Fleet composition in network optimization problems

Network optimization problems differ in structure from traditional vehicle routing problems. Rather than finding the optimal set of tours that visits every customer once, they are concerned with the selection of arcs in a graph in order to satisfy some flow requirements and to minimize the total system cost. The requirements are often expressed in the form of commodities that should be transported from an origin to a destination within a given time interval. Turnquist [148] describes several research opportunities in the area of supply of transportation services with focus on service design. The research topics are classified into three principal categories; vehicle scheduling, vehicle control and capacity provision which includes fleet sizing. A basic

classification of approaches to fleet sizing problems are developed using three factors; type of traffic, shipment size relative to vehicle size, and deterministic vs. stochastic analysis.
Fleet dimensioning in network optimization problems is considered several times in the literature. Gertsbach and Gurevich [69] describe a general method to compute the optimal fleet size for a transportation schedule in a network environment. They define the deficit function for each terminal as the difference between the number of departures and arrivals over the time interval considered. The same deficit function was used in an approach for the vehicle scheduling process for Egged (The Israel National Bus Carrier) proposed by Ceder and Stern [21]. Here the main optimization criterion was to minimize the vehicle fleet investment and the idle time. Sim and Templeton [135] developed a queue-dependent vehicle-dispatching rule with options to use special vehicles for relieving long waiting lines. They derived an efficient recursive algorithm to analyze the performance of the system and used an average cost criterion to determine the fleet size and dispatching strategy. The problem of fleet sizing and redistribution of empty equipment in a hub-and-spoke transportation network is considered by Du and Hall [46]. They treated the problem from an inventory theory point of view with decentralized stock control policies for empty equipment. Their methodology uses a mathematical model to determine the fleet size and control variables, and then the probabilities of stock-out as a function of these parameters. A heuristic methodology for solving large-scale fleet sizing problems is presented by McGinnis [96]. The heuristic uses two decision variables: varying resource capacity for meeting demand, and varying task duration.
The problem of sizing a fleet under uncertain conditions is considered by Turnquist and Jordan [149]. They developed a model for sizing a fleet of containers used to ship parts from a single manufacturing plant to a group of assembly plants. The parts are produced in a deterministic production cycle, but container travel times are stochastic. The optimal fleet size depends on the relative cost of owning containers compared to the cost of running out. This work is continued in a more strategic context by Beaujon and Turnquist [8] when formulating a model to optimize both sizing and utilization of the vehicle fleet simultaneously under dynamic and uncertain conditions. The model is designed to answer questions about the number of vehicles in the fleet, the location and time dependent size of the vehicle pools, and how available vehicles should be allocated between loaded movements, empty movements and vehicle pools. The paper presents a network approximation to this model and proposes a solution procedure. Another paper which incorporates strategic decisions on fleet composition and routing is Bookbinder and Reece [14]. They formulate a multicommodity capacitated distribution-planning problem as a non-linear MIP model. Distribution from factories to customers is two-staged via depots and the number and location of depots is another part of the decision process. The problem is solved as a generalized assignment problem within an algorithm for the overall distribution / routing problem based on Benders decomposition.
One class of network optimization problems considered in the literature is to determine the optimal fleet size and the resulting vehicle routes when external carriers are available. Ball et al. [5] consider this problem with a fleet of homogeneous vehicles and discuss some approximate solution strategies. Klincewicz et al. [84] look into the case of delivery of goods from a warehouse to local customers. They look into strategic decisions about whether to maintain a private delivery fleet, to use external carriers or a combination of these. Their algorithm divides the area of service into sectors and decides how to best serve each sector. They base their solution approach on a single-source capacitated facility location formulation that determines the private fleet size and the specific assignment of sectors to private or external carriers. The problem with use of external carriers in addition to the own fleet is also considered by Chu [29] and Bolduc et al. [12]. Each customer should be served either by one of the vehicles of a heterogeneous internal fleet or by external carriers, and the decision of when to use external carriers may bring significant cost savings to the company. Chu [29] presents a mathematical model and a heuristic algorithm for the problem, while Bolduc et al. [12] present a heuristic that obtains better results. Their heuristic first

selects which customers to be served by external carriers and then construct an initial solution that in turn is improved by 4-opt moves.
A synchronized production and distribution problem for a large-scale supply chain network with a fixed heterogeneous fleet is presented by Bolduc et al. [13]. It is a multi-period problem that takes the production schedule, inventory costs and the schedules for demand at the retailers into account. A mathematical model and four different heuristics for direct deliveries are presented, and Some extensions to deal with the multiple-retailer-route situation are also proposed.
List et al. [93] developed a formulation and solution procedure for fleet sizing in a transportation network with uncertainty in demand and operating conditions. The formulation focuses on robust optimization and incorporates the risk into the expected recourse function of a two-stage stochastic programming formulation. The fleet-sizing problem in the truck-rental industry is addressed by Wu et al. [156]. Trucks that vary in capacity and age are utilized in a time-space network to meet customer demand. Operational decisions about demand allocation and truck repositioning and tactical decisions about purchasing and selling vehicles are explicitly examined in a linear programming model to determine the optimal fleet size and mix. A solution approach using Benders decomposition and Lagrangian relaxation is developed to solve large scale instances of the problem.
One specific case of a network design problem is presented by Lai and Lo [86] when simultaneously considering the optimal fleet size, routing and scheduling for the ferry services in Hong Kong. A mixed integer multiple origin-destination network flow model with ferry capacity constraints is developed. In addition to the model, a two-phase heuristic algorithm is presented. Phase one determines a set of paths, and phase two searches for solution improvements using the path set as base. The study considers a single ferry type, but the authors argue that the model and algorithm can be extended to include multiple ferry types with different service characteristics.

Table 7. Fleet dimensioning in network optimization problems

		Year	Method	Problem	Modality
1	Gertsbach and Gurevich [69]	1977	Statistical method	Fleet sizing	Generic
2	Ceder and Stern [21]	1981	Statistical method	Fleet sizing	Road-based
3	Sim and Templeton [135]	1982	Statistical method	Fleet sizing	Generic
4	Ball, Golden, Assad and Bodin [5]	1983	Mixed integer programming, constructive heuristics	Fleet sizing	Road-based
5	Turnquist [148]	1985	Classification of research opportunities in transportation	Fleet sizing	Generic
6	Turnquist and Jordan [149]	1986	Statistical method	Fleet sizing	Generic
7	Bookbinder and Reece [14]	1988	Non-linear mixed integer programming	Fleet composition and routing	Road-based
8	Klincewicz, Luss and Pilcher [84]	1990	Mixed integer programming, Lagrangian relaxation	Fleet sizing	Road-based
9	Beaujon and Turnquist [8]	1991	Integer programming	Fleet sizing	Generic
10	Du and Hall [46]	1997	Statistical method	Fleet sizing	Generic
11	McGinnis [96]	1997	Constructive heuristics	Fleet sizing	Generic
12	List, Wood, Nozick, Turnquist, Jones, Kjeldgaard and Lawton [93]	2003	Stochastic programming	Fleet sizing	Generic
13	Lai and Lo [86]	2004	Mixed integer programming, constructive heuristics	Fleet composition and routing	Maritime
14	Chu [29]	2005	Integer programming, constructive heuristics	Heterogeneous fixed fleet routing	Generic
15	Wu, Hartmann and Wilson [156]	2005	Linear programming, Benders decomposition, Lagrangian relaxation	Fleet composition and routing	Road-based
16	Bolduc, Renaud and Montreuil [13]	2006	Mixed integer programming, constructive heuristics	Heterogeneous fixed fleet routing	Generic
17	Bolduc, Renaud and Boctor [12]	2007	Constructive heuristics	Heterogeneous fixed fleet routing	Generic

3.9 Other problems related to fleet composition and routing

In some routing problems it is easy to see how the fleet composition aspect will affect the solution, although fleet composition is not necessarily defined as a part of the problem. Some important examples are described in this section.

3.9.1 The truck and trailer routing problem

The truck and trailer routing problem (TTRP) is described by Chao [22] as a variant of the traditional VRP where the customers can be divided into two groups: vehicle customers who can be reached with a complete vehicle with a trailer, and truck customers who can only be reached by a single truck. The objective of this problem is to construct the best possible set of routes where all customers are visited once, and the customers' limitations are taken into account. Single trucks can serve some routes and a complete vehicle with a trailer can serve others. A truck carrying a trailer can also serve routes with both vehicle and truck customers, by uncoupling and park the trailer while the truck drives a subtour to serve truck customers. For these routes, deciding on the optimal parking place for the trailer is a part of the problem. We observe that in the case where there exist different types of trucks, trailers and combinations of those, the TTRP will also be a fleet composition problem. Figure 2 shows the structure of a solution to a TTRP. Here the thick lines describe routes which are served by a truck carrying a trailer, the squares illustrate parking places where the trailer is left, and the thin lines describe routes served by a single truck.

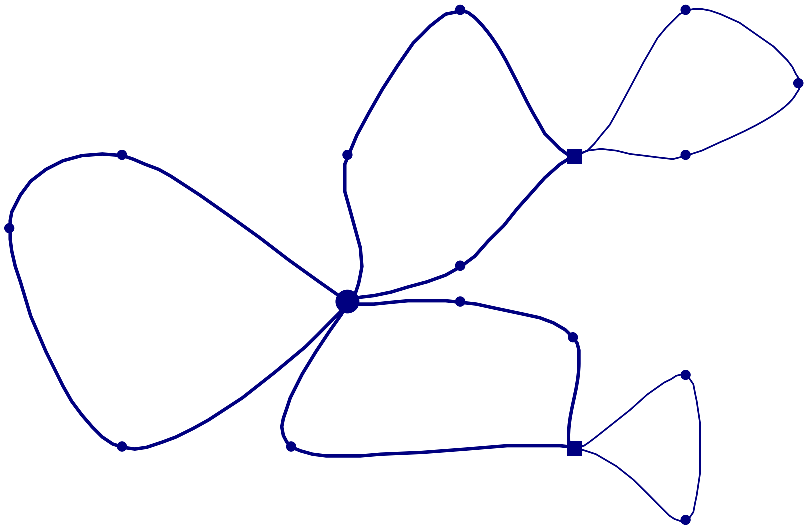

Figure 2. Structure of a TTRP solution

Nag et al. [105] describes the site-dependent VRP, a problem related to the TTRP as the company has a fixed heterogeneous fleet of vehicles and the customers have restrictions on which type of vehicle that can visit the customer. Semet [131] present a mathematical model of a partial accessibility constrained VRP related to the real-life case presented in Semet and Taillard [132]. This model assumes a heterogeneous fixed fleet of trucks, and trailers that all have the same capacity. All available trucks are used, but the number of trailers to use is a decision. An enumerative procedure in which bounds are obtained from a Lagrangian relaxation is presented. In Gerdessen [68] all customers are reachable by trailers, but some are located at places where maneuvering the complete vehicle with a trailer is very difficult. Thus, the need for parking the trailer is calculated as a function of the service time at the customers. All customers have unit demand, each tour consists of exactly one subtour without the trailer and all vehicles have the same capacity. Three different construction heuristics with an improvement method are tested on instances with different vehicle capacities. However, the problem treated is not exactly fleet composition as the fleet size is fixed. Chao [22] describes a model for the TTRP, and Scheuerer [130] addresses the same problem. The model uses a fixed number of trucks and trailers. The problem is to construct the best possible routes and decide whether to visit the truck customers on a single tour from the depot, or on a subtour with a truck from a parked trailer. Both papers describe a construction heuristic and a following tabu search method to solve the problem.

The literature also describes several real-life problems related to the TTRP. Vahrenkamp [151] describes the problem of milk collection in Western Germany where the truck uses the trailer as a mobile depot. The fleet size is determined with respect to trucks and trailers with a fixed capacity. Hoff and Løkketangen [79] address a similar case for milk-collection in Western Norway with multiple depots and different types of trucks and trailers. Del Pia and Filippi [38] describe a case for waste-collection in Italy as a generalization of the capacitated arc routing problem. Large vehicles (compactors) cannot traverse the narrowest streets of the town, and smaller vehicles (satellites) need to collect the waste in those streets. Tours for compactors and satellites are constructed in a way such that the satellites will meet the compactors at the same node within the same time interval. Then a satellite can unload its content to the associated compactor. The problem described in the paper assumes a fixed number of compactors and satellites of equal size, and does not take fleet dimensioning into account.

3.9.2 The rollon-rolloff vehicle routing problem

The rollon-rolloff VRP (RRVRP) is related to the TTRP and defined by Bodin et al. [11] as a combination of an asymmetric vehicle routing problem and a bin packing problem. In RRVRP,

tractors move large trailers between the locations where they are positioned and a disposal facility. Here, a tractor is defined as a vehicle without an own loading capacity. At the disposal facility, full trailers are emptied and empty trailers are attached to tractors. The problem is to minimize the total travel time of the tractors needed to service all of the trips requiring service. A secondary objective is to minimize the number of vehicles needed to provide the desired service. De Meulemeester et al. [37] consider a real-life case that falls under the definition of an RRVRP. A skip rental firm in Belgium delivers empty lift-and-carry containers (skips) and collects the full skips from the customers in addition to deliver the full skips to dump sites. The customers require different types of skips, and are assigned to special dumpsites depending on the type of skip. Two simple heuristics and an exact algorithm based on branch-and-bound are presented. Bodin et al. [11] present a mathematical programming formulation, and two lower bounds and four heuristic algorithms are developed and tested. The partial enumeration method (PEM), which is a truncated dynamic programming procedure, achieves the best and most reliable results. In a later paper, Baldacci, et al. [4] consider the RRVRP with multiple disposal facilities and multiple inventory locations. The problem is modeled as a time constrained VRP on a multigraph and formulated as a set partitioning problem with an additional constraint that limits the number of vehicles. An exact procedure for solving the problem is described. The procedure combines three lower bounds derived from different relaxations of the problem.
Tan et al. [139] describe a multi-objective problem that they call the truck and trailer vehicle routing problem (TTVRP), not to be confused with the TTRP described in the previous subsection. The problem is equivalent to the RRVRP. The solution to the problem consists of finding a complete routing schedule for serving the jobs with minimum routing distance and minimum number of trucks. A mathematical programming model is developed. A hybrid multi-objective evolutionary algorithm featured with specialized genetic operators, variable-length representations, and a local search heuristic is applied to find Pareto optimal solutions for the problem.

3.9.3 The vehicle routing problem with multiple use of vehicles

In classical VRP it is assumed that each vehicle is traveling only one route during the planning horizon. However, in real situations with relatively small vehicles and short distances, it is possible to assign several routes to the same vehicle and thus use fewer vehicles. One variant of the VRP is the VRP with multiple use of vehicles (VRPM) where the same vehicle can be assigned to several routes during a given planning period. Ronen [124] shows how to develop algorithms that assign a set of trips to a heterogeneous fixed fleet of trucks. He proposes a two-step heuristic based on the assignment of trips to vehicles first and slide-and-switch of trips between vehicles second. Taillard et al. [138] describe a tabu search heuristic for the same problem. The method first generates a set of vehicle routes where each route obeys the VRP constraints. Then it makes a selection of a subset of these routes using an enumerative algorithm. Finally, it assigns the selected routes to the vehicles using a bin packing heuristic.
A real-life distribution problem for a British biscuit manufacturer is considered in Brandão and Mercer [16]. In addition to the common VRP constraints, the problem is a multi-trip problem with restricted access to customers for some vehicles. The customers impose delivery time windows, and the schedules must respect legal driving time rules and contain time breaks for the drivers. The company owns a heterogeneous fleet of vehicles, but can hire other vehicles if the capacity is insufficient. A heuristic that combines nearest neighbor and insertion concepts with a two-phase tabu search is developed to solve the problem. In Brandão and Mercer [17], the heuristic is adjusted for the basic VRPM and used on the instances generated by Taillard et al. [138]. The algorithm by Taillard et al. [138] presents slightly better results, but it appears to be slower and does not balance the tours as well as the algorithm of Brandão and Mercer [17]. Prins [117] presents several heuristics for the HFFVRP with the optional possibility of each vehicle to perform several trips. The most efficient one progressively merges small starting trips while ensuring that the fleet can perform them. A secondary objective of the heuristic seeks to minimize

the number of required vehicles. The paper presents a real-life case of a French manufacturer of furniture.

Table 8. Other problems related to fleet dimensioning and routing

		Year	Method	Problem	Modality
1	Nag, Golden and Assad [105]	1988	Constructive heuristics	Trucks and trailers	Road-based
2	Vahrenkamp [151]	1989	Constructive heuristics	Trucks and trailers	Road-based
3	Ronen [124]	1992	Mixed integer programming, constructive heuristics	Multiple use of vehicles	Road-based
4	Semet [131]	1995	Integer programming, Lagrangian relaxation, branch-and-bound	Trucks and trailers	Road-based
5	Gerdessen [68]	1996	Constructive heuristics	Trucks and trailers	Road-based
	Taillard, Laporte and Gendreau [138]	1996	Tabu search	Multiple use of vehicles	Generic
6	Brandão and Mercer [16]	1997	Tabu search	Multiple use of vehicles	Road-based
7	De Meulemeester, Laporte, Louveaux and Semet [37]	1997	Constructive heuristics, branch-and-bound	Rollon-rolloff	Road-based
8	Brandão and Mercer [17]	1998	Tabu search	Multiple use of vehicles	Road-based
9	Bodin, Mingozzi, Baldacci and Ball [11]	2000	Integer programming, lower bounds, constructive heuristics	Rollon-rolloff	Road-based
10	Chao [22]	2002	Constructive heuristics, tabu search	Trucks and trailers	Road-based
11	Prins [117]	2002	Constructive heuristics	Heterogeneous fixed fleet routing	Road-based
12	Baldacci, Bodin and Mingozzi [4]	2006	Set partitioning	Rollon-rolloff	Road-based
13	Del Pia and Filippi [38]	2006	Constructive heuristics, variable neighborhood search	Trucks and trailers	Road-based
14	Scheuerer [130]	2006	Constructive heuristics, tabu search	Trucks and trailers	Road-based
15	Tan, Chew and Lee [139]	2006	Integer programming, hybrid evolutionary heuristic	Rollon-rolloff	Road-based
16	Hoff and Løkketangen [79]	2008	Constructive heuristics, tabu search	Trucks and trailers	Road-based

3.10 Road-based industrial cases

Real-life routing problems often consist of a large number of different constraints and objectives. They cannot necessary be classified into one specific group of VRPs. Several papers listed in other subsections do also have aspects which relate them to real-life problems. This subsection lists some papers regarding fleet composition in real-life applications with a rich problem structure that are difficult to classify elsewhere.

Some papers describe the implementation of decision support systems (DSS) used to assist managers in the planning process. Avramovich et al. [1] describe a DSS used to plan the fleet configuration of North American Van Lines. The DSS use a large-scale LP-model to find a suggested mix of vehicles which is used by the company when forecasting and planning activities. Another DSS is presented by Couillard [31]. The DSS was developed with reference to the trucking industry in Quebec, Canada, but it is not reported whether it has been used in practice. It is designed to assist managers in every step of the planning process and it can also be used to plan the composition of the fleet. By using a multicriteria approach for evaluating plans, it helps the

manager to forecast the demand, choose relevant criteria and to generate and evaluate the alternative plans with respect to the criteria. A stochastic programming (SP) model based on a model first presented in Couillard and Martel [32] is used to generate different plans.
The determination of the optimal fleet size and types of vehicles for an Indian oil company is considered by Saksena and Ramachandran [125]. The problem consists of three different aspects: transportation of workers to static operating points, transportation of workers to operating points whose location change over time, and transportation of children to schools. The authors propose a methodology that uses a cluster-first route-second strategy to find a surplus or a deficit in each vehicle category.
Another real-life case for a Swiss grocery chain distributing goods to its stores is presented by Semet and Taillard [132]. In addition to capacity constraints of the vehicles and time windows for deliveries, the problem takes the heterogeneous character of the fleet into account. The paper also deals with the fact that due to accessibility restrictions, not all stores can be served by a road train consisting of a truck carrying a trailer. Thus, the stores are defined to be either a truck-store that only can be accessed by a single truck, or a trailer-store that can be accessed by a road train. The truck-stores can be served on tours operated by a single truck, or on subtours on a tour operated by a road train where the trailer is parked at a trailer-store. The paper shows a tabu search approach that obtains solutions significantly better than those implemented in practice. A related problem is presented by Rochat and Semet [122] when considering a problem that occurs in a major Swiss company producing pet food and flour. The company uses a heterogeneous fixed fleet that serves its customers from one depot. A large variety of restrictions are considered; accessibility, time windows, capacity of the vehicles, duration of routes, drivers' breaks, and so on. The paper proposes two heuristic methods; a fast straightforward insertion procedure and a method based on tabu search.
Privé et al. [118] studied the problem that arises from the distribution of soft drinks and collection of recyclable containers in a Quebec-based company. The problem is modeled as a FSMVRPTW with additional volume constraints, and a modified objective function that also considers revenues from selling recyclable material. Three construction heuristics and an improvement procedure are suggested for the problem. The nearest neighbor heuristic (NNH) iteratively constructs vehicle routes by adding unvisited customers to routes according to the nearest neighbor criterion and using the smallest vehicle available. The first petal heuristic (FPH) constructs a set of feasible routes based on several runs of NNH and then makes an optimal selection by solving a generalized set partitioning problem. The second petal heuristic (SPH) is different from FPH in the way seed customers for the tours are selected. A subsequent improvement phase using the 3-opt neighborhood in each route, a restricted 2-interchange between the routes and a possibility to merge routes if there exists an available vehicle able to serve all customers are added to each of the construction heuristics. The results obtained show that the second petal heuristic with the improvement procedure performs best, and it achieves a 23% distance reduction on the real-life case.
A real-life mail collecting problem in an urban area is considered by Mechti et al. [97, 98, 99]. The problem is modeled as a FSMVRPTW. For all three references, a tabu search approach for the problem is introduced. The search alternates between local moves involving only one route, and global moves which can change the solution structure more dramatically and allow a wide exploration of other possible solutions. The authors tried the method on a data set given by the French Postal Services Company, with good results. In addition to the tabu search, an exact algorithm with set partitioning, dynamic programming and branch-and-bound is described in [98].
Leung et al. [89] present a paper considering distribution from production facilities in mainland China and to the company headquarters in Hong Kong. The problem includes decisions about whether to use own vehicles or to hire vehicles of different capacities and costs in China or in Hong Kong. In addition, routing decisions about where to cross the border have to be made. The authors present a goal programming model for solving the problem and use different economic scenarios to show the robustness of the model.

Demand-responsive paratransit service is a mode of passenger transportation typically for elderly people or people with disabilities that offers door-to-door service from any origin to any destination in a service area. Fu and Ishkhanov [63] address the fleet related decision problems for such services, i.e., what type and how many vehicles to use. By using a real-life example, the paper illustrates the performance of the service system with respect to the size and type of its vehicles. A greedy search heuristic identifies the optimal fleet mix that maximizes the operating efficiency of a service system. Diana et al. [43] present a continuous approximation model to forecast the number of vehicles needed to provide a predetermined quality on a demand responsive transit service.

Perrier et al. [113] present a survey of models and methodologies for winter road maintenance. The survey considers problems on strategic, tactical, and operational level, and refers to several industrial cases. One part of the survey is devoted to removal of snow from roads. For plowing, they refer to ARP models arc routing models, while for snow disposal the referred models are of VRP type. The survey presents simulation methods, rule-based construction, and metaheuristics. There are a number of references to optimization and analytical models that include the fleet sizing aspect in the context of winter road maintenance.

Table 9. Road-based industrial cases

		Year	Method	Problem	Industry
1	Avramovich, Cook, Langston and Sutherland [1]	1982	Linear programming	Fleet sizing	Van lines
2	Saksena and Ramachandran [125]	1986	Constructive heuristics	Fleet composition and routing	Oil company
3	Couillard and Martel [32]	1990	Stochastic programming	Fleet sizing	Trucking industry
4	Couillard [31]	1993	Stochastic programming	Fleet sizing	Trucking industry
5	Semet and Taillard [132]	1993	Constructive heuristics, tabu search	Trucks and Trailers	Grocery chain
6	Rochat and Semet [122]	1994	Constructive heuristics, tabu search	Heterogeneous fixed fleet routing	Pet food and flour
7	Mechti, Poujade, Roucairol and Lemarie [97]	1999	Tabu search	Fleet composition and routing	Mail collection
8	Mechti, Poujade, Roucairol and Lemarie [98]	2001	Dynamic programming, set partitioning, branch-and-bound	Fleet composition and routing	Mail collection
9	Mechti, Poujade, Roucairol and Lemarie [99]	2001	Tabu search	Fleet composition and routing	Mail collection
10	Leung, Wu and Lai [89]	2002	Goal programming	Fleet composition and routing	Cross-border logistics
11	Fu and Ishkhanov [63]	2004	Greedy search heuristic	Fleet composition and routing	Paratransit service
12	Diana, Dessouky and Xia [43]	2006	Statistical method	Fleet composition and routing	Transportation service
13	Privé, Renaud, Boctor and Laporte [118]	2006	Constructive heuristics	Fleet composition and routing	Soft drink distribution
14	Perrier, Langevin, and Campbell [113]	2007	Survey paper, simulation methods, heuristics	Fleet composition and routing	Winter road maintenance

3.11 Maritime industrial cases

A survey on ship routing and scheduling is published by Christiansen et al. [26]. The objective of the paper is to show the status of research regarding ship routing and scheduling and numerous papers with industrial cases are presented. The paper starts at the strategic fleet planning level, and discusses the design of fleets and sea transportation systems. It then continues with the

tactical and operational fleet planning level and considers problems that comprise various ship routing and scheduling aspects.
Several papers regarding the optimal fleet design in maritime transportation has been published since the classical paper of Dantzig and Fulkerson [34] on the minimal number of tankers required for meeting a fixed schedule. Cho and Perakis [23] consider the case of deciding the fleet size and design of optimal liner routes for a container shipping company. They generate a number of candidate routes for the different ships and use an LP model to find the optimal route mix and service frequency. A MIP model which in addition considers possible investments for expanding the fleet is also presented. Xinlian et al. [158] consider a similar problem and present a fleet planning model aiming at determining both the ship types to add and the optimal fleet deployment plan. The paper shows both the mathematical programming model of the dynamic fleet planning and its algorithm.
Another model for determining the optimal number of ships and the plan for fleet deployment is given in Bendall and Stent [10]. They consider a short-haul hub and spoke feeder operation based in Singapore. Fagerholt [55] presented the problem of deciding an optimal fleet mix of ships and corresponding routes for each ship for a liner shipping system along the Norwegian coast. The solution method presented consists of three phases. In phase one all feasible single routes are generated for the largest ship available. In phase two the single routes generated in phase one are combined into multiple routes, and in phase three a set partitioning problem is solved. The method cannot handle ships with different speed, and thus Fagerholt and Lindstad [58] proposed a new solution algorithm considering this aspect. The algorithm was tested on a real-life problem for offshore supply ships in the Norwegian Sea and considerable savings were identified compared to the solution used at that time.
Pesenti [114] addresses the problem of resource management for a merchant fleet. The problem involves the employment of the shipping company's fleet and decisions on purchase and utilization of own ships to satisfy customer demands. The paper shows a hierarchical model for the problem and describes heuristic techniques for solving problems at different decision levels.
Sigurd et al. [134] discuss an application of advanced planning support in designing a transportation system for Norwegian companies who depend on maritime transportation between Norway and Central Europe. To achieve faster and more frequent transportations by combining tonnage, the possible construction of up to 15 new ships are considered. The problem is a variant of the general pickup and delivery problem with multiple time windows, and the paper shows how to solve it by using a heuristic branch-and-price algorithm.
Some papers consider the fleet dimensioning aspect within maritime supply chains. Larson [87] provides a model used by the City of New York to design a new logistic system to transport sewage sludge from city-operated wastewater treatment plants to an ocean dumping site 106 miles offshore. Richetta and Larson [121] present an extension of the problem when considering the refuse maritime transportation system in New York. Here, trucks unload their cargo at road-based transfer stations where the refuse is placed in barges and then towed by tugboats to the Fresh Kills Landfill on Staten Island. A computer-based event simulation model for decision support in fleet sizing and operational planning is developed. Fagerholt and Rygh [59] describe another simulation model regarding the design of a maritime supply chain. The problem considered is about designing a maritime system for fresh water transportation from Turkey to Jordan in the Middle East. The fresh water is transported to discharging buoys by the Israeli coast and then in pipelines from the buoys to a tank terminal ashore and finally through a pipeline from Israel to Jordan. The analysis aimed at answering questions about the total supply chain, i.e. the needed number, capacity and speed of ships and design and capacity of buoys, pipelines and tank terminals. Another crucial question is how sensible the chain is to failures of each component in the chain.
Mehrez et al. [100] consider a real industrial ocean cargo-shipping problem. They reduce the large scale, dynamic and stochastic problem to a deterministic model and use commercial optimization software to solve the problem. The number and size of ships to charter in each time period are

among the decisions to make in addition to the number and location of transshipment ports to use and the transportation routes from discharging ports to customers. A simulation study for ferry traffic in the Aegean Islands that is used as a decision support system for regional development is presented by Darzentas and Spyrou [36]. The model has built-in flexibility to consider additional variables and parameters based on data availability and scenarios to be examined. The problem of determining the size of a refrigerated container fleet is addressed by Imai and Rivera [81]. A simulation model is developed for fleet sizing and various scenarios are analyzed to determine the most convenient composition of the fleet. Crary et al. [33] present the use of quantitative methods in conjunction with expert opinions in their paper regarding the size of the U.S. destroyer fleet. They use an analytic hierarchy process to gather expert opinions. Distributions based on these expert opinions are derived and integrated into a MIP model for determining the effectiveness of a fleet with a particular mix of ships. The ideas are applied to the planning scenario for a potential conflict on the Korean Peninsula.

In industrial and tramp ship routing and scheduling, the number of ships in the fleet is fixed. In industrial shipping all cargoes have to be assigned to a ship and picked up at its origin and delivered at its destination, while in tramp shipping the transporter lift all committed cargoes and the profitable optional ones. In some cases the controlled fleet may have insufficient capacity to serve all cargoes for an industrial ship scheduling problem or all committed cargoes for a tramp ship scheduling problem during the planning horizon. In such a case some of the cargoes can be serviced by spot charters, which are ships chartered for a single voyage. There exists several applications described in the literature for both tramp and industrial shipping where some of the cargoes might be serviced by spot charters, see for instance [7, 19, 25, 56, 133]

Table 10. Maritime industrial cases

		Year	Method	Problem	Industry
1	Larson [87]	1988	Constructive heuristics	Fleet composition and routing	Sewage sludge transport
2	Mehrez, Hung and Ahn [100]	1995	Mixed integer programming	Fleet composition and routing	General cargo-shipping
3	Pesenti [114]	1995	Constructive heuristics	Fleet composition and routing	Container shipping
4	Cho and Perakis [23]	1996	Linear programming	Fleet Sizing	Container shipping
5	Darzentas and Spyrou [36]	1996	Simulation	Fleet composition and routing	Ferry traffic
6	Richetta and Larson [121]	1997	Simulation, constructive heuristics	Fleet composition and routing	Solid waste transport
7	Bausch, Brown and Ronen [7]	1998	Simulation	Spot charters	Liquid bulk transport
8	Sherali, Al-Yakoob and Hassan [133]	1999	Mixed integer programming	Spot charters	Oil-tanker industry
9	Fagerholt [55]	1999	Constructive heuristics, set partitioning	Fleet composition and routing	Container shipping
10	Fagerholt and Lindstad [58]	2000	Integer programming, set partitioning	Fleet composition and routing	General cargo-shipping
11	Xinlian, Tengfei and Daisong [158]	2000	Linear programming, dynamic programming	Fleet composition and routing	Container shipping
12	Bendall and Stent [10]	2001	Mixed integer programming	Fleet composition and routing	Container shipping
13	Imai and Rivera [81]	2001	Simulation	Fleet Sizing	Container shipping
14	Christiansen and Fagerholt [25]	2002	Set partitioning	Spot charters	Maritime transportation
15	Crary, Nozick and Whitaker [33]	2002	Mixed integer programming	Fleet composition and routing	US destroyer fleet
16	Fagerholt and Rygh [59]	2002	Simulation	Fleet composition and routing	Fresh-water transport
17	Christiansen, Fagerholt and Ronen [26]	2004	Survey paper	Various routing and scheduling problems	Maritime transportation
18	Fagerholt [56]	2004	Simulation	Spot charters	Maritime transportation
19	Sigurd, Ulstein, Nygreen and Ryan [134]	2005	Set partitioning, heuristic branch-and-price	Fleet composition and routing	General cargo-shipping
20	Brønmo, Christiansen, Fagerholt and Nygreen [19]	2007	Constructive heuristics, set partitioning	Spot charters	Maritime transportation

4 Critique, trends and directions

In Section 2 the importance of combined fleet composition and routing in industry was presented by describing aspects and issues in maritime and road-based transportation. In Section 3, there is a comprehensive survey of the relevant research literature. A number of mismatches between industrial aspects and focus in the research community have already been pointed out. In this section we give a constructive critique of the research efforts in view of industrial needs. Moreover, trends in industry are discussed and fruitful areas for further research are indicated.

4.1 A critique of the research literature

The first observation to mention is the scarcity of papers on the combined fleet composition and routing problem. Other variants of the VRP are clearly more popular among academics, such as the VRPTW. This is certainly related to he increased complexity and size of the combined problem.

It is quite common in industry to have a non-uniform composition of vehicles, that is, a fleet of vehicles with different characteristics (i.e. it is heterogeneous). Many commercial VRP solvers can handle the simpler problem of having a VRP and a (given) mixed fleet, solving the day-to-day routing problems using the available vehicles. Questions are often asked along the lines of: ”What is the best fleet size and mix to maximize my profits for the next period”, with period being half a year, with a daily routing problem underneath. Even if fixed daily routes are used, as might be the case for a service network design problem, this is a stochastic problem with an associated expected value.

A large part of the literature focuses on operational questions, along the line of “what to do given a certain fleet mix and a given set of service requests”. This is in contrast to the more tactical, or strategic, “which vehicles should we acquire to best solve our daily routing problem for the next half year”. There is a big absence of papers addressing these more tactical questions, and also on how to make robust or resilient solutions. The treatment of stochastic problems, along with the treatment of risk and flexibility, is virtually non-existent in the literature.

One fundamental problem in the research literature is that most of the relevant research is quite general and with idealized (or simplified) problem models. The simplifications are done in most aspects of the problem: vehicles are uniform, travel cost is equated to the Euclidian distance, drivers are integrated with the vehicles, etc. Reporting is also along the same simplified dimensions. This approach is not adequate for industrial applications, where the vehicle fleet seldom is uniform, drivers need to be treated separately, driving speed is related to the time of day, etc.

The reporting of experimental results is not standardized enough, as different researchers have different ways of reporting, and are using different idealized models. This makes comparisons difficult and confusion easy. The lack of good benchmarks is addressed in the final part of this chapter.

Although many papers are formulated as being transportation mode independent, the bulk seems to be inspired by aspects of road-based transportation. It may be argued that fleet composition and routing problems are more acute for the waterborne fleets, as the capital costs, lead time and life time of vessels are typically much larger than for trucks.

It should be emphasized that many of the papers surveyed do contain real world aspects in their models and problem descriptions. This trend is increasing and is a step in the right direction for the industry. We would at this point like to give credit to the scientific journals for their generally positive attitude towards publishing articles about real world problems and associated research.

4.2 Industrial trends

The transportation industry is seeing many of the same trends as other industrial areas. There are a lot of mergers and acquisitions with increased competition, and thus increased focus on profitability. With larger companies often come larger and more complex problems. There is in this context a shift of focus from the routing of vehicles in the individual companies, to a focus on the whole supply chain. It is also clear that most transportation companies will have a heterogeneous fleet, both because of the flexibility this gives when servicing customers in different locations or with different demands, and because of the natural diversity that arises when vehicle acquisitions are made over time, as the company grows.

In society in general, there is a trend towards wanting and rewarding lower emissions and increased sustainability. This might induce a shift in the modality of the transportation by the introduction of bonus or penalty systems, e.g. reflected in the relative cost of different energy types.

When looking at the information available for decision making relevant for the fleet composition and routing problems in the industry, there is more information, and more types of information, available to the decision maker than ever before. Historical data are collected in data warehouses. This can be customer related patterns, driving times (or speeds) on road segments at different times of the day, etc., as well as positional data acquired from GPS aboard the vehicles and RFID for easier tracking of goods. Also available are electronic guidance systems (on electronic maps) and electronic orders, giving rise to an increasingly dynamic environment where routing plans need to be remade on the fly due to incoming orders needing more immediate response.

The world of transportation management is thus becoming increasingly more dynamic. The environment that a typical company operates in can change rapidly, and plans need to be remade. Typical examples are express pick-up orders that require rerouting or changing traffic conditions altering the amount of resources required. Hence, there is an increasing need for more robust, or resilient, plans that can adapt to a diverse set of changes to the input data in a graceful and contained way, usually focusing on minimal disruption of the current plan, at least for the near future.

All this requires the decision support systems to be flexible and based on many sources of information. Some of the problems (at the operational level) need very fast answers, while others (at the strategic level) can be allowed significantly longer response times. The combined problems of fleet composition and routing will in this setting mostly be on the tactical or strategic level, while the operational variants are more like VRPs with a given, heterogeneous fleet, this is called the HFF – heterogeneous fixed fleet problem.

What industry needs are decision support systems able to handle these extra requirements. We envision systems that rely much more on historical (and stochastic) data, and where the plans are developed interactively with a decision maker. In this setting, the system will typically also give the decision maker a set of solutions to choose from, where the decision maker can choose one, or parts of one, as a basis for further refinement.

These new decision support systems will also lead to the need for a new type of planners, or decision makers. Historically, most planners have been recruited from within the company, where the planners would have gained experience in the way the company works. These types of planners usually are used to manual planning, and may have problems in a rapidly changing company, and world. The new breed of planners will typically have a much stronger theoretical background, being able to understand the strengths and limitations of the tools they have available.

Dynamic planning must be supported in a better way. This is for instance the case for return cargos with trucks, and spot cargos for tramp shipping. In these cases the planner will try to utilize (i.e. sell) spare capacity that arises due to the dynamics of the planning. Most of the capacity over time might be locked into contracts, but the extra capacity might also be utilized on an ad-hoc basis.

4.3 Future Research

A clear trend in the research literature is that the problems addressed are becoming more complex, containing more real world detail. The associated models thus become richer, and larger, containing more constraints. These models can in general not solve the problems to optimality with today's methodology and equipment. This holds true for most of the problem instances that are of a size that is relevant for the industry. As can be seen from the survey part, the use of metaheuristic and combined methods for solving these problems dominates the solution methods, even though some work using exact methods are reported.

Progressively larger instances of a given model can be solved. This is to be expected, and reflects both the general advancement in computing power and optimization method developments. Applying richer models means that the actual size of the problem, in terms of customers and

vehicles, that can be solved to optimality typically gets smaller. The models are also becoming more integrated, focusing e.g. on both fleet size and mix and supply chain management and other aspects of the supply chain, such as location.
DSSs are becoming increasingly important as containers for optimization kernels or "black boxes", having access to all relevant data, and a proper GUI for user interaction. One trend here is also to let the optimization module suggest several good, diverse, solutions, rather than just the best solution found according to the implemented model (see e.g. [57] and [95]).
What is conspicuously lacking in the literature is the treatment of uncertainty, and the associated concepts of risk and flexibility. Many of the underlying data used when solving real-world fleet size and mix problems are stochastic, being only known by their probability distributions or approximations thereof. This can for instance relate to travel times or customer demand. Historical data are also a source of stochastic information that can be used to make predictions about the future.
We believe that this stochasticity is an inherent part of many of the problems related to fleet composition and routing. One simple example is to decide on a fleet size and mix for the next half year for an underlying VRP problem. The solution will be the set of vehicles that minimizes the expected cost (or maximizes the expected profit) over the set of future daily problems, where the stochasticity typically lies with the expected customer location, customer demand, or traffic conditions.
There is also a need to look at contracts together with fleet composition and routing, and more specifically, how to integrate routing into more strategic models apart from at a generalized level. This also indicates that models (and methods) combining the fleet composition and routing problem with game theory, auction theory and investment theory would be of relevance and value.

4.3.1 A need for better benchmarks

To be able to evaluate new methods and systems fairly, good and diverse test data should be available to the research community. Unfortunately, for the types of problems addressed in this paper, the set of test-case instances based on these problems leaves much to be desired. The problems might be based on real-world cases, but are then usually simplified somewhat. Often the test-cases are modified versions of previously published test-case portfolios. A large amount of randomization is often used. This randomization is usually assuming independent problem instance variables, so that possible correlations between the variables in the corresponding real world problem are lost.
Even if the testdata is available, it will be in a user-defined format, and special I/O routines need to be written for each case.
The common test cases should have the following attributes:

- They should be real-world based
- They should be as rich as possible, with rich meaning containing sufficient details
- They should adhere to a common format based on a structured language, for instance XML
- Solutions (and not just the objective function values) should be published
- Solutions (or solution sets) may have the following attributes: best solution, good solution, diverse set of solutions
- The publishing medium for test-cases should be the Web.

5 Summary and conclusions

Efficient transportation is becoming more and more important to society. Economic growth, increasing consumption, and globalization increase the need for transportation. Strong competition between transportation providers and between goods owners, partly due to globalization, leads to higher demands on efficiency, customer service, timeliness, reactivity, and cost reduction in transportation. The industry faces fleet composition challenges at all decision levels. For transportation providers and goods owners alike, a goal is to strike an optimal balance between owning and keeping a fleet and subcontracting transportation, as well as deciding on the right overall fleet composition seen in relation to the transportation requirements. This gives rise to a family of combined fleet composition and routing problems, which has been the focus of this article.

This paper has first given an overview of the industrial aspects of combined routing and fleet composition problems in transportation, showing the importance of the field and the difficulties associated with solving these types of problems.

The main difference between road-based and maritime transportation modes is that road-based transportation very often is based around the use of one or more depots. The transported goods are very often consolidated at the depots, and the depots are usually also the base for the vehicle fleets. This can be contrasted with the maritime mode which is usually more like an endless sequence of PDPs, with no central depot.

Three basic mathematical models from the literature have been presented. A thorough survey of most relevant papers on combined fleet composition and routing has been given with a view to their industrial aspects. Around 120 articles have been reviewed. Of these around 50% are of a general nature with no explicit focus on a specific transportation mode, while 25% specifically discuss land-based operations. The remaining 25% consist of papers on maritime transportation. It should be stated that the general papers seem to be mostly inspired by land-based applications. Most of the papers discuss tactical decision-making. The bulk of papers investigate metaheuristics due to the difficulty of solving larger instances exactly.

In the survey part, the papers are grouped according to the major classification scheme within the combined fleet composition and routing, as defined in section 3. These include the standard *fleet size and mix VRP*, and important variants like *heterogeneous fixed fleet VRP*, problems with time windows, and problems with multiple depots, as well as other minor variants of the problem. Finally, industrial cases are surveyed, both for land-based and maritime applications.

The major critique of today's research into combined fleet composition and routing problems can be regarded as two-fold. First, there is a tendency to describe problems that are too idealized and far from the requirements of the real world. The second, but related issue is the lack of treatment of stochastic aspects, together with concepts of risk and robustness. We believe that both of these shortcomings will be handled better in future research.

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Appendix

Table 12. Abbreviations used in the paper

AMP	Adaptive memory procedure
ARP	Arc routing problem
BATA	Backtracking adaptive threshold accepting
BB	Branch and bound
CARP	Capacitated arc routing problem
CG	Column generation
CPP	Chinese postman problem
CVRP	Capacitated vehicle routing problem
DP	Dynamic programming

DSS	Decision support system
EIA	Energy Information Administration
EU	European Union
FPH	First petal heuristic
FSM	Fleet size and mix
FSMVRP	Fleet size and mix vehicle routing problem
FSMVRPMD	Fleet size and mix vehicle routing problem with multiple depots
FSMVRPTW	Fleet size and mix vehicle routing problem with time windows
FTL	Full truckload
GA	Genetic algorithms
GAP	Generalized assignment problem
GDP	Gross domestic product
GENIUS	Generalized insertion procedure – unstringing and stringing
GEROCA	Generalized route construction algorithm
GPS	Global positioning system
GRASP	Greedy randomized adaptive search procedure
GUI	Graphical user interface
HFF	Heterogeneous fixed fleet
HFFVRP	Heterogeneous fixed fleet vehicle routing problem
IP	Integer programming
LBTA	List based threshold accepting
LNG	Liquefied natural gas
LP	Linear programming
MA	Memetic algorithms
MD	Multiple depots
MIP	Mixed integer programming
MP	Mathematical programming
MRPERT	Modified route perturbation procedure
NNH	Nearest neighbor heuristic
NP	Non-deterministic polynomial time
OR	Operations research
PDP	Pickup and delivery problem
PEM	Partial enumeration method
PPA	Passenger pickup algorithm
RFID	Radio frequency identification
RPERT	Route perturbation procedure
RPP	Rural postman problem
RRVRP	Rollon-rolloff vehicle routing problem
SA	Simulated annealing
SND	Service network design
SP	Stochastic programming

SPH	Second petal heuristic
SS	Scatter search
TA	Threshold accepting
TS	Tabu search
TSP	Traveling salesman problem
TTRP	Truck and trailer routing problem
TTVRP	Truck and trailer vehicle routing problem
TW	Time windows
VNS	Variable neighborhood search
VRP	Vehicle routing problem
VRPM	Vehicle routing problem with multiple use of vehicles
VRPTW	Vehicle routing problem with time windows

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