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Report

A Review of the Physics of Aircraft Trailing Vortices

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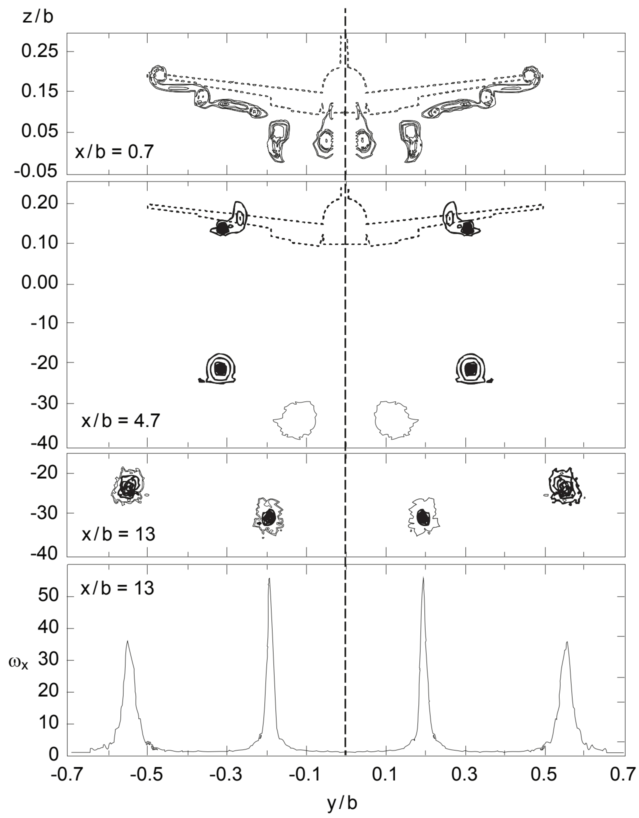

Streamwise vorticity in the wake of an airliner (de Bruin et al 1996).

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ABSTRACT

In this review the formation, movement and persistence of aircraft trailing vortices are discussed. The focus is on parameters that are important for the safety and productivity of air travel and transport. This review investigates the predictability of the vortices, in particular in terms of the lifespan and durability of vortex pairs. The strong interplay between the rotational and axial velocity components in the vortex is analyzed. Different types of turbulence that can influence the airflow within the vortices are discussed, including the various atmospheric and ground-related factors that dominate the vortex behavior. Finally, instances of wakes rising back, and the still unexplained bursting of the vortices are discussed. The review also briefly covers the potential for wake vortex detection and control.

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Nomenclature

ϵ	turbulent dissipation rate
η	nondimensional intensity of atmospheric turbulence
Γ	vortex circulation
τ	nondimensional time
b_0	vortex pair spacing
ATC	Air Traffic Control
CFD	Computational Fluid Dynamics
CM	Confined Model
DM	Diffuse Model
LES	Large-Eddy Simulation
MTOW	Maximum Take Off Weight
NASA	National Aeronautical Space Agency (US)
PD	Predictable Decay
PDF	Probability Distribution Function
PIV	Particle-Image Velocimetry
VLCT	Very Large Commercial Transport

1 Introduction

1.1 Motivation

Unless an aircraft is cleared for visual approach to an airport, specific spacing rules must be followed. This is particularly important for the approach when aircrafts have nearly identical flight paths. In some cases the spacing is set by wake turbulence constraints which depend on the weight of the leader and the follower aircraft. There is a small number of weight classes and a *matrix* of separation distances that are used as *Wake Avoidance Separation Criteria* (see Table 1). An example is that a Boeing 737-800 (~ 80 ton) airplane following an Airbus 330-300 (~ 210 ton) airplane must be no closer than $5\,nm$ ($\sim 9.3\,km$). The wake turbulence rules vary somewhat from country to country and are adjusted now and then. Increasing the distances may benefit safety. However, at many airports that will increase costs and delays. A coordinated scientific effort towards an accepted and rational process by which to set the *optimal distances* appears very worthwhile, but 40 years of scientific work have had little impact on *Air Traffic Control* (ATC) practice. That may change in the years to come. Both US and European research programs [14] have been initiated and are likely to end up with new *dynamic ATC rules* regarding optimal distances between aircraft that take into account the dynamic atmospheric conditions and the aircraft characteristics in a more detailed manner than the existing static ones.

Even though the equations of airflow motion, i.e. the Navier-Stokes equations are known, an analytical approach to the ATC wake turbulence problem faces two problems. First is the difficulty in calculating the flow around an airplane in landing configuration, due to limitations in numerics and turbulence modeling or in measuring the flow. To describe the vortices in enough detail, even relatively small components such as the landing gear may need to be included; a full solution is years away. An even larger problem is that the wake environment, which dictates boundary conditions for a solution of the equations that describe downstream wake development, is ever changing. The boundary conditions reflect the winds, atmospheric turbulence, and stratification, which all have a strong effect on the motion and persistence of the vortices.

With such stochastic inputs, a theory giving the trajectory and lifespan of the vortices behind an airplane in a given weight class to within $\pm 30\%$ accuracy, for instance, would be very impressive. However, in ATC rules, the difference between 4 and 5 nm is significant. This conflict, and the insufficient confidence level for CFD (Computational Fluid Dynamics) , explain why spacing rules appear to be based on flight tests and wake encounter statistics, rather than on airflow predictions.

The state of the art, and the challenge of a *transition to physics-based ATC rules*, is illustrated in Figure 1. It shows the nondimensional time (τ) until initial destruction of pairs of trailing vortices versus the nondimensional intensity (η) of the surrounding atmospheric turbulence. Here $\tau \equiv t\,\Gamma/2\pi b_0^2$ and $\eta \equiv 2\pi\,\epsilon^{1/3} b_0^{4/3}/\Gamma$, where t is time, Γ is the vortex circulation, b_0 is the vortex pair spa-

Table 1: ICAO *Wake Avoidance Separation Criteria*. Aircraft categories *Light*, *Medium* and *Heavy* are defined by MTOW $\leq$ 7000 kg, 7000 kg $<$ MTOW $<$ 136000 kg and MTOW $\geq$ 136000 kg, respectively (MTOW = Maximum Take Off Weight).

Leading Aircraft	Following Aircraft		
	Heavy	Medium	Light
Heavy	4 NM	5 NM	6 NM
Medium	3 NM	3 NM	5 NM

cing, and ϵ is the turbulent dissipation rate (the larger ϵ is, the more turbulent the atmosphere feels when flying). Γ and b_0 are the dominant characteristics of a wake. A correlation between η and τ was proposed by Tombach [11] and Crow & Bate [23], who collected flight tests and created a theory. Laboratory tests by Sarpkaya & Daly [28] and Liu [10] and numerical simulations by Spalart & Wray [19] are also shown. With logarithmic axes, the figure indicates that a correlation exists (higher turbulence causes faster destruction), that all predictions agree, and that the theory is successful. η can be measured, and τ could then be predicted. However, the same figure in linear axes demonstrates that (even with modest statistical samples, below 100 cases each) the scatter in τ is so large it makes the information nearly useless for ATC rules. One τ unit roughly corresponds to 1 nm, for very large airplanes. The only use of this material may be to indicate gross trends for wake lifetime versus airplane size.

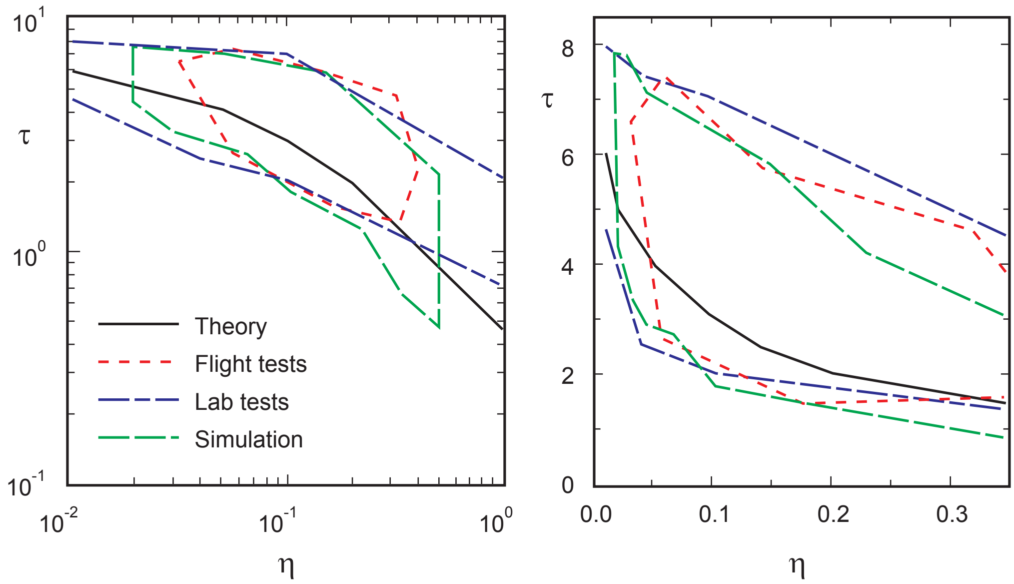

Figure 1: Lifespan (τ) of airplane wakes in different turbulence intensities (η). Logarithmic plot to the left, and linear plot to the right.

Theory and empirical models suggest a correlation between encounter frequency and time of day, season, and latitude, which control the atmospheric conditions. Such a correlation is far from obvious from the statistics; this is a concern, quite apart from the current capabilities of the theories. Field studies in which atmospheric stratification was estimated lead to the same concern (Kopp [5], Rudis et al [21]): The correlation between stratification and wake lifetime is very weak (possibly, in both studies, ground effect dominated over stratification).

The analytical approach further suffers from vague criteria about desirable wake characteristics; in particular, it has been taken for granted that alleviating the wake by reducing the circulation of the vortices is good. However, a vortex pair with higher circulation descends faster (the velocity is $\Gamma/2\pi b_0]$), thus reducing the probability of an encounter with a following aircraft. By that standard, the Concorde wake, with large circulation and small pair spacing, is excellent. This suggests a trade between encounter frequency and severity. Today designers receive no guidance when trading circulation for spacing (their product Γb_0 is fixed by the airplane's weight and speed).

Two reasons for scientists to increase efforts in later years are the introduction of airliners with capacities beyond 600 passengers and the congestion at many airports. Presently, the Airbus 380-800 is the world's largest passenger airliner, and the airports at which it operates have upgraded facilities to accommodate it. Airbus 380-800 provides seating for 525 people in a typical three-class configuration or up to 853 people in an all-economy class configuration. The so-called *Very Large Commercial Transport* (VLCT) aircraft will be less attractive if larger ATC separations cancel any advantage in terms of arriving seats per hour and per runway. As for the congestion in the existing system, any procedure to increase arrival frequencies with the same level of safety would be most welcome, and could save the construction of new runways or entire airports. Trailing vortices also control the motion of exhaust gases such as nitrous oxides (Gerz & Erhet [26], Jacquin & Garnier [15]). This is more meaningful in cruise flight, especially for a supersonic transport at high altitudes, than in approach. Conversely, there is concern about roof damage under the airport approach corridor; this may result from the vortices forming half-rings and thus extending a low-pressure core to the ground like a small tornado (Lee [7]; see Section 3.6).

1.2 Past Reviews, Textbooks, and Dedicated Conferences

The year 1975 saw the clear Annual Review by Widnall (brief and of wider scope than this one), a useful survey by Lee, and the extensive monograph of Donaldson & Bilanin. The latter, along with a turbulence model that failed to grow into a proven tool, offers a mine for clear mathematics and thoughtful warnings, some of which have been ignored. Donaldson & Bilanin's belief that *the corner of rationality has been turned* now appears optimistic. The 1970s were a time of vigorous effort, partly motivated by the Boeing 747, with the theoretical advances seeming more impressive. The last 40 years have not provided much progress in fundamental knowledge, and deep analytical advances are unfortunately not foreseen. Improvements in instrumentation have taken taken place, but the vast improvements in numerical simulation capabilities have yet to be fully exploited.

Reasons for the lower level of activity in the 1980s appear to be that the ATC rules were viewed as satisfactory and that much heavier airplanes did not enter service. Exceptions were the experimental work of Phillips & Graham [31] and Sarpkaya ([27], Sarpkaya & Daly [28]) and the widely recognized empirical model of Greene [6].

Saffman's [17] book, again of a much wider scope, is an excellent source for analytical studies of vortices. They are inviscid or laminar, and many assume special forms for the inputs, for instance the lift distribution along the span of the wing. Some of these limitations can be overcome with brute force numerical work. He provides brief discussions of the physics. Green's [25] chapter on wing tip vortices is longer than the present article but covers basic topics not covered in this article, such as lifting-line theory. It also emphasizes the near field more deeply. Two valuable sets of conference proceedings are those of the Federal Aviation Administration (FAA) [4] and of the Advisory Group for Aerospace Research and Development (AGARD) [30], although it is not clear either meeting had any effect on ATC practice. The Web site http://www.volpe.dot.gov/wv offers an excellent list of vortex turbulence references, with abstracts, thanks to Dr. James Hallock, the head of the Volpe Center's Aviation Safety Division. A few stimulating recent findings, and a need to better articulate the key open questions, have motivated this article.

1.3 Study Tools

Wake turbulence theories should be pursued vigorously, although the word turbulence by itself reduces the hope of a complete solution. Typically, inputs such as wing features or atmospheric stratification are studied one at a time since they have effects of similar magnitude, with no reason to expect that they are additive. The best role for theories is to explore scale effects and to suggest *active* systems for wake destruction (Crow & Bate [23]). Active systems are easier to analyze in that the controlled input dominates the stochastic ones, at least when control is successful.

Numerical simulations, boosted by computer improvements, will contribute more and more as they can be free of assumptions needed by the theory. Typically, the simulation will cover nonlinear regimes. However, simulations are still restricted to greatly simplified situations and are time consuming. Those shown in Figure 1 assumed simple vortex pairs with little detail in the cores and no axial flow; standard Kolmogorov turbulence; and no stratification, shear, or ground plane. The geometry of the airplane was not considered at all; only Γ and b_0 entered the description.

Wind-tunnel tests produce flow fields of good quality and accurate measurements, but only to about 15 spans downstream, and that only in unusual facilities. There is renewed interest in wake surveys [9], helped by computer-driven systems and by arrays of 5-hole probes. Details of the near-wing flow field matter because some features survive in the far wake (Section 2.1). Two other industrial incentives are (a) to attribute drag or lift changes to local geometry changes and (b) to define separate components for the drag, usually the *viscous* drag and the *induced* or *vortex* drag, which scale differently with Reynolds number. Unfortunately, that distinction remains too ambiguous; efforts to date base the distinction on some equations of lifting-line theory, which are defeated by separation and viscosity. *Apparent induced drag* becomes *apparent viscous drag* as the survey plane is moved downstream [16], whereas the total drag calculated is quite accurate. This ambiguity creates an opportunity for a theoretical leap forward; the definitive theory would deal with multiple nonplanar lifting surfaces and work without the light-loading approximation.

A concern in wind tunnels is vortex meandering, which artificially diffuses the time-averaged vorticity. The strategies devised to correct for this effect (Devenport et al [29]) may not be definitive. In particular, the probability distribution function (PDF) of the vortex position is often assumed to be Gaussian. However, meandering caused by a large-scale mode of oscillation in the tunnel could give the vortex the shape of a sine wave, in which case the PDF would be M-shaped instead of bell-shaped. Conditional-sampling and spectral-filtering techniques are also useful.

Tow tanks allow long nondimensional lives for the vortices, but at the expense of the Reynolds number. There are also concerns about residual turbulence and stratification, and the analysis is limited to flow visualization and particle-image velocimetry (PIV) , which is less mature than 5-hole probes. The extrapolation to flight situations requires extreme care.

Flight tests contain the complete physics and are essential to the validation of any prediction or control method. Deliberate tests such as tower fly-bys are expensive, if only because of the revenue potential of an airliner for a day, and allow little control over atmospheric conditions (a problem NASA is addressing through an extended test campaign. Using commercial flights is cost-effective, but restricted to a narrow range of flight conditions. Quantitative measurements are difficult, and often impossible out of ground effect, as the instrumentation is on the ground (Kopp [5], Rudis et al [21]). In a typical test, an airplane with a span of $40\,m$ flies $80\,m$ above ground level; the vortices have an initial descent velocity of 2 m/s and are followed for 100 s. These numbers speak for themselves, but abusive generalizations have been made from such tests (Section 3.6). Flow visualization, by condensation or smoke, is very valuable but not fully reliable. In particular, condensation disappears when the minimum temperature increases past a threshold value that does not depend only on the vortex characteristics. Many statements in the literature regarding vortices being *destroyed* or *cut* are probably erroneous.

A predictive ATC technology will, clearly, draw on many sources of knowledge. The physical model will be semi-empirical and may be very complex, more than Greene's [6]; the system is likely to use weather predictions and may depend on real-time measurements of the vortices. Progress will depend on effective efforts with each of the above tools and on constant discussions aimed at clear concepts and consensus.

2 Vortex Formation

2.1 Overall Features

The initial organization of the trailing vorticity, or *roll-up*, has received much attention the lat 80 years. Recent analysis has been stimulated by new measurement techniques and by high-performance numerical simulations. Accurate distributions of velocity are sought for several purposes. One is to simulate encounters with a following airplane. Another is the possible influence on later phenomena, such as the behavior in a stratified atmosphere and the instabilities and destruction. For both purposes, the issue is under reexamination whether the vortex system *rapidly rolls up into two counter-rotating vortices*.

For high-lift configurations with part-span flaps, many near-field measurements and near-inviscid simulations have produced multiple vortices (Donaldson & Bilanin [3], Krasny [20]). However, most of the experts believe all the vortices on one side of the plane of symmetry do merge, based on visualizations and on the rarity of *multiple hits* on instrumented towers. Also, simulations today are not conclusive for such effects of turbulence and of viscous velocity defects. If the vortices do not merge, the aspect of the *mature* wake is quite different. Instead of descending in a quasi-steady manner, two or more vortex pairs *tumble down* together. The wake is not followed by an oval of fluid from the initial altitude; instead, it periodically exchanges fluid with the atmosphere (Spalart [18]).

Experimental support for the idea that vortices remain separate long after roll-up for some configurations was strengthened by tests in the Duits-Nederlanse wind tunnel (DNW) (de Bruin et al [1]). The wakes generated by a airliner model are shown in Figure 2 through the streamwise vorticity. The inboard flap is highly deflected and the outboard flap partly retracted, which is not the exact design setting. In the first survey plane, many features of the wing can be identified: edges of the inboard flap, outboard flap, and tip (the model had neither horizontal tail, engine, nor landing gear). The vortices from the tip and the edge of the outboard flap merge about 7 spans behind the model; the merging begins at 5 spans as the weaker vortex winds around the stronger one (top of second frame). The circulation shares for the tip-region vortex, flap vortex, and side-of-body vortex are 61%, 75%, and negative 36%, respectively.

At 13 spans, the vortex from the inboard flap is still isolated from the tip vortex. The tip vortex is now inboard at $y/b = \pm 0.2$ as part of the tumbling. The side-of-body vortex has, unfortunately, left the survey area. The tip and flap vortices are both so tight that there is no reason to expect merging in the near future.

Tests at $18°$ angle of attack instead of $7°$ showed full merger after 9 spans, giving a single pair, apparently as a result of creation of the flap vortex as more diffuse and with a large axial velocity deficit. The multiple-vortex wake may be limited to a range of angle of attacks (and some flap types), but that range seems to include the approach value, which is the most relevant.

There is a common misunderstanding among researchers, regarding the initial behavior of the vortex sheet. Consider the two-dimensional (2D) sheet created by an elliptical circulation distribution: $\Gamma(y) = \Gamma_0(1 - 4y^2/b^2)^{0.5}$, if $|y| \leq b/2$, and 0 otherwise. Initially the vertical velocity component w at the vortex sheet equals $w_0 = -\Gamma_0/b$ for $|y| < b/2$. This is the classical *uniform downwash* of lifting-line theory. Some authors then expect the exact sheet to descend, without deformation, at

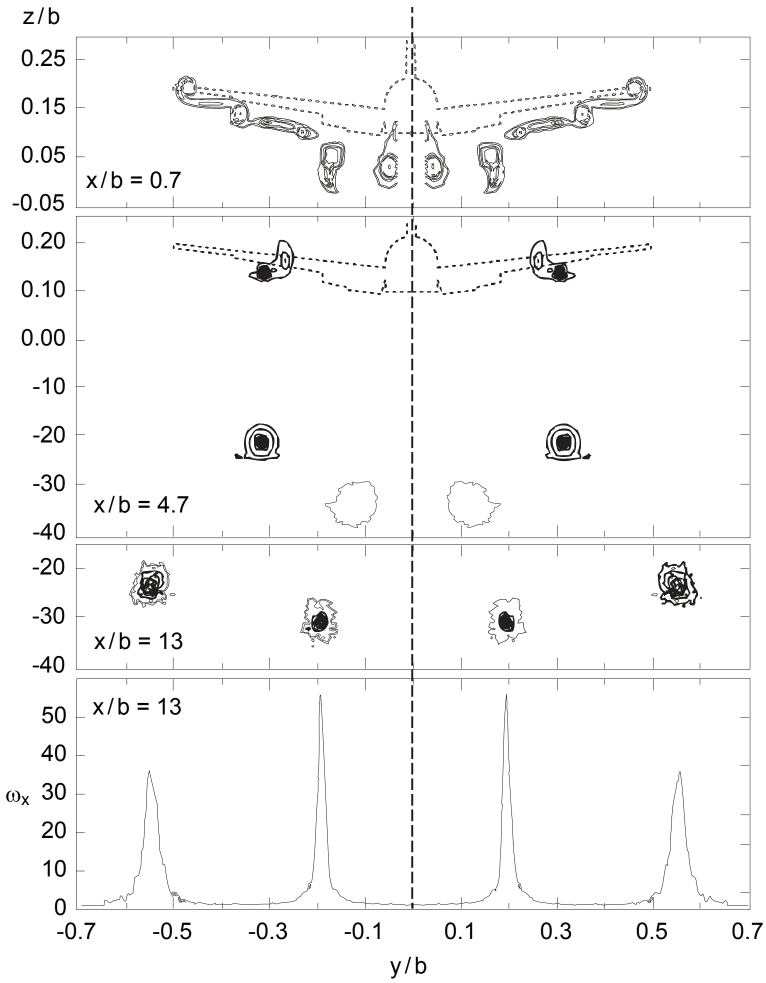

Figure 2: Streamwise vorticity in the wake of an airliner (de Bruin et al 1996). Flap deflection $35°$ inboard, $5°$ outboard; angle of attack $7°$. Contour levels ± 2.5, ± 7.5, and so on, normalized with freestream velocity and span.

the rate w_0. *Real* sheets would roll up because of *imperfections* at the tip or an *instability*. This is incorrect, as shown most simply by Spreiter & Sacks [12] (the velocity field is singular as $|y| \to b/2$, making elementary arguments elusive). Spreiter & Sacks derive the velocity of the centroid of vorticity from an integral that is insensitive to the tip singularity. That velocity equals $w_1 = -(1 - \pi/4)\Gamma_0/b$, which is much smaller than w_0. Therefore, all the vorticity cannot be descending at w_0. At early times the central part of the sheet does descend at w_0, but its tips move upwards, initially at an infinite velocity. This is clear in Krasny's [20] calculations for a regularized sheet, and he views the motion as deterministic, not the result of an instability. Rapid upward motion of the tip vortex is seen in actual three-dimensional (3D) wakes, particularly in highly loaded rectangular wings such as that of Chow et al [13]. This means that modeling the wakes as flat, as in many panel methods, is crude.

2.2 Rotational Velocity

In flows that are two dimensional or at least slender, vorticity tends to organize itself into distinct *poles*, which each become close to axisymmetric as a result of winding and are loosely called *vortices* even if weaker vorticity is distributed around them (Figure 2). This rotational motion is easiest to detect and has a dominant influence in case of a wake-airplane encounter. Outside the core of an axisymmetric vortex, the rotational velocity at a distance r is $u_0 = \Gamma/(2\pi r)$; this is the *point-vortex* field.

The notion of vortex core still suffers abusive simplifications. Real vortices are far from the Rankine vortex, which has a unique *core radius* r_c at which the velocity peaks: $u_\theta = r\Gamma/(2\pi r_c^2)$ for $r \leq r_c$. Figure 3 shows the velocity profile from a roll-up calculation (Spalart [18]; w is shown, which equals the combined u_θ of the two vortices on the $z = 0$ line). It is consistent with measurements (Widnall [24]), in that the radius r_1 at which the velocity peaks and the radius r_2 at which the profile blends with the point-vortex profile differ by an order of magnitude. The circulation at r_1 is less than 30% of the total, compared with 100% for a Rankine vortex and 72% for a mature viscous vortex (Oseen vortex, see [8]).

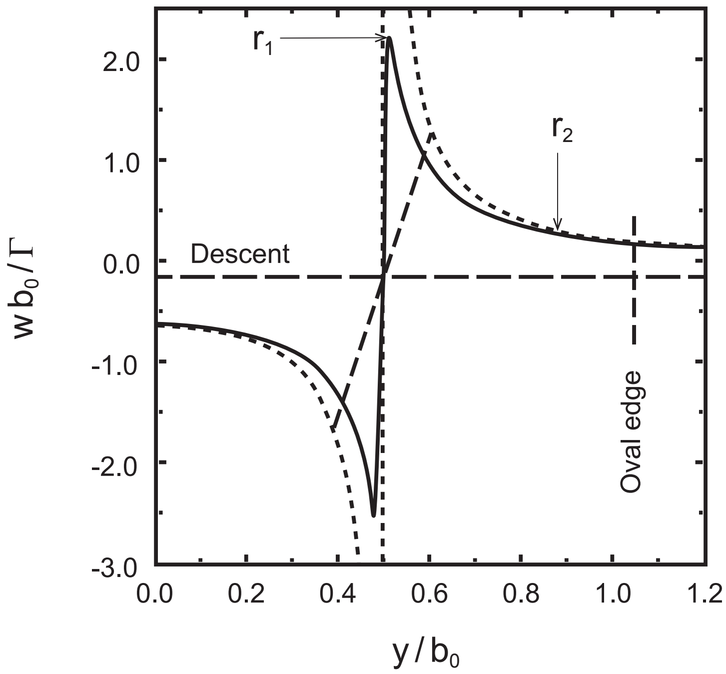

Figure 3: Profiles of vertical velocity at $z = 0$ behind clean wing. *Lines*: , calculation; - - -, point vortices; - -, Rankine-Prandtl. The oval edge and wake descent velocity ($-1/\pi$) are shown.

The velocity near $r = r_1$ has little leverage to cause a rolling moment on a following airplane. r_1 is also rather dependent on viscous diffusion or turbulent mixing, as well as axial-flow effects, near the axis (see Section 3.3). It is difficult to measure when any meandering takes place; r_1 is often surprisingly small, about 1% of the span, requiring an extremely fine spacing of sensors. The vortices of airplanes with spans up to $47\,m$ were measured with sensors spaced $0.6\,m$ apart (Garodz & Clawson 1993). Most of the measured peak profiles were discontinuous, with the highest value often four times larger than that returned by the next sensor. McCormick et al (1968) measured a radius r_1 equal to 2% span, 200 spans behind the airplane. The slow growth of r_1 could only be relevant to a following airplane of much smaller size. I conclude that r_1 and the associated *peak velocity* u_1 have received too much attention (Section 3.1); r_1 cannot be called the core radius.

The radius r_2 at the edge of the vortical region, although not sharply defined, is of interest. Figure 3 shows it to be fairly close to the centerline ($y = 0$) and to the *separating streamline* that bounds the oval of fluid that descends with the vortex pair. The figure derives from an elliptical lift distribution;

real-life distributions are more triangular, which locates even more vorticity (although weak) away from the vortex center. Measurements need to be extended to the centerline. If the vortical region touches the dividing streamline, which can result from stratification or surrounding shear, slight vorticity cancelation and detrainment occur. Finally, the gradual rise of the circulation from $r = 0$ to $r = r_2$ has a damping effect on centrifugal instabilities, so that turbulence is unlikely, unless sustained by axial flow (Section 3.3). The large ratio between r_2 and r_1 also raises questions about simple models of the vortices' motion, such as the local induction approximation, which use only one core radius. There is an inner core with high vorticity, but its effective circulation is less than that of the whole vortex, which could weaken the instabilities.

The Rankine model still appears in new literature, long after it was shown to be a poor approximation (Donaldson & Bilanin 1975, Widnall 1975). The sensible procedure of adjusting r_c to obtain the correct kinetic energy, based on induced drag, is due to Prandtl in the 1930s and correctly presented by Milne-Thomson (1966, p. 209). Credit is more often given to Spreiter & Sacks (1951) and the elementary mistake between their equations 14 and 15 earnestly reproduced in every paper, 45 years later! The correct equation makes the core radius 11% larger than that from Spreiter & Sacks (1951). The energy argument could be applied to other simple profiles such as the Oseen one, but the results are not much better, as r_1 is still too large. The Betz approximation is much more accurate (Widnall 1975), but the problem it addresses is now fairly easy to solve numerically.

2.3 Axial Velocity

The axial flow has a rich behavior, may sustain small-scale turbulence, and could also be essential in detecting trailing vortices from behind. It has surprised many of us that the velocity relative to the atmosphere may be directed towards the airplane (*wake-like*, as behind a nonlifting drag-producing body) but also away from it (*jet-like*) ([2]). Jet-like flow gives an apparent thrust in a control-volume equation, which is offset by low pressure in the same region. I discuss two, mathematically equivalent, explanations of the jet-like flow. I consider inviscid incompressible flow.

The first explanation uses the shape of the vortex lines. After leaving the trailing edge, the weaker regions of the vortex sheet are wrapped around the vortices that are forming, primarily at the tip. This gives the vortex lines a helical shape, a right-handed helix for the right wing. This is what induces jet-like axial flow, peaking on the vortex axis and zero outside the outer turn of the vortex sheet. Such flow is related to rotational flow because in a steady flow the vortex lines and the streamlines coincide. This is after full roll-up, when the vortex is nearly axisymmetric and independent of x. In cylindrical coordinates, the various components depend only on r; the velocity u_x is with respect to the airplane reference frame. The coincidence of vortex lines and streamlines is expressed by $\omega_x u_\theta = \omega_\theta u_x$. This may be rewritten $d(u_x^2 + u_\theta^2)/dr + 2u_\theta^2/2 = 0$. If the circulation at radius r is denoted by $\tilde{\Gamma} = 2\pi r u_\theta$, we also arrive at $4\pi^2 r^2 d(u_x^2)/dr + d(\tilde{\Gamma}^2)/dr = 0$. Because we observe very generally that $|\tilde{\Gamma}|$ increases with r, it is clear that $|u_x|$ increases as we approach the vortex axis (larger velocity magnitude with respect to the airplane means a jet-like flow). Asymptotic inviscid solutions for small r and elliptical loading show that u_θ has a $r^{-0.5}$ singularity, leading to a $r^{-0.5}$ singularity for u_x also (Moore & Saffman 1973).

The other explanation uses the pressure. It has less visual appeal but can be extended to viscous flows by inserting a loss of total pressure (Batchelor 1964). Under the same conditions as above, the r momentum equation is $dp/dr = u_\theta^2/r$ and the Bernoulli equation is $d(p + [u_x^2 + u_\theta^2]/2)/dr = 0$. They lead again to $d(u_x^2 + u_\theta^2)/dr + 2u_\theta^2/r = 0$. The core has low pressure (sometimes causing condensation) and high velocity. From here on, I refer to the jet-like axial flow as a *Bernoulli effect*. In a real flow, the fluid around the vortex is layered with boundary-layer fluid and a larger proportion of *clean* fluid that was engulfed between the turns of the vortex sheet. Therefore, the average total pressure is close to the freestream value.

The equations provide the order of magnitude of the Bernoulli effect. The radius r_2 of the vortices scales with the span b (Figure 3); therefore the variations of u^2 scale with Γ^2/b^2. As a result the velocity deviations relative to the freestream velocity are proportional to $\Gamma^2/(U_\infty b^2)$. This is why they are neglected in lifting-line theory, which assumes light loading, $\Gamma/(U_\infty b) \ll 1$. The scaling also helps reconcile experiments. Chow et al (1997) measured $u_x/U_\infty = 1.78$ for a rectangular wing at 10° angle of attack, with $\Gamma/(U_\infty b) \approx 0.2$; Green (1995) reports $u_x/U_\infty = 1.62$, with $\Gamma/(U_\infty b) \approx 0.14$. In contrast, Devenport et al (1996) report no jet-like flow; however, they had 5° angle of attack and a higher aspect ratio, giving $\Gamma/(U_\infty b) \approx 0.028$, a much lower value. This probably explains why viscous effects overcame the Bernoulli effect. Figure 20 of Devenport et al is strongly suggestive of the Bernoulli effect at 7.5° angle of attack. They also began their measurements 5 chord-lengths downstream, in contrast with Chow et al. Finally, de Bruin et al (1996) measured only $u_x/U_\theta = 1.06$ in the first frame of our Figure 2; again, the loading is lighter and the measurement is not close to the trailing edge.

Batchelor's (1964) paper illuminates the Bernoulli effect and the subsequent interaction with the rotational velocity (viscous effects are compounded by that interaction to suppress the jet-like flow, see Section 3.3). However, his viscous solution lacks in realism, and it is unfortunate that this *q-vortex* is used almost universally in stability studies. Saffman (1974) already pointed out that it neglects any interaction with the opposite vortex; it corresponds to an unrealistic *rectangular* lift distribution. Behind real wings, the initial core size r_2 is not small relative to the span (Figure 3). As a result, there is no regime in which viscous (or even turbulent) diffusion has dictated the velocity profiles and overlap has not started with the other side. Thus the q-vortex property that the axial-velocity and axial-vorticity profiles have the same shape is far from satisfied (recall the $r^{-1/2}$ behavior for axial velocity, when the vorticity had $r^{-3/2}$ behavior).

3 Motion and Persistence

3.1 Decay or Collapse?

Two radically different concepts of the long-term behavior of trailing vortices have been held for years. One is more empirical and rooted in the government/industry community; the other is more theoretical and rooted in the academic community. They are sketched, in their simplest form, in Figure 4, where the total time interval is of several minutes, as is of interest in airline practice.

The *predictable decay* (PD) view is that measures of the strength of the vortices (often, their circulation) gradually decrease under the effect of various dissipation mechanisms, which sap the kinetic energy. The decay is sufficient for the strength to drop to an acceptable level, several times smaller than the initial value, within a useful time; thereafter, an encounter is not hazardous (Vicroy et al 1997). The most influential PD model is by Greene (1986), who also includes factors such as stratification and destruction by the Crow instability (Crow 1970). While correctly identifying most of the relevant mechanisms, that model is controversial and appears to generally underestimate the wake strength. The analogy between the vortex-pair oval and a solid body with noslip conditions, which leads to a viscous drag and Greene's equation 1, is particularly weak (the tangential velocity at the oval boundary is not zero, and no vorticity is created there). The circulation and impulse decay driven by ambient turbulence, attributed to Donaldson & Bilanin (1975), also conflicts with basic theorems. Only the

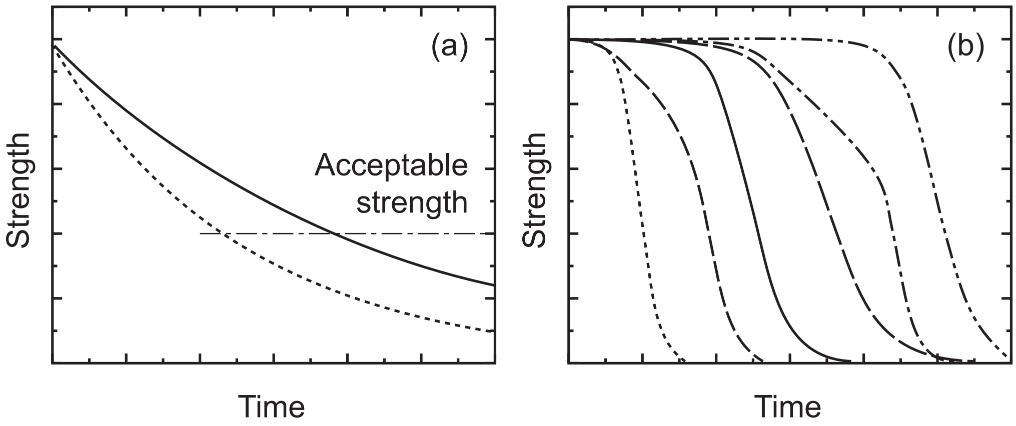

Figure 4: Two views of the evolution of a vortex system. (a) *Predictable Decay*; the two curves correspond to two weather conditions. (b) *Stochastic Collapse*; the different curves correspond to different realizations.

initial effect of stratification is supported by measurements shown in the paper itself. No other proposals have appeared since, showing how extremely challenging the creation of a comprehensive model is.

The PD view is consistent with current ATC regulations, expressed by a separation matrix (Section 1.1). In Figure 4a, the leading airplane would provide the initial strength and (in combination with the atmospheric conditions) the decay rate; the following airplane would provide the acceptable strength. Numerous experimental studies have discussed vortex decay, of which an influential one is that of Iversen (1976). However, Iversen did not allow decay of circulation or impulse, and the level of effective viscosity he predicted gives a Reynolds number Γ/ν_{eff} as high as 10^4 for flight (he and some contemporaries also failed to incorporate the initial size of the cores, through a virtual origin in his equation 9, which would further depress the deduced effective viscosity). In a few minutes, viscosity this weak falls far short of causing a decay commensurate with Figure 4a. The peak velocity u_1 may decay noticeably, but, again, it occurs at a radius too small to be relevant (Section 2.2). Tower fly-by measurements do imply a high level of decay (Garodz & Clawson 1993, Rudis et al 1996) but only in ground effect (see Section 1.3), if not in stable stratification. A generalization to other conditions cannot be defended.

Numerical studies have attempted to reproduce the decay. Unless marked by strong viscous or numerical dissipation, they failed to show decay of the true circulation; other measures have been tried until decay could be reported, so dominant has the PD view been. Such a measure is the *core circulation*, measured at the peak velocity, $\Gamma_1 \equiv 2\pi r_1 u_1$ (Section 2.2). It is as legitimate as any other and decays in some situations. However, in the laminar solution $u_\theta(r,t) = [2 - \exp(-r^2/[4\nu t]) - \exp(-r^2/[4\nu t + \sigma^2])]\Gamma/[4\pi r]$, Γ_1 steadily increases. Clearly, Γ_1 is not a reliable indicator of vortex aging.

The stochastic collapse (SC) view respects the classical theorem that, unless specific mechanisms are active, circulation and impulse are conserved. In that view, the left and right halves of the wake are too segregated to cancel vorticity on the plane of symmetry (Figures 22 and 3). Angular momentum is approximately conserved for each wake half (the strain caused by the other half being weak), which limits the growth of the vortex cores. Little decay of kinetic energy takes place, as turbulent mixing is suppressed by rotation (molecular diffusion is minute on this time scale, of course). As a result, the strength is constant after roll-up, until a crisis takes place that greatly deforms the vortices. The three-dimensionality then allows a cascade to small scales and the dissipation of the kinetic energy (Van Dyke 1982, Figure 116). The crisis results from instabilities, themselves initiated by random atmospheric turbulence (and slight deviations by the airplane from a straight flight path), so that the crisis time is stochastic. This is consistent with Figure 1.

The average of the curves in Figure 4b may resemble Figure 4a, but such an average is meaningless in the field of safety. The probability that the strength is still over the acceptable level is meaningful. However, the time for that probability to reach useful values, such as 10^{-6}, is both too long to explain the current matrix entries and extremely difficult to calculate. It appears that (unless maybe the following aircraft is heavier than the leading one) safety is obtained by avoiding encounters through flight-path control, rather than by flying so far back that no active wake could ever be encountered.

The SC view is gaining credibility as sources of spurious decay, such as measurement errors, excessive viscous or turbulent diffusion in laboratory tests, ground effect, or wing geometries that allow plane-of-symmetry vorticity cancelation, are identified. Heinrichs & Dasey (1997) explore measurement errors due to the angle between the lidar beam and the vortex pair; the errors lead to a spurious decay, by as much as one third in one case. Recent flight tests at altitude also failed to indicate much decay at all, whether visually or in measurements; stochastic collapses were evident. Decay studies are far from extinct, however, although with more mentions of opposite vorticity lifted from the ground or injected by buoyancy (Hallock & Burnham 1997). It is unfortunate that SC thinking is unable to provide concrete numbers for ATC rules; however, no separation matrix has ever been explicitly based on a PD curve.

3.2 Sources of Turbulence

An area of disagreement, related to the PD/SC question, is sketched in Figure 5. In the diffuse model (DM) , the oval of fluid that approximately follows the vortex system is filled with vorticity and turbulence; both diffuse slightly through the interface with outside fluid and are progressively left (*detrained*) in a curtain above the vortex system. Of course, detrainment is a cause of decay for the circulation of the primary vortex pair. In contrast, in the confined model (CM) , vorticity does not touch either the oval or the line of symmetry, and there is no detrainment. The vortex strength is conserved. Turbulent effects may take place inside the vortex cores but have not caused enough diffusion to cause contact. Detrainment must be attributed to an additional mechanism, such as stable stratification (Gerz & Ehret 1996, Spalart 1996).

The DM receives more support from tradition and small-scale experiments (especially on vortex rings) than from flight tests or high Reynolds-number, high aspect-ratio experiments. See Figure 2 in this review and figure 84 of Van Dyke (1982). Delta wings or wings with aspect ratios of 2 may fit the DM. Accurate laminar simulations fit the CM, but many turbulence models predict a strong diffusion, leading to the DM fairly rapidly. Invariably, these turbulence models were calibrated in flows quite removed from free vortices; in my opinion, their accuracy when applied to trailing vortices is in question (Zeman 1995).

People with different interests come with strikingly different expectations of which turbulence is dominant in and near trailing vortices. Whether two types of turbulence can interact and trade energy can become a matter of debate. Normally, interaction requires their length and time scales to be of the same order of magnitude. In Table 2, I attempt to list all the types of turbulence that can be envisioned, along with their source and typical eddy size, to predict possible interactions. Here, ΔU is the velocity difference supporting the turbulence, and U_∞ the flight velocity; L is the length scale of the large turbulent eddies, and b the span. A time scale would be $L/\Delta U$.

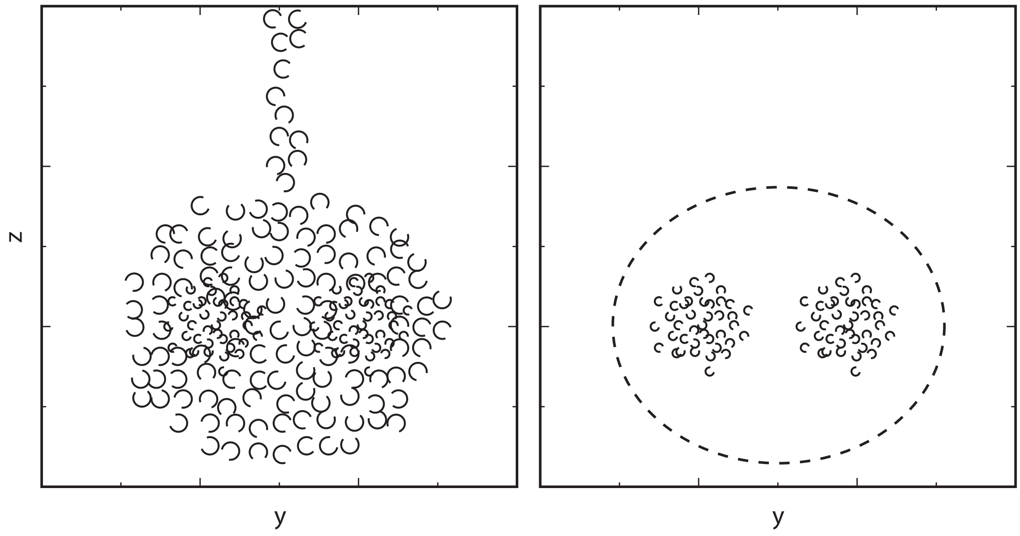

Figure 5: Two models of a vortex system, seen in cross-section. Small crescents indicate vorticity and turbulence. Left, Diffuse Model; right, Confined Model.

Table 2: Types of turbulence.

Source	$\Delta U/U_\infty$	L/b	Extent
Boundary layers (no-slip condition)	1	0.001	To trailing edge
Viscous wakes (streamwise shear)	0.1	0.01	A span
Vortex sheet (lateral shear)	0.1	0.01	A few spans
Rolled-up vortex (circulation)	1	0.1	Possibly many spans
Atmosphere (thermal, shear)	0.01	10	Everywhere

It appears that significant gaps exist, even after we recall that turbulence has a wide range of scales. Thus, severe atmospheric turbulence contains fluctuations on the $0.001\,b$ scale, but the kinetic energy in that band is very small. Fluctuations over distances much smaller than the dominant length scale vary roughly with the power $5/3$ of the distance (Kolmogorov scaling). Scaling towards larger lengths is more delicate. The table may be a framework for discussions.

3.3 Dynamics of Vortex Cores

Line vortices

These are vortices with a negligible dependence on x, the streamwise coordinate. Their gradual diffusion has received much attention, but conservation of angular momentum has often been ignored long after Govindaraju & Saffman (1971) exposed its importance. To illustrate it, consider an axisymmetric line vortex with circulation Γ and integrals from the axis to a radius R that is well outside the vortical region. The angular momentum out to R is given by

$$2\pi \int_0^R u_\theta r^2\, dr = \Gamma \frac{R^2}{2} - \pi \int_0^R \omega_x r^3\, dr,$$

The integral on the right is half of the second moment of the vorticity and will be denoted by $I/2$. In laminar flow, we have $dI/dt = 4\nu\Gamma$; circulation is conserved, but angular momentum decays (the torque at large r is not zero). Squire (1965) proposed a simple model of the turbulent vortex with a uniform eddy viscosity ν_e, which is proportional to Γ on dimensional grounds. The ratio ν_e/Γ has been called the Squire constant; the torque at large R equals $-2\nu_e\Gamma$. The core radius is proportional to $(\nu_e\, t)^{0.5}$. Similar analyzes are successful in many turbulent flows, such as mixing layers or wakes: A dominant invariant quantity is identified, and the turbulence scales with that quantity and time. This gave Squire's hypothesis credibility.

Govindaraju & Saffman (1971) give a convincing argument why the turbulent torque at radius R (proportional to $R^2\overline{u'_r u'_\theta}$, where $\overline{u'_r u'_\theta}$ is the Reynolds stress) is negligible.

In that case, we have $dI/dt = 0$ and the flow has two invariants: Γ and I. Together they imply the length scale $R_0 = \sqrt{2I/\Gamma}$, raising the possibility of a mature state with a radius proportional to R_0, instead of unbounded growth of the core (here I disregard the molecular viscosity, as is usual in free shear flows). Govindaraju & Saffman (1971) explore the converse situation and prove that if unbounded growth occurs, so that $R_1 \gg R_0$ a circulation overshoot must develop (relative to the point-vortex profile), which removes R_0 as a meaningful length scale. In Figure 6, a loss of angular momentum at small r is offset by a gain at larger r, eventually requiring an overshoot. A circulation overshoot implies that the turbulence creates a *sleeve* of negative vorticity around a core of positive vorticity. This would be unusual and has not shown up in any measurements. Note that an airplane could generate a negative sleeve, if opposite vorticity from the horizontal tail or wing-body junction were wound around the stronger vorticity from the wing tip. It is the subsequent creation of such a sleeve by turbulence that would be remarkable.

The structure of a mature vortex remains a matter of some conjecture. *There is no satisfactory treatment of the turbulent vortex* (Saffman 1974). Calculations with typical turbulence models, with zero torque at large radii, of course produce the overshoot, which adds nothing to the work of Govindaraju & Saffman (1971). Each turbulence model has its own Squire constant (typically 10^{-2}, when Iversen's value is 10^{-4}, thus inspiring little confidence). Zeman's (1995) study led to an atypical evolution, dependent on molecular viscosity.

Many experiments report no discernible diffusion [see [2]; *no substantial reduction in outfield rotary speed* from 63 to 110 spans and Phillips & Graham (1984)] or a diffusion consistent with viscous diffusion (Devenport et al 1996). Uberoi (1979) already stated that *the axial velocity difference... is necessary for the sustenance of turbulence*. The turbulent viscosity scale in a classical axisymmetric wake decays like $x^{-1/3}$, on dimensional grounds; therefore, it eventually becomes much smaller than the vortex circulation, and *Squire behavior* is not sustained. Saffman (1974) further estimated, based on the shared time scale b_0^2/Γ, that if the vortices are turbulent, the Crow instability *is probably masked by turbulent dissipation*. In other words, a wake that exhibits that instability is probably not turbulent.

Large-eddy simulations (LES) and direct numerical simulations (DNS) of line vortices with axial flow have produced valuable results (Ragab & Sreedhar 1995; G Blaisdell, F Coppens, personal communications). Turbulence is amplified only by sufficiently strong axial flow; once that component decays, so does turbulence. Furthermore, the peak axial vorticity and rotational velocity increase with time, in contrast with our plausible Figure 6. This indicates a slight transfer of kinetic energy from the axial flow, through the turbulence, to the rotation. Of course, the failure to sustain turbulence in a

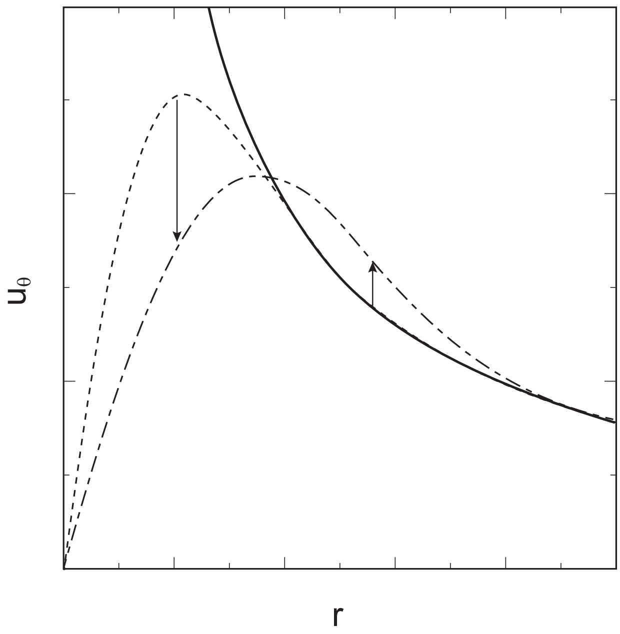

Figure 6: Sketch of an evolution of rotational velocity profile permitted by conservation of angular momentum. Lines: , point vortex; - - - earlier time; - -, later time (lower kinetic energy).

DNS/LES study only shows that the basin of attraction of a turbulent solution has not been found. This could be due to the constraints placed on the flow, including domain size, grid resolution, Reynolds number, and level of initial disturbances. Nevertheless, the three studies differ in enough respects to represent independent evidence, and each group has made serious efforts to vary the parameters in search of sustained turbulence.

A finding by the author (1996, profile shown in Figure 3 here) may also disturb views of turbulence in vortices. Calculations of the 2D near-inviscid roll-up of the vortex sheet from an elliptically loaded wing yield a logarithmic dependence of the circulation $\tilde{\Gamma}$ on radius. This dependence, confirmed by experiments (Iversen 1976), was attributed by Hoffmann & Joubert (1963) to turbulence, which would impose a mixing length proportional to the radius. Other, equally tentative, explanations have been proposed [that of Govindaraju & Saffman (1971) conflicts with the original one]. The simulation result suggests that the logarithmic dependence is in fact established early by an inviscid mechanism, and owes nothing to turbulence.

An intriguing exercise is to minimize the kinetic energy of the mean rotational flow, $\pi \int_0^R u_\theta^2 r\, dr$, simply based on the idea that the most certain effect of turbulence is to extract energy from the mean flow. If Γ and I are given and we add the constraint that *the vorticity ω_x does not switch sign* (no circulation overshoot), minimum energy occurs for a core in solid-body rotation, a Rankine vortex of radius R_0 [[2]Brown (1973) had a similar exercise but allowed overshoots, leading to a core of arbitrary radius and to discontinuities]. The flow has exhausted the kinetic energy accessible without creating negative vorticity. Some experimental papers state that *the core approaches solid-body rotation*, but the property is not clearly beyond what a simple Taylor expansion at small radii requires. An exception is Case A of Phillips & Graham (1984), their figure 4: The trend towards solid-body rotation is striking. However, that case had axial flow, and we have just seen that the energy-depletion argument is unconvincing in such cases.

Another reason for interest in the rotational kinetic energy is that it controls the cutoff length l in self-induced motion of thin filaments (Widnall 1975). For a fixed value of R_0, the solid-body core gives $l = 0.64\,R_0$; the Gaussian core has more energy, leading to a slightly smaller cutoff length: $l = 0.62\,R_0$. The above exercise indicates that (without opposite vorticity) aging of a single vortex cannot raise the cutoff length beyond a limit dictated by the angular momentum.

Trailing vortices

The figure of Phillips & Graham (1984) mentioned above leads to our next topic. The evolution from $z = 45$ (their axes) to $z = 109$ appears to violate conservation of angular momentum, as u_θ decreases everywhere. This is partly explained by what I call the Uberoi effect from here on (Uberoi 1979). Phillips & Graham's (1984) Case A had a strong positive axial flow, which decreases in the streamwise direction: $\partial u_z/\partial z < 0$. To conserve volume, the radial velocity u_r is positive, thus transporting vorticity away from the axis and increasing I by a mechanism that is not diffusion. The magnitude does not appear sufficient, however: $u_r/r \approx -(\partial u_z/\partial z)/2 \approx 0.001$, which explains an apparent growth of only about 15%, compared with about 100% in the measurements.

Uberoi (1979) made a severe critique of the literature and showed that the effects of radial transport ($u_r > 0$) could well be mistaken for diffusion, resulting in a spurious Reynolds-number dependence, especially when different levels of maturity existed. He gave the correct equations and emphasized the difference between line vortices and trailing vortices (in a trailing vortex with zero torque, it is the flux of angular momentum $2\pi \int_0^R u_x u_\theta r^2\, dr$ that is conserved). In his example, radial convection dominated true diffusion; I show below that it may be a little artificial. Uberoi discarded the possibility that the viscous term would balance the pressure term in his equation (24). Nevertheless, his point is essential that extracting the Squire constant (or other estimates of diffusion) without including radial convection is inaccurate.

These effects are illustrated by a slight extension of a model by Leonard (1994) for axisymmetric vortices. Leonard describes the azimuthal and axial flow profiles with simple functions and obtains evolution equations from weighted averages of the governing equations, much like integral methods in boundary layers. The two variables are the length σ, a measure of the core size, and the velocity γ, essentially the peak excess axial velocity. For a trailing vortex with outside velocity U_∞ and circulation Γ, a viscous version of the model is as follows:

$$\begin{aligned} U_\infty \frac{d\sigma^2}{dx} + \frac{d(\tilde{q}\sigma^2)}{dx} &= C_3 \nu \\ U_\infty \frac{d\gamma}{dx} + \frac{d(q\gamma)}{dx} &= -\frac{\Gamma^2}{2\pi^2 C \sigma^3}\frac{d\sigma}{dx} - C_4 \frac{\nu\gamma}{\sigma^2}. \end{aligned}$$

Here, $q = C_1\gamma$ and $\tilde{q} = C_2\gamma$ are weighted averages of the axial flow; ν is the viscosity. The constants, for Gaussian vorticity cores, are $C = 1/\log(2)$, $C_1 = 1/2$, $C_2 \approx 0.41$, $C_3 = 4$, $C_4 \approx C_3$. The Γ^2 term in the second equation reflects the pressure variations in the core when the vortex *spins down*, inducing axial flow. Conversely, the $\sigma^2 d\tilde{q}/dx$ term in the left equation reflects radial convection; σ and γ are coupled.

I next invert the above two-by-two linear system for $d\sigma^2/dx$ and $d\gamma/dx$. I define the axial-flow ratio $r_{ax} \sim \gamma/U_\infty$ and the azimuthal-flow ratio $r_{az} \sim \Gamma/(3\pi U_\infty \sigma)$. The peak azimuthal velocity is close to $U_\infty r_{az}$, and this is how the experimental data were processed. The trajectories r_{ax} vs r_{az} are shown in Figure 7, as well as the ratio of the true growth of σ^2 to the growth directly due to viscosity, $G \sim (U_\infty/C_3\nu) d\sigma^2/dx$. For a line vortex, $r_{az} = r_{ax} = 0$, and $G = 1$. Experiments are shown, although they cannot be strictly laminar (no correction for meandering was made either).

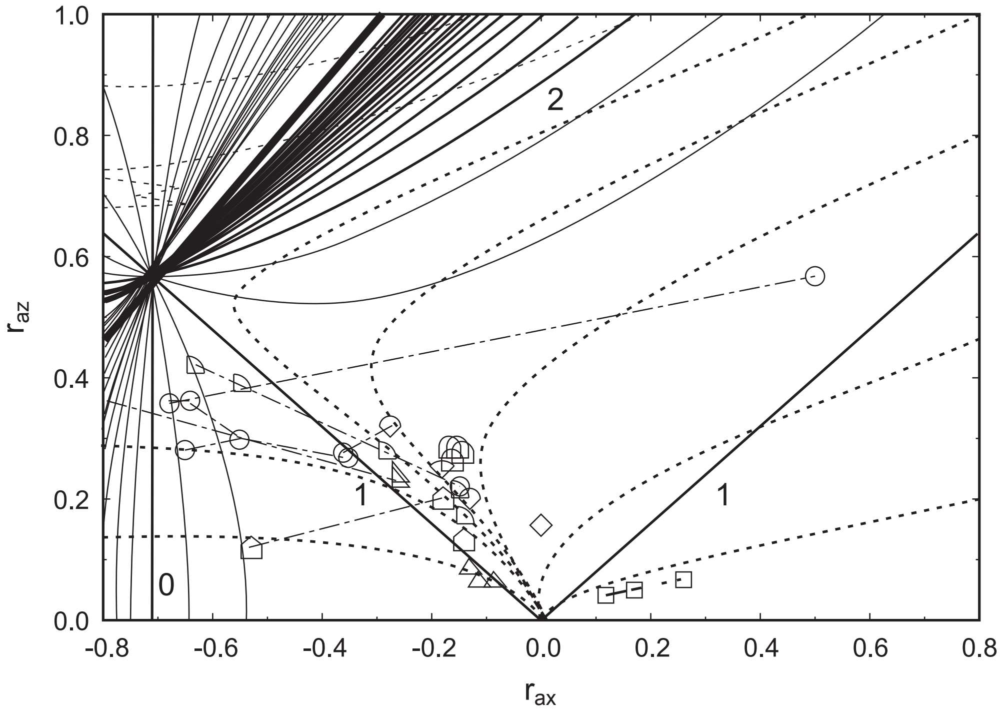

Figure 7: Trajectories and growth ratio G for a Leonard-model viscous trailing vortex. Lines: , G contours, by 0.25 (thick lines with labels: integer values); - - - model trajectories; - -, experimental trajectories (Singh & Uberoi 1976, Phillips & Graham 1984, Chow et al 1997, de Bruin et al 1996). All trajectories are towards the (0,0) point.

The model trajectories converge to the line-vortex limit at $(0, 0)$ with a vertical tangent because in a viscous line vortex, r_{ax} decays as $1/t$ and r_{az} as $1/\sqrt{t}$. The model predicts a form of vortex breakdown, as found by Leonard, in the upper left region of the figure, which is not visited by the experiments. In the lower corners ($|r_{ax}| < r_{az}$), the flow is stabilized by rotation over the axial-flow instability (Lessen & Paillet 1974), loosely indicating that turbulence is unlikely. Uberoi considered states with $r_{ax} = 0$; for a fairly strong azimuthal ratio $r_{az} = 1$, the growth ratio G is about 2.7; thus, the Uberoi effect can be large (although I do not find ratios near 200 as he did). However, that region is not visited either.

In a vortex without total-pressure loss, the Bernoulli equation gives $r_{ax} \approx 1.75\, r_{az}$. Real flows have losses, giving $r_{ax} < 1.75\, r_{az}$; the lower right corner is not accessible to a trailing vortex [Case A of Phillips & Graham (1984) is in that corner, as a result of adding axial momentum with a jet]. The growth ratio is less than 1 in both lower corners and negative for r_{ax} less than 0.71, as a result of a strong $d\gamma/dx$ stretching the vortex lines. For most of the experimental points, G is between 0.7 and 1.2. The trajectories seem to *seek* the left line on which G is near 1, which is the equilibrium between the pressure and the viscous term I mentioned above. It also has roughly neutral stability ($r_{ax} \approx -r_{az}$). The experimental trajectories are quite consistent with those of the simple laminar model. In particular, r_{ax} tends to drop from positive to negative and then rebound; therefore, not only can the axial flow be wake- or jet-like, but it can switch sign within the same flow (Saffman 1974). The two isolated experimental points are that of Phillips & Graham (1984) without axial flow ($r_{ax} = 0$, $r_{az} = 0.16$), which shows no evolution, and that of Chow et al (1997) ($r_{ax} = 0.77$, $r_{az} = 0.97$), for which no streamwise evolution was measured.

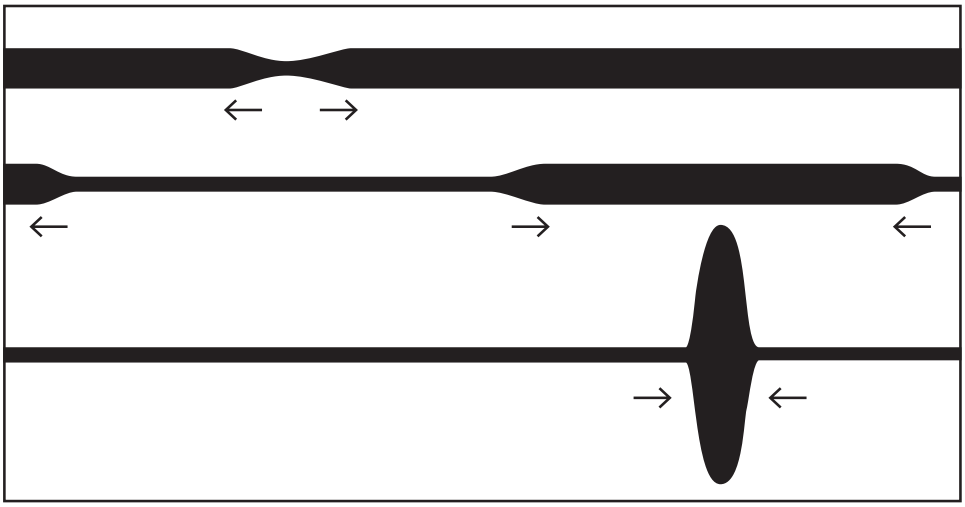

Figure 8: Sketch of vortex bursting, from a videotape of a NASA flight test. The black region represents smoke. Time is from (a) to (c).

Bursting

Gradual effects of axial flow seem to be fairly well understood, although the amount of turbulent diffusion is very uncertain. I now discuss a more elusive later phenomenon, which I call bursting (Sarpkaya & Daly 1987, Liu 1992). It is an apparently spontaneous crisis of the vortex core, which often travels. Although both are dramatic events affecting a vortex, the identification with vortex breakdown must be avoided. The final comment in Saffman's (1992) text is that the relation between the two *is unclear*. Sarpkaya & Daly (1987) wrote that *the causes and structure of the bursts remain unknown*. So far, bursting has been revealed only by flow visualization. This includes NASA flight tests of a Lockheed C-130 (G Greene, personal communication; see Vicroy et al 1997), which were consistent with photographs of Chevalier (1973) and others. A sketch is given in Figure 8. The region marked by smoke or dye rapidly contracts in portions of the vortex, and expands in others, sometimes making *pancakes*. Bursting is rarely simultaneous on the two vortices, indicating that it involves only one vortex.

The common statement that *bursting destroys vortices* is easy to challenge, especially if it involves only one vortex. In such a case, circulation cannot change; therefore vorticity may be displaced but not removed. In addition, the angular momentum, averaged over a large streamwise length, is also conserved. Therefore, if velocities are reduced at one station, they must increase at another (presumably, where the smoke region becomes thinner). Sarpkaya & Daly (1987) observe that *the vortices survive the core bursting and continue to rise*. Their statement that *one vortex is rapidly destroyed while its mate... continues to rise* appears contradictory and suggests that the first vortex merely vanished from the visualization.

Challenging the practical value of bursting to the travel industry does not remove our desire to explain the sudden occurrence of large axial flow on a stationary vortex with no apparent perturbation, such as compression in the streamwise direction (as is usually invoked in vortex breakdown). No explanation is available. In particular, models of vortex cores with axial flow (exemplified by the Leonard model, above) predict waves that travel at phase velocities proportional to Γ/σ and steepen if nonlinear but that are neutrally stable. The NASA flight tests show propagation at similar speeds towards the airplane and away from it, suggesting that preexisting axial flow does not play a strong role.

3.4 Dynamics of Vortex Systems

After the initial roll-up, vortices interact in three essential ways: They induce each other's basic nearly 2D motion; they can support 3D instabilities; and they can merge, particularly if co-rotating. In the classical single-pair model, basic motion is simply a translation, the instability is that of Crow (1970, first discussed by Scorer 1958), and merging can only be the pinching or reconnection that creates rings. All three processes are rather well understood, at least if we omit the details of the reconnection, which are not of great practical importance.

With multiple vortices on each side, much richer behavior takes place. Two vortex pairs tumble together in a periodic manner, and the distance between them is fairly constant. Three pairs (as seen in Figure 2) have a nonperiodic motion, and the vortex clusters on each side expand and contract very noticeably as they tumble, notwithstanding the impulse and kinetic-energy invariants. Double pairs have been shown to support instabilities with shorter wavelengths and higher growth rates than Crow's (Crouch 1996); these are promising for the purpose of wake destruction.

The merging of vortices from the same side of the airplane is delicate. It was seen in Figure 2 and purposely studied by Vogel et al (1995). We know that co-rotating vortices merge if the distance between them is about the same as the diameter of their cores. This merging is rapid and inviscid, resulting in a single vortex with the same total circulation and nearly the same total kinetic energy as the original cluster. However, the conditions for that inviscid merging can be gradually created by viscous or turbulent diffusion, not to mention radial transport (see Section 3.3). Diffusion is not well understood. The cores can also be brought closer by the cluster contraction. As a result, an accurate prediction of that sensitive motion is needed; 2D numerical approximations may not be sufficient.

The motion of the vortices after large distortions is of practical importance, because we need to know when the velocity field actually becomes benign in case of an encounter. Van Dyke's caption that the wake *quickly disintegrates* after rings are formed (1982, Figure 116) is challenged by the experiments of Delisi et al (1996) at rather low Reynolds number ($\Gamma/\nu \approx 5\,10^4$), and it is not strongly supported by his own photograph (see also Lewellen & Lewellen 1996). In all cases, coherent vortex rings are present for a time that is at least as long as the time from wake creation to first reconnection. This partly accounts for why Crow & Bate's (1976) curve is somewhat low, relative to tests, in Figure 1. This delay between reconnection and complete destruction is unfortunate and needs to be quantified with simulations and eventually with flight tests.

3.5 Environmental Influences

Stratification, surrounding turbulence, and shear are the primary influences present in the atmosphere, with comparable effect on typical airliner wakes and with ever-changing strengths. Theory, numerical simulations, and laboratory tests now provide a fair understanding of isolated effects, but accurately predicting combined effects remains very challenging.

The approach to stratification may evolve, in the sense that the nondimensional stratification number $N* \sim 1/F \sim 2\pi N b_0^2/\Gamma$ is no longer expected to be much smaller than 1 (N is the Brunt-Väisäla frequency, a measure of how stable the atmosphere is, and F is the Froude number). Some authors omit the 2π factor and use the semi-spacing $b_0/2$, making their number 8π smaller. Several references of the 1970s place $N*$ below 0.2, leading to a very weak effect of stratification (the effect scales with $N*^2$). However, the largest airliners clearly have time constants $T \sim 2\pi b_0^2/\Gamma$ larger than 30s, especially after a long-range flight and at the higher speeds of the early approach (smaller Γ). For the Brunt-Väisälä frequency, the mean worldwide winter value is about $N = 0.02\, s^{-1}$ (Charney & Drazin 1961); $N = 0.03\, s^{-1}$ is routinely observed; $N = 0.04\, s^{-1}$ has been measured, without seeking extreme values (Rudis et al 1996). Therefore, a complete treatment of stratification must include values of $N*$ somewhat beyond 1, which I term moderate although they derive from strong stratification (relatively high N).

My 2D numerical study (1996) indicates that weak stable stratification leads to a (counter-intuitive) acceleration of the descent (Scorer & Davenport 1970, Widnall 1975). It also shows that moderate $N*$ values cause a rebound, followed by a rapid descent, which is equally counter-intuitive. The rebounding vortices cross the initial altitude if $N*$ exceeds about 1.2, which would require a reexamination of ATC assumptions. To date, there is no experimental validation of such a rebound in which it is attributed solely to stratification; some experts doubt the relevance of 2D laminar results. Vaughan et al (1996) did measure a rebound to above the flight path, at a height of about 1.5 spans above ground level, whereas rebound from ground effect alone is expected only up to about 0.7 spans. They cite buoyancy as a possible cause; they emphasize that the case shown was not an isolated result. Vicroy et al (1997) show a definite rebound followed by a sharp drop, all out of ground effect. They invoke *local variations in the vertical wind, temperature, or turbulence*.

Simulation studies derived from engineering programs tend to use the simplest useful set of equations and boundary conditions and to emphasize nondimensional parameters (Robins & Delisi 1996, Spalart & Wray 1996). In contrast, groups with atmospheric backgrounds often propose to simulate the wakes as well as the whole atmospheric boundary layer, leading to much more complex equation sets and huge domains (Corjon et al 1996, Gerz & Erhet 1996, Schowalter et al 1997). On today's computers, this tends to preclude detailed unsteady 3D solutions.

Robins & Delisi (1993, 1996) have successfully addressed environmental effects two at a time in their simulations, for instance, stratification with shear, although sometimes with a modest numerical resolution. In particular, they found a favorable interaction between stratification and the Crow instability: Ring formation occurs earlier with stratification, owing to the vortices approaching each other. This could mitigate the rebound problem mentioned above, unless the stable atmosphere sets a much lower level of ambient turbulence. See also Corjon & Poinsot (1997).

3.6 Ground Effect

The vortex behavior near the ground is now understood qualitatively, but predictions accurate enough for ATC use are both difficult and crucial, since a sidewind can make the upwind vortex linger over the runway. In an inviscid fluid, the vortices part as they approach the ground and follow hyperbolas, eventually following the ground at an altitude half their initial separation. In a viscous fluid, the vortices rebound and nearly stop moving. This is due to separation of the boundary layer from the ground, which launches secondary vortices of opposite sign (Harvey & Perry 1971). Prediction is difficult because the size of the secondary vortices depends on the turbulence; as mentioned before, turbulence models that are well validated in vortices do not exist today. Another uncertainty is the profile of the side wind. The situation is further complicated by the atmospheric turbulence that accompanies winds and the possibility for the vortices to develop the Crow instability *with* their image vortices under the ground plane. The formation of half-rings has been observed (Bisgood et al 1971) but appears to be rare.

Figure 9 shows a rebound simulated in two dimensions (M Strelets, personal communication). The circulation Reynolds number is $\pi \times 10^7$. A one-equation turbulence model is used, tentatively

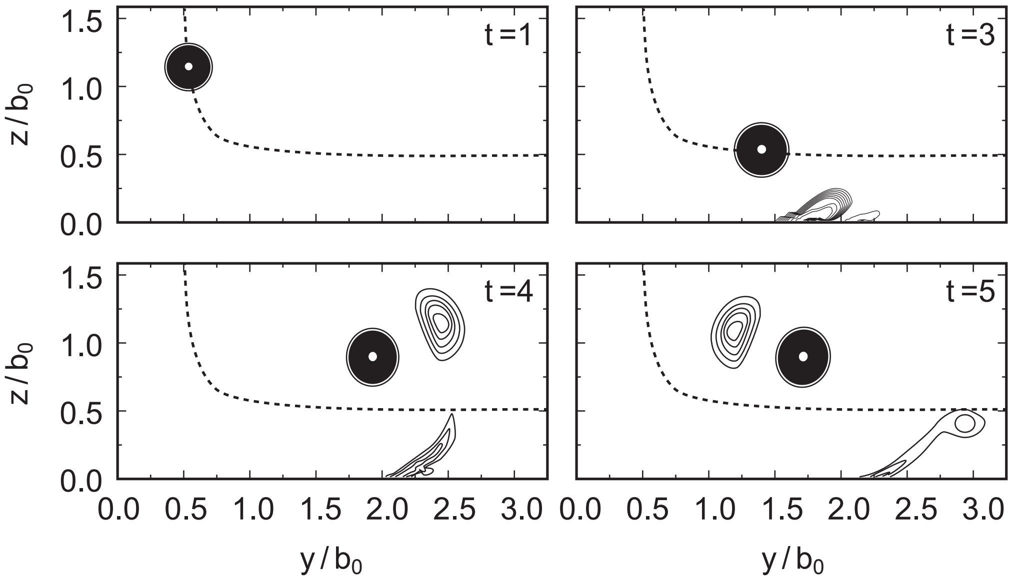

Figure 9: Vorticity contours for vortex-pair rebound. Results of the Strelets group. Turbulent boundary layer on a smooth ground plane. Line: - - -, inviscid flow.

modified to suppress the eddy viscosity in vortices once they are nearly axisymmetric (Dacles-Mariani et al 1995). Separation occurs later than in laminar flow; as a result, the first loop in the trajectory of the primary vortex ($t = 4$ in Figure 9) reaches $y/b_0 \approx 2$, as opposed to $y/b_0 \approx 1.5$. The secondary vortices orbit the primary vortices, as found by Orlandi (1990), Schilling (1992), Robins & Delisi (1993), Zheng & Ash (1996), and others, some of which also include side-winds. This orbiting was predicted by Harvey & Perry, but has yet to be seen experimentally, and is ignored in some empirical models. The secondary-vortex circulation is about one-third of the primary and opposite. Some authors expect the secondary vorticity to be wrapped around the primary vortex, instead of forming discrete vortices of its own (Hallock & Burnham 1997). Additional vorticity lifts up later. Many ground-based measurement systems would fail to distinguish the circulation of the primary and secondary vortices, indicating a spurious *decay* of the circulation (see Section 1.3). Vaughan et al (1996) do not observe circulation decay. Quantitative flow-field measurements are desirable, ideally in a large wind tunnel and with an oblique floor belt to simulate side-winds.

4 Outlook

4.1 Outlook of Prediction and Detection

Improvements in our ability to predict the motion and persistence of trailing vortices are required before analysis is of value to ATC in a significant manner. Numerical simulations have the largest potential. However, the simulations are most often narrowly focused and will remain research tools for some time. Every possible theoretical research line must therefore be explored. Laboratory experiments will be used to develop new concepts, and wind-tunnel tests to validate the roll-up phase. Flight tests will still be indispensable, since safety matters and scaling are challenging.

When a prediction system becomes available, one that track every wake near an airport, it is likely to be theoretical-empirical-numerical and probably readjusted by frequent sweeps of a detection system [14]. Such a system will be installed at selected sites, first to test its consistency and later to improve productivity at the most congested airports. Ground-based acoustic and laser instruments are much better at locating a wake than at providing details of its flow field. In [14] systems are described that can operate in all weather conditions.

Little has been done with on-board sensors, which would indicate to pilots that a wake is nearby. A coherent wake has a clear signature in the transverse plane; unfortunately, many sensors are sensitive to the velocity component parallel to their beam. The challenges will be to provide a clear warning with enough notice, and to factor in the weather aspects.

A critical situation arises if a moderate sidewind, especially combined with a slight tail wind, causes a vortex to stall on the flight path in final approach. A row of anemometers near the runway threshold has little trouble detecting this situation, at least in the absence of gusts [22]. A go-around could be made by the next arriving airplane. This is an inexpensive and visibility-independent concept, but it could be viewed as an extra burden on the controllers. Another concern at many airports is trees and buildings situated fairly close to the end of the runway, which would disturb the system with their wakes. Signals of about 1 m/s need to be reliably distinguished from the background.

Even as the economic incentives rise, the development of new ATC components to benefit productivity at the same safety level will be marked by a lengthy search for consensus and definitive validation, as well as by concerns of cost and crew workload.

4.2 Outlook of Control

Air traffic control would benefit greatly by the ability to intentionally make the vortices decay or collapse within an acceptable time interval. Because of its commercial and military potential, methods of wake-vortex control are not usually available to the public. However, the various known approaches are described in the following.

Again, the decay and collapse mindsets are focused. With decay of the circulation or at least of the rotational velocity as the objective, the most likely approach is *to enhance the turbulent diffusion*. This has been tried with drag-producing parachute-like devices and with unequal thrust on the inboard and outboard engines. These are passive systems, in the sense that they are time-independent. Active systems are aimed at an instability that has a nonzero frequency in the reference frame of the airplane, and therefore they require moving devices. Here it is distinguished between concepts aiming at the destruction of each vortex on its own and those based on a collaborative instability that allows the two vortices to cancel each other (Crow & Bate [23]).

The *destruction* of a single vortex is a questionable concept; vorticity and angular momentum can only be rearranged. It is certainly possible to reduce the peak rotational velocity u_1 without reducing the potential to induce rolling moments. That remark also applies to tip treatments such as winglets and turbines: Doubling the core radius r_1 of Figure 3 from 0.5 to 1 m is of no value to a following airplane with a 30-m span. It could be of value if roof damage is the issue, by reducing the pressure deficit in the core (Lee [7]).

Active systems based on the Crow instability may be revisited, especially with the assistance of fly-by-wire systems, but it is not clear that collapse can be obtained rapidly enough by ATC standards. Crow & Bate's [23] estimate of a 33-s lifespan for a Boeing 747 with active control is much too optimistic, for several reasons (low estimate of b_0, which enters to the third power; delay between vortex contact and actual dissipation of the rings). A significant improvement appears to be necessary.

Other forms of control, although they have no history, should be discussed. The first is the deliberate creation of multiple vortices, as in Figure 2. These reduce the harshness of encounters, provided the span of the following airplane is less than half that of the leader. This concept depends entirely on the persistence of separate pairs, as discussed above, and flight tests are needed. The second is to make the vortices descend faster by increasing their circulation and reducing their spacing (for the same impulse). This does not hinge on any serious unknown of the physics of the situation, but rather on ATC policy. The descent itself is favorable in almost any situation (the exception is vortex stall in ground effect, which will merely occur with a different wind level). Narrow high-circulation pairs also resist stratification better and amplify Crow waves faster. However, this concept clashes with the traditional idea of alleviating the wakes. Much further debate will occur.

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