# 2D-Packing with an Application to Stowage in Roll-on Roll-off Liner Shipping

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**Abstract.** Roll-on/Roll-off (RoRo) ships represent the primary source for transporting vehicles and other types of rolling material over long distances. In this paper we focus on operational decisions related to stowage of cargoes for a RoRo ship voyage visiting a given set of loading and unloading ports. By focusing on stowage on one deck on board the ship, this can be viewed as a special version of a 2-dimensional packing problem with a number of additional considerations, such as one wants to place vehicles that belong to the same shipment close to each other to ease the loading and unloading. Another important aspect of this problem is shifting, which means temporarily moving some vehicles to make an entry/exit route for the vehicles that are to be loaded/unloaded at the given port. We present several versions of a new mixed integer programming (MIP) formulation for the problem. Computational results show that the model provides good solutions on small sized problem instances.

**Keywords:** Maritime transportation, 2D-packing, Roll-on Roll-off

## 1 Introduction

Roll-on/Roll-off (RoRo) vessels are the preferred choice when transporting vehicles and other types of rolling material around the globe. However, due to more efficient short sea feeder traffic in and out of main ports, the containerized fleets are becoming more and more of a threat to the RoRo segment. Therefore, it is important for the RoRo industry to continuously improve and become more effective, maintaining the position as the leading maritime transportation method for this type of cargo.

A RoRo ship transports different types of vehicles, such as cars, trucks, heavy rolling machinery, and trains, as illustrated in Figure 1. During loading, the vehicles typically enter the ship through a ramp placed at the stern or the side and from there they are placed in one of several decks on the ship. A major problem that occurs when loading/unloading the cargo is shifting, which means temporarily moving some vehicles to make an entry/exit route for the vehicles that are to be loaded/unloaded at a given port. This forces the ship to stay longer in the port and increase the cost of workers. Therefore, it is important to develop a good stowage plan that brings as much cargo as possible, utilizing the available space on the decks, while at the same time keeps the cost and time spent on shifting as low as possible.


**Fig. 1.** RoRo vessel. Source: WWL

In the field of RoRo-transportation, strategic planning is concerned with a time horizon of several years, and typically involves decisions such as determining the fleet size and mix, see for example Pantuso et al. (2015). Andersson et al. (2015) consider fleet deployment in RoRo-shipping on a tactical level. At the operational level of planning, the greater part of research regarding RoRo-ships focuses on safety and stability, such as Kreuzer et al. (2007). Despite its importance, research within stowage on RoRo-ships is scarce, and to the authors' knowledge, only the research conducted by Øvstebø et al. (2011a,b) exists on stowage on board RoRo-ships.

In other fields of maritime transportation, stowage problems are more common, as e.g. tank allocation problems in maritime bulk shipping (Hvattum et al., 2009). However, the vast majority of literature regarding stowage in maritime transportation focuses on stowage problems for container ships. The containers are stacked on top of one another, and when dispatching a certain container, containers stacked on top of it needs to be removed. The objective in container stowage problems is therefore often to minimize the loading/unloading time of all containers (Ambrosino et al., 2004) or the number of container movements (Avriel et al., 1998). Where a container is lifted straight up from its position, a vehicle's entry/exit route needs to be calculated for each vehicle in the RoRo ship stowage problem (RSSP). This is a complicating factor, considering the deck layout and ramp placement, which makes the stowage plans difficult to evaluate. The RSSP presented by Øvstebø et al. (2011a), aims at deciding a deck configuration with respect to height, which optional/spot cargos to carry, and how to stow the vehicles carried during the voyage, given a predefined route. Øvstebø et al. (2011a) propose a mixed integer programming (MIP) model and a heuristic method for solving this problem, where the objective is to maximize the sum of revenue from optional cargoes, minus the penalty costs incurred when having to move cargoes when performing the stowage along the route. For modeling purposes, Øvstebø et al. (2011a) divide each deck into several logical lanes into which the vehicles are lined. The vehicles enter the ship at the stern, and are unloaded according to the last in-first out (LIFO) principle. However, dividing the decks into lanes may be too restricting, limiting the possibilities of finding good solutions. Therefore, the models presented in this paper does not rely upon this assumption.

As stowing vehicles on a deck may be seen as packing problem, a short review of cutting and packing problems is now presented. Wäscher et al. (2007) present a typology of cutting and packing problems, partially based on the original ideas of Dyckhoff (1990). According to this typology, the RoRo ship stowage problem is classified as either a two-dimensional knapsack problem (2KP) or a multiple heterogeneous large object placement problem (MHLOPP). Here, a fixed number of small items have to be allocated on a smaller number of large objects, where each item increase the profit by a specified value, if placed. This is transferable to the RSSP, where all vehicles (small items) from the mandatory cargoes and the carried spot cargoes have to be allocated on one of the ships decks (large objects). Hadjiconstantinou and Christofides (1995) present an exact tree-search procedure for solving the 2KP, where the algorithm limits the size of the tree search using a bound derived from a Lagrangean relaxation of a binary formulation of the problem. Hopper and Turton (2001) suggest two types of hybrid algorithms to solve the 2KP. Recently, Seixas et al. (2016) proposed a heuristic for solving a pickup and delivery allocation problem for offshore supply vessels. In terms of mathematical modeling, the resulting problem is seen as a rich variant of the 2KP, using a grid representation of the deck. Several constraints are evaluated, many of them comparable to the RSSP, such as packing constraints, weight limitations, adjacency of delivery/pick-up cargoes, positioning of dangerous and refrigerated cargoes.

The objective of this paper is to propose a new and more realistic mathematical model for the RoRo ship stowage problem. We focus on stowage of a single deck, which is an essential building block in solving the problem for multiple decks, i.e. for the whole ship.

The outline of the remaining of the paper is as follows: Section 2 describes the RoRo ship stowage and the shifting problems in detail. The proposed mathematical model is presented in Section 3. Computational results are reported in Section 4, while concluding remarks are provided in Section 5.

## 2 Problem description

In this section, the stowage challenges for a RoRo ship are presented. First, the general RoRo ship stowage problem (RSSP) is presented. Then, a detailed description of the two-dimensional RoRo ship stowage problem for one deck (2DRSSP) is given, which is the problem we aim to solve in this paper. Finally, the shifting problem is presented. This research is based on a collaboration with one of the world's largest RoRo-shipping companies, operating more than 50 RoRo ships all over the world.

The RSSP focuses on how to utilize the ships decks, carrying a number of cargoes along a voyage with a predefined given set of loading and unloading ports to visit. A cargo (or a shipment) is defined as a set of vehicles or units of some other rolling material that are to be loaded and unloaded at the same ports. In this work, the term vehicle is used to describe the content of the cargo. The cargoes are divided into two categories, mandatory cargoes and optional cargoes. Mandatory cargoes have to be transported due contractual terms, while optional cargoes are only desirable to transport if they can increase the profit on the voyage given available capacity on the deck. For every vehicle and deck, the weight, width, height and length are known. At each port, a fixed number of mandatory cargoes are present. There is also a given upper limit of optional cargoes the ship may take at each loading port. The objective is to maximize the revenue from optional cargoes while keeping the shifting cost to a minimum. Different factors complicate the problem, such as weight limits on the deck, stability considerations, and placement of vehicles.

In this paper, a simplification of the RSSP is addressed, namely the two-dimensional RoRo ship stowage problem (2DRSSP) that arises if we consider only one deck. The problem may then be reduced to a two-dimensional packing problem, where one has to stow all mandatory cargoes and then stow as much optional cargo as possible in the space that is left, and at the same time keep the shifting costs to a minimum. It is assumed that each vehicle is placed longitudinal to the deck, i.e. with its front facing the bow, which is most common. Stability constraints are not included in the model, as considering the stability of a single deck gives no real value. However, in the case where all decks are considered, the stability calculation becomes an essential part of the problem. Height and weight limitations are implicitly taken care of in the pregeneration of feasible areas of the deck for stowing each cargo.

To illustrate the problem, a small example is shown in Figure 2. Here, there are three mandatory cargoes, with four vehicles of different sizes. There is also one optional cargo, with two vehicles. It is assumed that no flexibility is allowed for the cargoes, meaning that one have to bring all or none of the vehicles of the optional cargo and all vehicles of the mandatory cargoes. There are four ports along the voyage, first two loading and then two unloading ports. The four ports indicate that the problem has a total of three sailing legs, where a leg is defined as the part of the voyage between two subsequent ports. The figure shows a feasible stowage plan for each sailing leg. It should be noted that a given cargo cannot be moved from one sailing leg to the next. From the solution one can also see that even though it is enough area on the deck to bring the optional cargo, the outline of the deck makes it impossible to include it. This example illustrates that allocating vehicles based only on a deck's area capacity, could give infeasible solutions.

Given a feasible solution from the 2DRSSP, as illustrated in Figure 2, the shifting costs associated with the stowage plan must be evaluated. The shifting costs reflect the costs and/or time used to move cargoes in order to access other cargoes that are to be unloaded at a given port. For each vehicle, both an entry

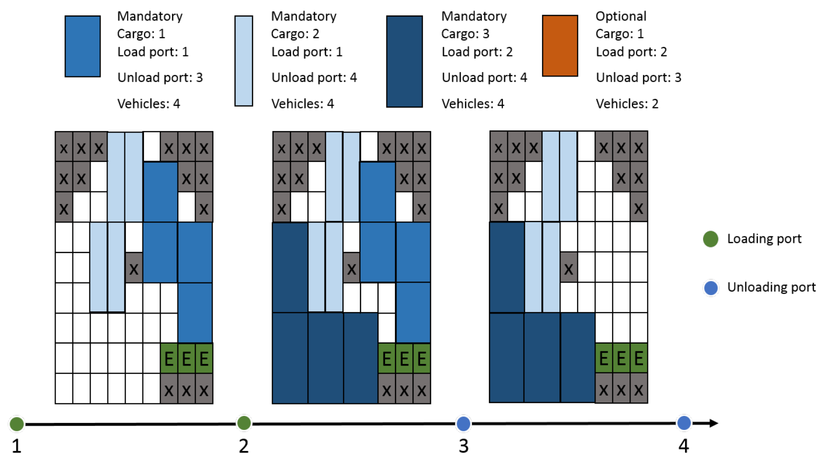


**Fig. 2.** A possible solution to the packing problem for each sailing leg during the voyage. Grey squares marked $X$ is unavailable space, and squares marked $E$ is the entry/exit point.

and exit route needs to be calculated. The total shifting cost of a voyage, is given by the sum of shifting costs for each entry/exit route for all vehicles along the voyage. The shifting model discussed in Section 3.3 is used to evaluate the total shifting cost for a stowage plan along a voyage.

## 3 Mathematical models

In this section, we propose a MIP model for the 2DRSSP. First, some modeling choices and definitions that are used in the mathematical model are introduced. Then, the objective functions and the constraints of the mathematical model are presented. Finally, the evaluation of the shifting is discussed.

### 3.1 Assumptions and modeling approach

Our approach to solve the 2DRSSP splits the problem in two phases. First, we solve the stowage problem for a given deck. Then, we evaluate the number of shifts needed when applying the resulting stowage plan for the voyage. This results in two models: A stowage model and a shifting model. It is a reasonable approach to deal with these two problems in sequence, since the results of the stowage, i.e. the extra revenue from optional cargoes that can be transported, is assumed more important than the shifting costs.

Still the results from the stowage influence the shifting costs. Therefore, to implicitly take into account the shifting when determining a stowage plan, different objective functions are proposed and tested. Two concepts are introduced with expectation to reduce the shifting costs, namely *grouping* and *placement*.

Placing vehicles from the same cargo next to each other is denoted as grouping. By grouping vehicles together, the shifting costs may decrease, as vehicles from the same cargo can use the same entry/exit route. The example in Figure 2 shows how vehicles from the same cargo are grouped together. Placing cargoes which are on the ship for the most number of sailing legs farther away from the entry/exit than cargoes with shorter time on the vessel, is known as placement. This is introduced based on the expectation that vehicles placed farthest away from the entry/exit, is probably less exposed to shifting, and those squares should therefore be more costly to use.

Instead of dividing the deck into lanes such as Øvstebø et al. (2011a), we suggest a grid representation of the deck, as illustrated in Figure 3. This enables us to represent real deck layouts in a better way, and the resulting stowage plan becomes more realistic. This is done by defining a set of rows $\mathcal{I}$ and columns $\mathcal{J}$. Square $(1, 1)$ is defined as the square located at stern, on the ship's port side (bottom left corner in Figure 3). All squares are assumed to be of the same size.

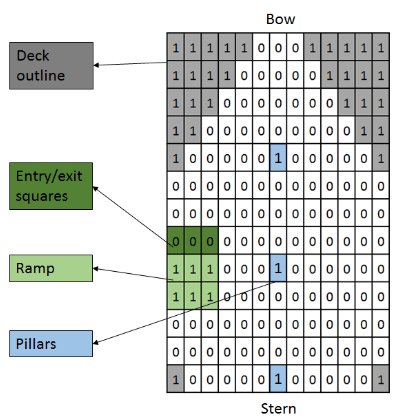


**Fig. 3.** Illustration of the grid representation of a deck. The 1's indicate that the corresponding square is unusable.

Each cargo $c \in \mathcal{C}$ consists of $N_c$ identical vehicles. If, in practice, one is to carry a cargo consisting of heterogeneous vehicles, this cargo is split into several cargoes consisting of identical vehicles. When all vehicles in a cargo are identical, the number of squares needed to place a vehicle from that cargo is equal for all vehicles in the cargo. For a given grid representation of the deck, each vehicle in a cargo needs $S_c^L$ length squares, and $S_c^W$ width squares to be placed on the deck. These parameters will vary with the grid resolution chosen, given by the number of rows times the number of columns ($|\mathcal{I}||\mathcal{J}|$). The area of the resulting square usage always gives an overestimation of the actual area usage. Increased resolution will give a more detailed representation of the deck and the vehicles, but increases the number of variables in the model.

The ports are assumed to be separated into two regions, one supply region and one demand region, where the loading ports are visited before the unloading ports. This is how most voyages are in RoRo-shipping. Also following common practice, it is assumed that once a vehicle is placed, it stays in the same location during the whole voyage. From this it follows that all carried vehicles are to be placed on the deck on the sailing leg between the last loading port and the first unloading port. Hence, by generating a stowage plan for this sailing leg, the vehicle placements for all other sailing legs can be derived from this stowage plan.

### 3.2 2DRSSP stowage model

**Indices**

$c$ : cargo
$i$ : row
$j$ : column

**Sets**

$\mathcal{C}$ : set of all cargoes
$\mathcal{C}^{\mathcal{O}}$ : set of all optional cargoes
$\mathcal{C}^{\mathcal{M}}$ : set of all mandatory cargoes
$\mathcal{I}_c$ : set of all rows where the corner of a vehicle in cargo $c$ can be placed $\mathcal{I}_c = \{1, ...|\mathcal{I}| - S_c^L + 1\}$
$\mathcal{J}_c$ : set of all columns where the corner of a vehicle in cargo $c$ can be placed $\mathcal{J}_c = \{1, ..., |\mathcal{J}| - S_c^W + 1\}$

**Parameters**

$L^D$ : length of deck
$W^D$ : width of deck
$C_c^L$ : length of one vehicle in cargo $c$
$C_c^W$ : width of one vehicle in cargo $c$
$B$ : minimum clearance between vehicles
$N_c$ : number of vehicles in cargo $c$
$S_c^L$ : number of length squares needed to place one vehicle from cargo $c$ $S_c^L = \lceil \frac{(C_c^L + B)|\mathcal{I}|}{L^D} \rceil$
$S_c^W$ : number of width squares needed to place one vehicle from cargo $c$ $S_c^W = \lceil \frac{(C_c^W + B)|\mathcal{J}|}{W^D} \rceil$
$P_c^L$ : loading port of cargo $c$
$P_c^U$ : unloading port of cargo $c$, $P_c^U > P_c^L$
$R_c$ : revenue earned if optional cargo $c$ is taken
$U_{ij}$ : 1 if square $(i, j)$ is unusable, 0 otherwise
$E_{ij}$ : 1 if square $(i, j)$ is an exit square, 0 otherwise
$D$ : A small positive number that will increase the value of the objective function if vehicles from the same cargo are grouped together
$C_{ij}^S$ : The artificial cost of using square $(i, j)$ for a vehicle

**Decision variables**

$x_{ijc}$ : 1 if the lower left corner of a vehicle from cargo $c$ is placed in square $(i, j)$, 0 otherwise

$y_c$ : 1 if optional cargo $c$ is taken, 0 otherwise

$u_{ijc}$ : Number of vehicles from the same cargo $c$ placed next to square $(i, j)$, if a vehicle from cargo $c$ is placed in $(i, j)$

**Objective functions**

The objective of the 2DRSSP is to maximize the revenue from optional cargoes, minus the penalty costs incurred when shifting vehicles. Since the stowage model does not explicitly evaluate shifting cost, four objective functions are proposed and tested in an effort to place vehicles in a way that reduce the need for shifting.

$$\max\ z = \sum_{c \in \mathcal{C}^O} R_c y_c \tag{1}$$

$$\max\ z = \sum_{c \in \mathcal{C}^O} R_c y_c + \sum_{c \in \mathcal{C}} \sum_{i \in \mathcal{I}_c} \sum_{j \in \mathcal{J}_c} D u_{ijc} \tag{2}$$

$$\max\ z = \sum_{c \in \mathcal{C}^O} R_c y_c - \sum_{c \in \mathcal{C}} \sum_{i \in \mathcal{I}_c} \sum_{j \in \mathcal{J}_c} \sum_{i'=i}^{i+S_c^L-1} \sum_{j'=j}^{j+S_c^W-1} (P_c^U - P_c^L) \frac{C_{i'j'}^S x_{ijc}}{S_c^L S_c^W} \tag{3}$$

$$\max\ z = \sum_{c \in \mathcal{C}^O} R_c y_c + \sum_{c \in \mathcal{C}} \sum_{i \in \mathcal{I}_c} \sum_{j \in \mathcal{J}_c} (D u_{ijc} - \sum_{i'=i}^{i+S_c^L-1} \sum_{j'=j}^{j+S_c^W-1} (P_c^U - P_c^L) \frac{C_{i'j'}^S x_{ijc}}{S_c^L S_c^W}) \tag{4}$$

The objective function (1) maximizes the revenues from optional cargoes. The objective function (2) maximizes the revenues from optional cargoes and the artificial value of placing vehicles from the same cargo together. The objective function (3) maximizes the sum of revenues from optional cargoes minus the placement cost of each vehicle in all carried cargoes. The placement cost for each vehicle is a function of the number of sailing legs a vehicle is placed on the ship, multiplied with the cost of using the chosen square placement. The cost of using a square should reflect the square's probability of being exposed to shifting. The objective function (4) combines objectives (2) and (3).

**Unusable space and entry/exit squares**

Some squares are unusable due to ramp placement, deck outline, pillars, etc. These constraints are handled in the variable declaration of the model. For all squares $(i, j)$, if the corner of a vehicle from cargo $c$ cannot be placed in that square due to unusable space ($U_{ij} = 1$) or entry/exit squares ($E_{ij} = 1$), then $x_{ijc}$ is fixed to zero for the given cargo and square.

**Common constraints**

$$\sum_{i \in \mathcal{I}_c} \sum_{j \in \mathcal{J}_c} x_{ijc} = N_c, \qquad c \in \mathcal{C}^{\mathcal{M}} \tag{5}$$

$$\sum_{i \in \mathcal{I}_c} \sum_{j \in \mathcal{J}_c} x_{ijc} = N_c y_c, \qquad c \in \mathcal{C}^{\mathcal{O}} \tag{6}$$

$$\sum_{i'=i}^{i+S_c^L-1} \sum_{j'=j}^{j+S_c^W-1} x_{i'j'c} \leq 1, \qquad c \in \mathcal{C}, i \in \mathcal{I}_c, j \in \mathcal{J}_c \tag{7}$$

$$\sum_{i'=max(i-S_{c'}^L+1,1)}^{min(i+S_c^L-1,|\mathcal{I}_{c'}|)} \sum_{j'=max(j-S_{c'}^W+1,1)}^{min(j+S_c^W-1,|\mathcal{J}_{c'}|)} x_{i'j'c'} \leq M_{cc'}(1 - x_{ijc}),$$
$$c \in \mathcal{C}, c' \in \mathcal{C} \backslash \{c\}, i \in \mathcal{I}_c, j \in \mathcal{J}_c \tag{8}$$

$$x_{ijc} \in \{0, 1\}, \qquad c \in \mathcal{C}, i \in \mathcal{I}_c, j \in \mathcal{J}_c \tag{9}$$

$$y_c \in \{0, 1\}, \qquad c \in \mathcal{C}^{\mathcal{O}} \tag{10}$$

Constraints (5) guarantee that all the mandatory cargoes are placed on the deck. Constraints (6) ensure that all vehicles in an optional cargo are placed on the deck, if the optional cargo is taken. Constraints (7) guarantee that at most one vehicle from the same cargo uses the same place on the deck. Constraints (8) make sure that different cargoes do not use the same place on the deck. Min and max expressions are included to ensure that the constraints do not include squares outside the deck area. An upper bound on $M_{cc'}$ is given by $(S_c^L + S_{c'}^L - 1)(S_c^W + S_{c'}^W - 1)$. Constraints (9) and (10) force the variables to take binary values.

**Grouping constraints**

$$u_{ijc} \leq x_{i+S_c^L,jc} + x_{i-S_c^L,jc} + x_{i,j+S_c^W,c} + x_{i,j-S_c^W,c}, \qquad c \in \mathcal{C}, i \in \mathcal{I}_c, j \in \mathcal{J}_c \tag{11}$$

$$u_{ijc} \leq M x_{ijc}, \qquad c \in \mathcal{C}, i \in \mathcal{I}_c, j \in \mathcal{J}_c \tag{12}$$

$$\sum_{i \in \mathcal{I}_c} \sum_{j \in \mathcal{J}_c} u_{ijc} \geq 2N_c - 2, \qquad c \in \mathcal{C}^{\mathcal{M}} \tag{13}$$

$$\sum_{i \in \mathcal{I}_c} \sum_{j \in \mathcal{J}_c} u_{ijc} \geq (2N_c - 2) y_c, \qquad c \in \mathcal{C}^{\mathcal{O}} \tag{14}$$

$$u_{ijc} \geq 0, \qquad c \in \mathcal{C}, i \in \mathcal{I}_c, j \in \mathcal{J}_c \tag{15}$$

Constraints (11) force $u_{ijc}$ to take a value equal to the number of vehicles from the same cargo placed next to the vehicle in square $(i, j)$. Constraints (12) ensure that the number of neighboring vehicles is only calculated for the squares where a vehicle is placed. The upper bound on $M$ is 4, which is the maximum number of neighboring vehicles, defined as a vehicle placed exactly in front, behind, left or right of a vehicle. Thus, a vehicle placed in $x_{i+S_c^L,j+1,c} = 1$ is not defined as a neighbor, even though it could be interpreted as a neighbor in practice.

This modeling choice is made to reward practical stowage solutions. Constraints (13) and (14), in addition to (11) and (12) enforce vehicles from same cargo to have a total number of neighbors greater than or equal to the weakest form of compactness. Given that every vehicle is placed next to a vehicle from the same cargo, the weakest form of compactness is a line. In this case, all vehicles would have two neighbors, except the vehicles at each end of the line, which will only have one neighbor. The lower bound on the total number of neighboring vehicles for a cargo in this case is given by: $2N_c - 2$. Finally, non-negativity requirements for the variables related to grouping are given in (15).

### 3.3 2DRSSP Shifting model

Based on a given feasible solution from the stowage model described in Section 3.2, we want to evaluate the solution with respect to the shifting cost. The cost of shifting a given vehicle is set as a function of the area of the vehicle, since the cost of moving a large vehicle, e.g. a semi-trailer, is assumed higher than the cost of moving a small vehicle, e.g. a 3-door car. The shifting cost could also be based on other considerations than the area, e.g. expected time usage or shifting distance. It is assumed that a vehicle that is shifted is moved out of the deck during the port call and returned to the exact same square when the loading/unloading is done. We assume that each vehicle can move one square horizontally or vertically. In practice, vehicles have a given turning radius and can therefore not move sideways. However, sideways movement is assumed possible, as the inclusion of turning radius would drastically increase the modeling complexity of the shifting evaluation.

The most apparent shifting evaluation method is to treat the stowage solution as a node network, and solve it as a shortest path problem (SPP). For each port, an entry or exit route for all vehicles in every loaded or unloaded cargo could be calculated. However, this approach would only give an upper bound on the number of shifts, since it does not take into account the shifts made for other entering/exiting vehicles. In order to determine the entry/exit routes for all exiting vehicles simultaneously, a shifting model for the 2DRSSP has been developed. As the shifting model only evaluates a given stowage solution, the model is only briefly discussed in the following paragraph.

The objective of the shifting model is to find an optimal entry and exit path for each vehicle $v$ in cargo $c$ for the related loading and unloading ports of the cargoes, in order to minimize the total shifting cost. The problem is solved for every port, and the sum of the shifting cost for all ports along the given voyage is reported as the objective value. A small example for a given port is given in Figure 4. An exit path for both of the vehicles V1 and V2 is to be decided. The shortest path problem gives a shifting cost of 6 for V1, and 2 for V2, which gives a total shifting cost of 8. The 2DRSSP shifting model provides a better result. By taking into account that each vehicle only is shifted once, both V1 and V2 could use the squares where the shifted vehicle were placed. This gives an optimal solution of shifting cost equal to 6.

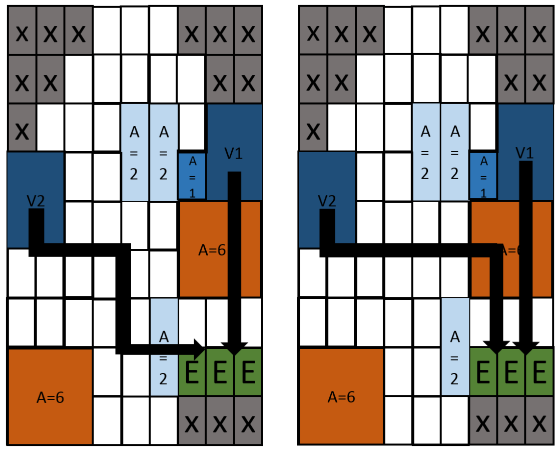


**Fig. 4.** Solution to the SPP for each vehicle to the left, and the optimal solution from the shifting model to the right. V1 and V2 indicates the vehicles that are to be unloaded, while A is the number of squares the other vehicles are occupying and indicate the cost to move those vehicles.

## 4 Computational study

This section presents a computational study performed on a number of test instances generated from real data from the case company. The mathematical models are implemented in Mosel and solved using the commercial optimization software Xpress. The test instances were run on a computer with Intel Core i7-3770 (3.40GHz) CPU and 16 GB RAM, running on Windows 7 Enterprise 64-bit Operating System. Section 4.1 describes the test instances, while the computational results are presented and discussed in Section 4.2

### 4.1 Test instances

The test instances are generated based on cargo data provided by the company. A typical real-sized deck has a length greater than $100m$ and width greater than $40m$. Using decks with areas of this size and a practical grid resolution, the stowage model is most likely not going to provide a solution within a reasonable amount of time. Hence, two smaller deck layouts are used to test the model. These layouts are created based on a scaled outline of a typical real sized deck. A deck measuring $45m$ x $20m$ and a deck measuring $20m$ x $10m$ are used, named decks 1 and 2, respectively. The cargo sets are randomly generated subsets of a real cargo list provided by the company. For each cargo set, the number of mandatory cargoes is low enough to ensure a feasible solution, and the number of optional cargoes is set such that the total area usage for all cargoes at least exceeds the decks area capacity. This is done to ensure that the 2DRSSP stowage model has to evaluate which optional cargoes to carry. For each instance, the number of length and width squares needed for each vehicle is pre-calculated, based on the vehicles length, width and the minimum clearance required between the cars, as well as the grid resolution. The discretization process from a real deck layout to a grid representation reduces the available area, due to an overestimated area usage of the unusable space. The resulting area available of total area and test instances are provided in Table 1.

**Table 1.** Test instances characteristics.

| Test instance | Deck # | Length of deck (m) | Width of deck (m) | Grid resolution | Cargo set | Area available of total area |
|---|---|---|---|---|---|---|
| $i10j10c1d1$ | 1 | 45 | 20 | 10x10 | 1 | 80 % |
| $i15j15c1d1$ | 1 | 45 | 20 | 15x15 | 1 | 88 % |
| $i20j20c1d1$ | 1 | 45 | 20 | 20x20 | 1 | 96 % |
| $i10j10c2d1$ | 1 | 45 | 20 | 10x10 | 2 | 80 % |
| $i15j15c2d1$ | 1 | 45 | 20 | 15x15 | 2 | 88 % |
| $i20j20c2d1$ | 1 | 45 | 20 | 20x20 | 2 | 96 % |
| $i10j10c3d2$ | 2 | 20 | 10 | 10x10 | 3 | 90 % |
| $i20j20c3d2$ | 2 | 20 | 10 | 20x20 | 3 | 95 % |
| $i10j10c4d2$ | 2 | 20 | 10 | 10x10 | 4 | 90 % |
| $i20j20c4d2$ | 2 | 20 | 10 | 20x20 | 4 | 95 % |
| $i10j10c5d2$ | 2 | 20 | 10 | 10x10 | 5 | 90 % |
| $i20j20c5d2$ | 2 | 20 | 10 | 20x20 | 5 | 95 % |

### 4.2 Results 2D stowage model

A goal in this computational study is to evaluate the performance of different model versions with regard to revenue generated, shifting cost and solution time. Even though the instances used is a scaled down version of real sized decks, the provided examples give valuable information of the performance of the different objectives for further study on the RoRo stowage problem.

The different objectives presented in Section 3.2 aim at influencing the vehicle placements so that the shifting cost is reduced. From this, five versions of the stowage model are presented in Table 2. Common for all the model versions is that the objective is to maximize the revenue generated from optional cargoes. For the basic model version N, this is the only objective. Model version P additionally influences the vehicles placement by introducing square costs. This results in placing vehicles carried for the most sailing legs furthest away from the exit, where the probability of being exposed to shifting is less. Model version H enforces a weak form of compactness to each cargo, placing the vehicles together. Model version S rewards grouping of vehicles. For each vehicle in a cargo, a higher number of neighboring vehicles from the same cargo increases the objective value. Finally, model version SP penalizes placement and rewards grouping.

**Table 2.** Model versions

| Model version | Objective | Constraints |
|---|---|---|
| Normal (N) | (1) | (5)-(10) |
| Placement (P) | (3) | (5)-(10) |
| Hard grouping (H) | (1) | (5)-(10), (11)-(15) |
| Soft grouping (S) | (2) | (5)-(10), (11)-(12), (15) |
| Placement + Soft grouping (SP) | (4) | (5)-(10), (11)-(12), (15) |

Each of the 12 instances from Table 1 was tested on the following versions of the MIP model: N, P, H, and S and SP. A maximum running time of 7200 seconds was set for the MIP model. If optimality was not proven within that time, the best solution is reported together with the gap from the upper bound. If the absolute gap between best bound and best solution was less than 0.01% the search was terminated. The clearance between vehicles was set to $0.15m$, $D = 0.001$, and the square cost, $C^S_{ij}$, was set to one thousand of the minimum number of squares to reach an exit for each square $(i, j)$. Table 3 shows the average results over all instances, obtained within the time limit of 7200 seconds.

**Table 3.** Average results for all test instances for the stowage model.

| Extension | Gap (%) | Time (s) | # optional cargo | Revenue optional | Area used (%) | # of shifts |
|---|---|---|---|---|---|---|
| N | 59.72 | 3600 | 1.33 | 17.67 | 78.53 | 9.91 |
| P | 24.03 | 3075 | 1.58 | 20.33 | 81.23 | 7.42 |
| H[1] | 35.34 | 3212 | 1.42 | 18.08 | 65.32 | 11.10 |
| S | 17.38 | 3432 | 1.58 | 20.50 | 81.75 | 8.36 |
| SP | 0.01 | 1778 | 1.58 | 20.50 | 81.75 | 5.64 |

[1]Two instances did not provide a feasible solution

The main objective of this problem is to maximize the revenue from the optional cargoes, while minimizing the shifting cost can be considered as a secondary objective. Since the extra terms in the objective functions (2)-(4) have a minor contribution to the objective value, the revenue of bringing an extra cargo always exceeds the cost of where to place or/and group the vehicles. The model versions N, P, S, and SP would therefore generate the same optional revenue in their optimal solutions, but the vehicles' placement could differ. The H-version is a bit different, as constraints (13)-(14) reduce the solution space. This model could therefore give an optimal solution which generates less revenue than the optimal solution for the other four models, or even give infeasible solutions, as for the two instances. The infeasible solutions may indicate that constraints (13)-(14) are too strict, excluding possible good stowage solutions.

Without evaluating the shifting cost of the solutions, there are some interesting findings regarding the performance of the different model versions. Model versions S and SP provide the best average revenue generated within the time limit. The stowage plans are not necessarily identical, but they do at least carry the same set of optional cargoes for every instance. The average gap for model version SP is 0.01, which implies that the optimal set of optional cargoes is carried for every instance. Based on the average gap, and the solution time, we conclude that version SP performs best on the given test instances.

For each model version and each instance, the shifting costs for the resulting stowage plans are calculated, using the 2DRSSP shifting model, briefly described in Section 3.3. This is done in order to evaluate the placement strategies used by the different versions of the stowage model. In Table 3, the average number of shifts for each model version is reported instead of the shifting costs. The two measures have a high correlation, and number of shifts is chosen for readability purposes. When evaluating the solutions it is important to consider the number of optional cargoes carried. As the revenues generated using the different model versions vary, the number of vehicles on the deck differ. With more vehicles on the deck, the number of shifts is expected to be higher. The computational results from the stowage model showed that the SP version of the model achieved the highest optional revenue on average. This implies that the resulting stowage plans from SP carry the largest number of vehicles. Despite this, the stowage plans from SP actually give the best results with regards to the total number of shifts. On average, model version SP gives the stowage solutions with the lowest number of shifts, lowest computational time, and carries the most optional cargoes. From this, it is reasonable to conclude that both grouping and placement modifications is preferable to incorporate in a RoRo stowage model.

## 5 Concluding remarks

The RoRo stowage problem is an essential part of the operational decisions for RoRo-operators in order to maintain their competitive position in the vehicle transportation market. We have proposed a mixed integer programming model for the two-dimensional RoRo ship stowage problem for one deck (2DRSSP).

Five alternative version of the 2DRSSP stowage model have been evaluated using 12 test instances. Test results showed that the inclusion of both grouping and placement objectives in the stowage model was preferable. This model version provided the overall best results, both regarding the revenue generated from optional cargoes, and the number of shifts.

However, the complexity of the problem limits the use of the models for real-life problems. We believe, however, that the research presented in this paper provide both important insights and modeling components that can be used in future research. A heuristic solution method is currently being tested, and preliminary results show that it provides feasible solutions to realistically sized problem instances for one deck. The natural extension to multiple decks is a promising venue for future research.

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